Formal Languages and Computation Theory (Spring 2005)

Examination 2

This is a **closed-book** examination. It is to be taken in a period of time no longer than 50 minutes.
On my honor, I have neither given nor received unauthorized aid in completing this examination:

_____.
(Sign your name above if you agree and wish to receive a grade for this examination.)

NOTE: Complete exactly four (4) of the following sic (6) questions. Clearly note which questions you do NOT want graded.

1. Is a Turing Machine with a two-way infinite tape more powerful than a regular Turing Machine? Justify your answer.

2. Say that a ***write-once Turing Machine*** is a single-tape TM that can alter each tape square at most once (including the input portion of the tape). Show that this variant Turing machine model is equivalent to the ordinary Turing machine model. (Hint: As a first step consider the case whereby the Turing machine may alter each tape square at most twice. Use lots of tape.)(Second hint: think about configurations.)(Third hint: use twice as much tape for the write-once machine.)

3. Show that the intersection of a context-free language with a regular language is context-free. (Correct construction is more important than a formal proof for this question.)

4. Using Ogden's lemma, show that the language $L = \left\{ a^n b^n c^i \mid i \neq n \right\}$ is not context-free. Could you use the normal context-free language pumping lemma to show this?

5. Show that a 2-PDA, that is, a pushdown automaton having 2 pushdown stacks instead of one, is equivalent to a TM. Remember that this is a *dual-containment* problem! Make sure to show how all moves of any TM can be converted to moves in the 2-PDA. Use the following formalism: $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where Q, Σ, Γ, q_0, and F are as in a typical PDA, and the transition function $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \times \Gamma_\varepsilon \rightarrow Q \times \Gamma_\varepsilon \times \Gamma_\varepsilon$, that is, it maps a state, input symbol or epsilon and one symbol or epsilon for each stack into a new state and a new stack symbol or epsilon for each stack.

6. Show that the context-free languages are closed under reversal, this is, if L is a context-free language, then so is L^R.