Prediction of EEG Signal by Digital Filtering

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***Abstract*: Prediction of EEG signal from past samples is needed for early diagnosis of patients, suffering from frequent epileptic seizure and/or psychotherapeutic treatment of psychiatric patients. This paper compares the performance of various digital filter algorithms to identify the right candidate for application in EEG prediction. The study includes variation of filter order and past sample size, and finally re-affirms the Kalman filter as the solution for its very low RMS prediction error in comparison to LMS, NLMS and RLS filter algorithms.**

I. INTRODUCTION

Electroencephalography (EEG) is one of the well known (and perhaps the oldest among all) [2] brain imaging techniques that provides cognitive underpinnings of various brain processes, reasoning, learning, perception-building and emotion arousals. An EEG system usually includes non-metallic electrodes such as carbon and carbon fiber. These electrodes are placed on the scalp at specific locations, determined by internationally agreed system. In a typical "10/20 system" [2], the distance from the patients' nasion to inion and between the preauricular points is measured. Each electrode is placed at 10/20 percent intersections along these distances. Fig.1 [5] describes a multi-channel EEG electrode array. Signals obtained by the EEG electrodes are amplified and then fed through a low-pass-filter (<100Hz).

Fig. 1 A multi-channel EEG electrode array

The EEG signals represent the electric potential differences in neuronal dendrites from the transmembrane currents. The primary source of EEG signals is presumed to be the current flow in the apical dendrites of pyramidal cells in the cerebral cortex [1]. These cells are located solely in gray matters and are oriented perpendicularly in the cortex. The values of the post-synaptic potentials continually vary, even when the brain is "at rest". The fluctuation in post-synaptic potentials is described as different frequencies in the EEG- waveform.

In order to localize the source of EEG samples, current research is moving towards a increasing density of electrodes. This is well-known as topographic mapping of source localization in the EEG literature [1] [Michel et. al.], where the smaller the distance between electrodes, the more accurate the mapping.

Localization of the source is important for its practical utility. The anatomy and physiology of human brain reveals specific lobe in the frontal brain are responsible for arousal of specific emotions. For instance, the amygdale is responsible for fear conditioning, threat perception sadness and disgust and the chaotic behavior in EEG for epileptic seizure [3]: Prediction of EEG from past samples thus has significance to detect possible abnormalities remedial measure for psychotherapeutic treatment of mental patients.

In this paper, we present different digital filter algorithm to predict EEG signal from past samples and identify the best algorithm from the performance of the filters.

Section 2 is denoted to studying LMS filter and its predictive capability through computer simulation. The NLMS filter is undertaken in section 3. The RLS filter and its predictive capability to EEG is studied in section 4. The Kalman filter is undertaken in section 5. Conclusions are listed in section 6.

II. EEG PREDICTION BY LMS ADAPTIVE FILTER

A Least Min Square (LMS) [6,7] finite impulse response (FIR) digital filter can be represented by Fig. 2.

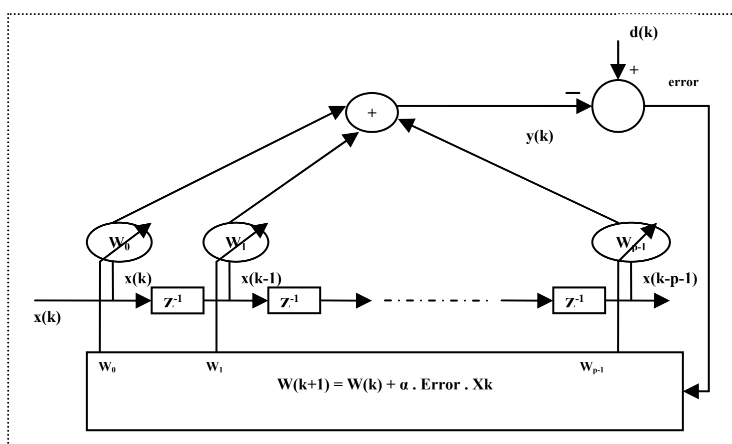

Fig. 2: An LMS adaptive FIR filter.

