

- Write the correct form of the activation for a bidirectional associative memory (BAM) with threshold vectors $\vec{\theta}_r$ and $\vec{\theta}_l$.

One possible answer:

The update of \vec{y} is given by $\vec{y}_i^T = \text{sgn}(\vec{x}_i W - \vec{\theta}_r^T)$, or, $y_{ij} = \text{sgn}\left(\sum_{m=1}^n x_{im} w_{mj} - \theta_{rj}\right)$

and the update of \vec{x} is given by $\vec{x}_{i+1} = \text{sgn}(W \vec{y}_i - \vec{\theta}_l)$, or

$$x_{i+1j} = \text{sgn}\left(\sum_{m=1}^k y_{im} w_{mj} - \theta_{lj}\right).$$

- Select an arbitrary, nontrivial, 2×2 weight matrix W and find a stable solution to the BAM described by W , namely, find vectors (\vec{x}, \vec{y}) that minimize the energy function

$$E(\vec{x}, \vec{y}) = -\frac{1}{2} \vec{x} W \vec{y}^T.$$

One possible answer:

Selecting $W = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$ and $\vec{x} = (-1 \ 1)$ yields $\vec{y}_1^T = \vec{x}_1 W = \text{sgn}\begin{pmatrix} -4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. And in turn, we get $\vec{x}_2^T = W \vec{y}_1^T = \text{sgn}(1 \ 7) = (1 \ 1)$. Finally, we get $\vec{y}_2^T = \vec{x}_2 W = \text{sgn}\begin{pmatrix} -2 \\ 6 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and we see that the vector pair $(\vec{x}, \vec{y}) = ((1 \ 1), (-1 \ 1))$ is stable.