The experiment was conducted by sampling an EEG signal and submitting the discrete samples as x0, x1 …xp-1 to the input of an LMS filter. The original EEG and the predicted (estimated) signal for a filter of order 30 (i.e. p=30) are given in Fig. 2 and 3 below. The error plot given in Fig. 4 demonstrates that the average error is around 0.5 units with large variance. It is noteworthy from Fig. 5 that % RMS error falls off significantly with increasing filter order until 45. In the above computer simulation, we took α = 0.0002, and the initial weight vector was presumed to be zero.

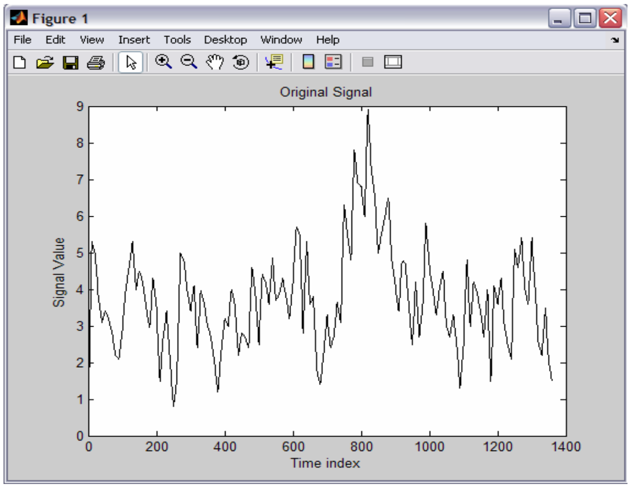

Fig. 3: A sample EEG signal.

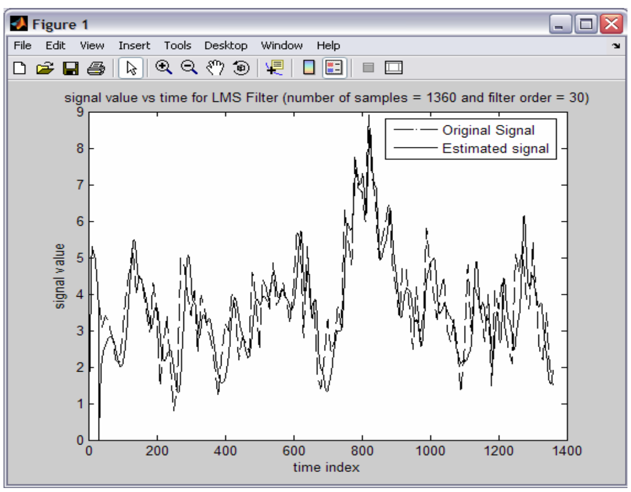

Fig. 4: EEG prediction by LMS filter algorithm.

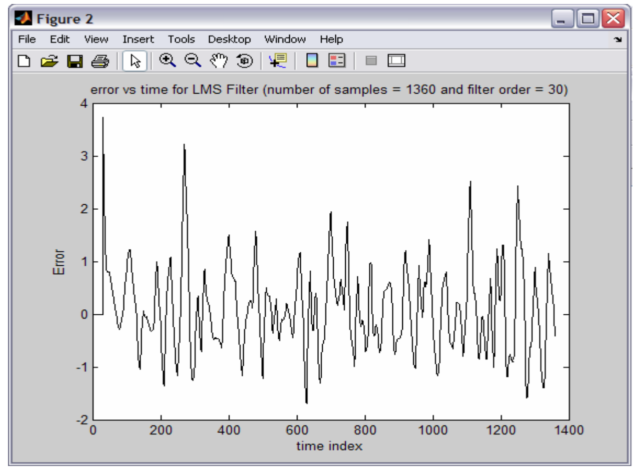

Fig. 5: The error in EEG prediction by LMS filter.

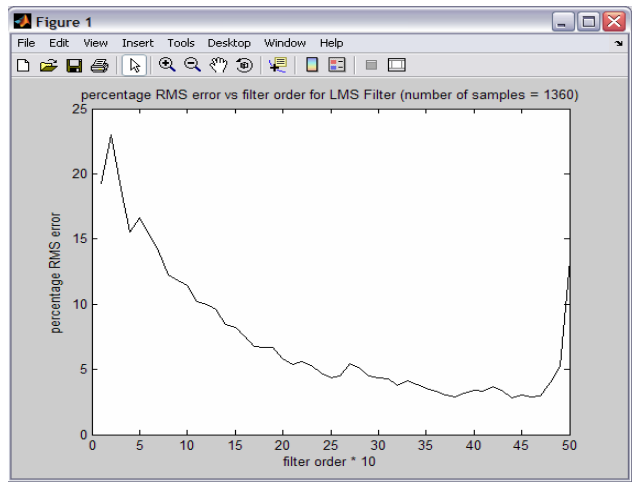

Fig. 6: The percentage RMS error versus Filter order plot for LMS filter.

III EEG PREDICTION BY NLMS ALGORITHM

The Normalized Least Min Square (NLMS) [7] filter has similar structure with the LMS filter with a modified weight adaptation rule, given by

W(k +1) = W(k) + α . error. Xk / (mean-square (Xk) + offset).

The offset is kept to ensure no division by zero. We used an offset settings of 50, and α = 0.005 for the computer simulation.

The original EEG and the predicted EEG by NLMS filter is given in Fig. 6, and the error in each sampling instant is plotted in Fig. 7. Here, the average value of error is around 0.2, but the variance is large. It is important to note that variance here is less than that of LMS filter. An illustrative weight variation by NLMS algorithm is given in Fig. 8. The error versus sample size and order of the filter given in Fig. 9 reveals that with both increase in filter order and sample size, the prediction error diminishes to as low as 0.05.

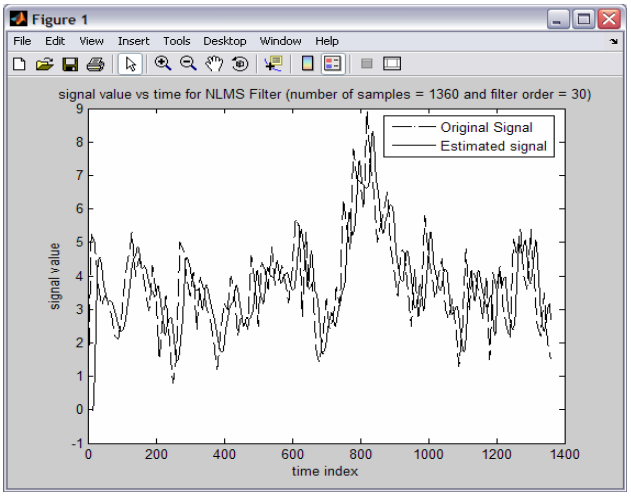

Fig. 7: EEG prediction by NLMS filter algorithm.

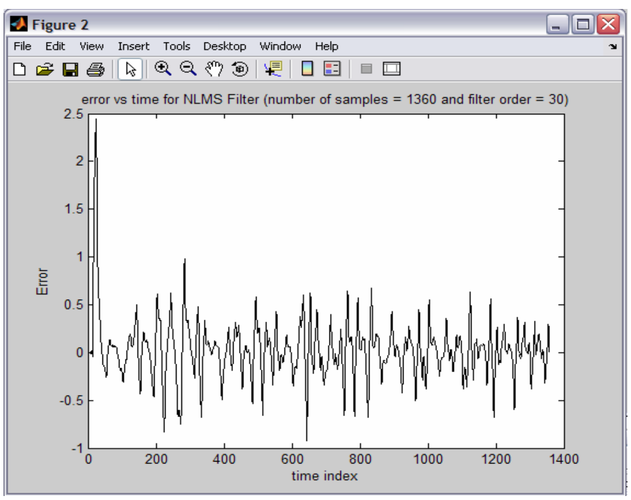

Fig. 8: The error in EEG prediction by NLMS filter.

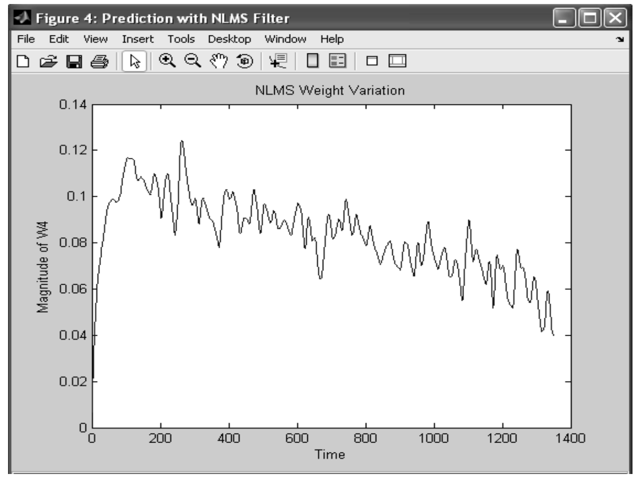

Fig. 9: Sample weight variation for NLMS filter prediction

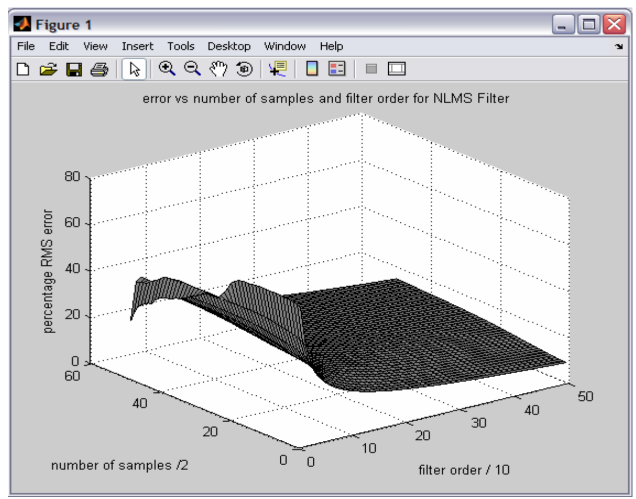

Fig. 10: Error versus sample size and order of the filter

IV. THE RLS FILTER FOR EEG PREDICTION

The Recursive Least Square (RLS) [6] filter employs the following algorithm for prediction.

Here $X(n) = [\; x(n-1) \;\; x(n-2) \;\; - \;\; - \;\; - \;\; x(n-p) \;]$ for a p order adaptive filter

and $X_k = X(n)$ at $n = k$.

while |error| not less than a predefined $\varepsilon > 0$

do begin

evaluate

i) error $e(n) = d(n) - W^T_n X(n)$

ii) $g(n) = P(n-1)X^*(n)\{\lambda + X^T(n)P(n-1)X^*(n)\}^{-1}$
iii) $P(n) = \lambda^{-1} P(n-1) - g(n)\, X^T(n)\, \lambda^{-1} P(n-1)$
iv) $W_n = W_{n-1} + e(n)\, g(n)$
end While;

Here P(n) is the inverse of the weighted auto correlation matrix of Xk weighted by the forgetting factor λ.
i.e. $P(n) = R_x^{-1}(n)$ where

$$R_x(n) = \sum \lambda^{(n-i)} X^*(i)\, X^T(i)$$

d(n) = desired signal, e(n)= error, and Wn = weight vector all at time sample t = n.

With $W_0 = 0$, $P(0)= \delta^{-1}I$, where I is a (p+1) ((p +1) identity matrix, and λ = forgetting factor = 0.99. We run the above algorithm until $\varepsilon = 10^{-3}$.

It is indeed interesting to note from Fig. 10 that the original and estimated signal have very negligible difference. The error plot in Fig. 11 also ensures the same point. A sample weight adaptation is shown in Fig. 12.

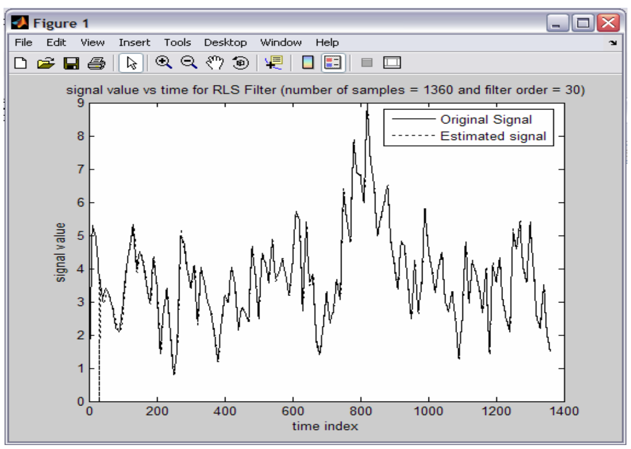

Fig. 11 EEG prediction by RLS filter algorithm.

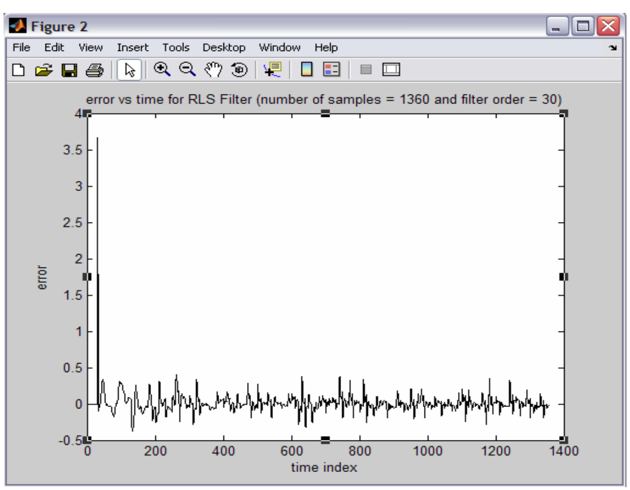

Fig. 12 The error in EEG prediction by RLS filter.

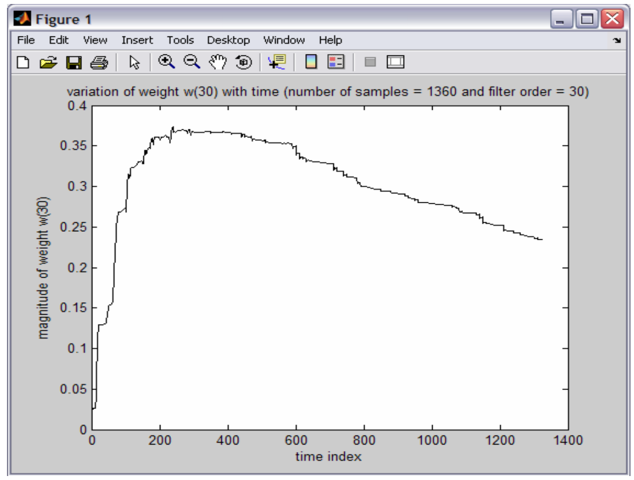

Fig. 13 Weight variation for RLS filter prediction

V. THE KALMAN FILTER FOR EEG PREDICTION

A Kalman filter [4] is a recursive p-order adaptive digital filter with filter input

$$X = [\, x(n-1)\; x(n-2) \ldots\ldots\ldots x(n-p)]$$

With a measurement equation

$$fi = W_n^T X - x(n) = 0$$

where W_n is the weight vector at the nth time instant.

$$W_n = [w(1)\;\; w(2) \ldots\ldots\ldots w(p)]$$

We estimate the signal x(n+1) as $y(n) = W_n^T X$.

Also let R_i be the expected value of $W_i W_i^T$
i.e. $R_i = E[W_i W_i^T]$.

The estimator vector in the present context is given by

$$A = [w(1)\;\; w(2) \ldots\ldots\ldots w(p)]T = W_n^T$$

Let S_i be the expected value of the estimation noise.
i.e. $S_i = E[(A_i - A_i^*)(A_i - A_i^*)^T]$
where A_i^* is the updated value of the estimator vector

For the signal estimation by the filter we need to determine the following derivatives:

dfi/dx= [w(1) w(2) ....w(p)] at ith instant,…….1

dfi/dA = [x(n-1) x(n-2) ....x(n-p)] at ith instant…2

Now the algorithm is as follows:

Begin

1. **Initiatize:**
   - **a)** $R_0 = (df_0/dx)(df_0/dx)^T$ in this case R_0 = a matrix of dimension p by p with all its entries zero.
   - **b)** S_0 = a diagonal matrix with large positive value of the diagonal terms which for this case is taken to be 50*I where I is a p by p identity matrix.
   - **c)** W_0 = [0 0 0 0 ……. p terms]
2. **Repeat:**
   - **a)** Input new signal sample x(n) and evaluate $y(n) = x(n+1)^* = W_n^T X$.
   - **b)** Update K_i as
     $K_i = S_{i-1} M_i^T (W_i + M_i S_{i-1} M_i^T)$
     where $M_i = df_i/dA$.
   - **c)** Update A_i as
     $A_i^* = A_{i-1}^* + K_i [x(n+1) - M_{i-1} A_{i-1}^*]$
   - **d)** Update S_i as
     $S_i = [I - K_i M_i] S_{i-1}$
   - **e)** Update df_i/dx and df_i/dA as in equations 1 and 2

Until prediction error is less than a preset value .

The EEG input and the prediction error using Kalman filter are plotted in Fig. 14 and 15 respectively. One sample weight adaptation is included in Fig. 16. For all the results filter order is taken to be 30 while the number of samples is taken to be 1360.

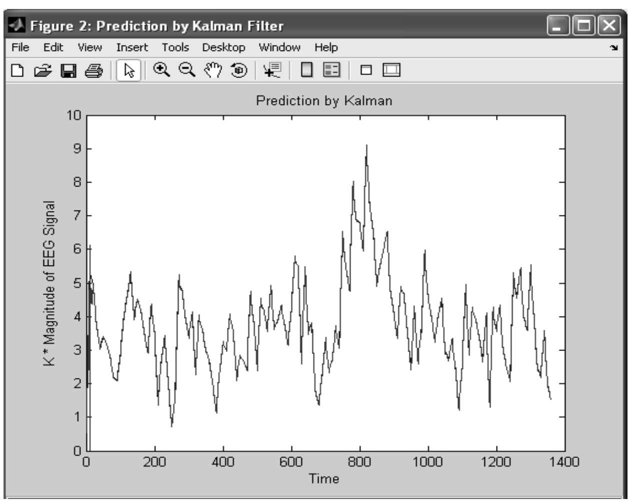

Fig. 14 EEG prediction by Kalman filter algorithm.

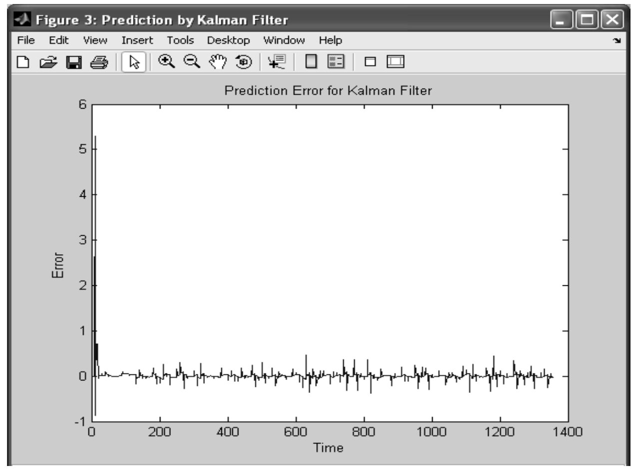

Fig. 15 The error in EEG prediction by Kalman filter.

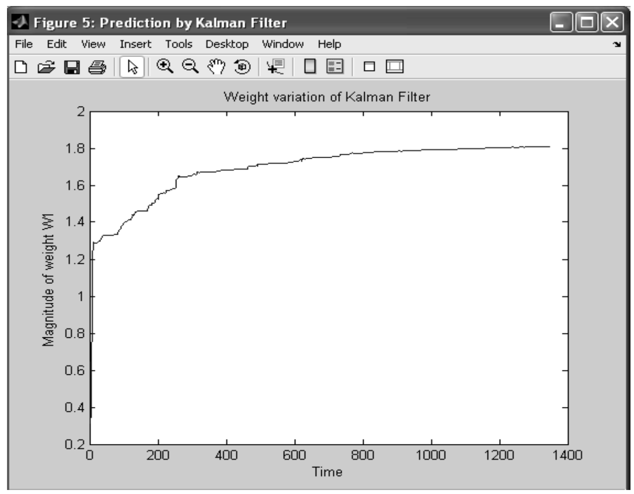

Fig. 16: Weight variation for Kalman filter prediction

VI. CONCLUSIONS

A comparison of the RLS and Kalman filter in EEG prediction is undertaken in this research. The computer simulation (Fig. 17) envisages that the Kalman filter yields less % RMS error in comparison to RLS filters, irrespective of filter order and sample size.

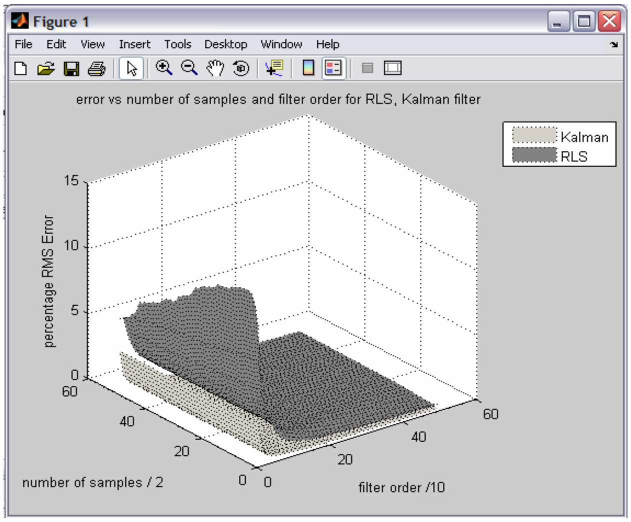

Fig. 17 Performance comparison of Kalman and RLS filters

The LMS and the NLMS Filters provide poorer performance than the Kalman and RLS filters which is can be understood from the following table. In the table M is the filter order while S is the number of samples. It is clearly evident that Kalman filter performs the best followed by RLS Filter.

Table 1: Performance comparison of all the 4 filters

	Percentage RMS Error in Prediction				
Filter Name	M = 30 S = 1360	M = 60 S = 2720	M = 90 S = 4080	M = 120 S = 5440	M = 150 S = 6800
LMS	10.687	5.775	3.023	2.430	1.711
NLMS	7.615	4.972	3.782	3.060	2.566
RLS	3.577	2.433	1.754	1.481	1.256
Kalman	0.375	0.167	0.109	0.082	0.063

It can be also observed from the table that in some cases LMS filter performs better than the NLMS Algorithm. This happens at a high filter order and for a high number of signal samples. However if we examine the RMS error vs. Number of samples plot of the LMS and the NLMS filters as shown in Fig 18 we see that for a small range of sample numbers the performance of the LMS filter betters that of the NLMS Filter. However after that range when the number of samples becomes 8704 and the filter order is 96 then the performance of the LMS filter deteriorates rapidly while that of the NLMS filter remains steady.

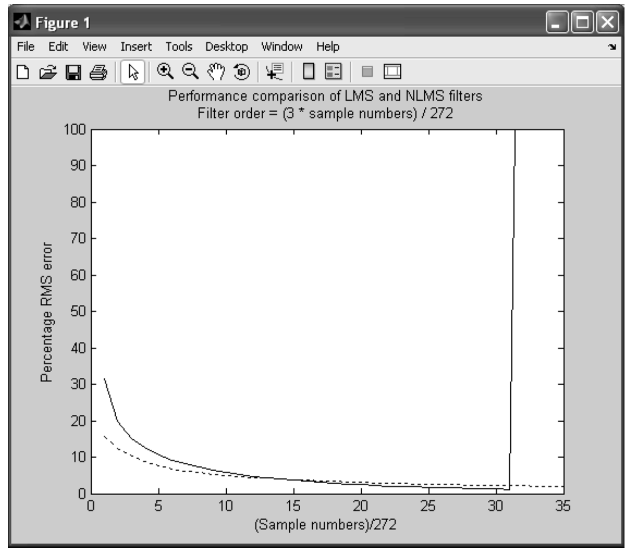

Fig. 18: Performance comparison of LMS and NLMS filters

The paper thus re-emphasizes the utility of Kalman filter for EEG prediction. Such prediction will help early diagnosis and prognosis of epileptic seizure and emotional outbreak/discharge for psychiatric patients.

ACKNOWLEDGMENTS

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