1. Let $\phi(v)=\frac{1}{1+e^{-av}}$.

   Show that $\frac{d\phi}{dv}=a\phi(v)\left[1-\phi(v)\right]$.

   What is the value of this expression at the origin?
   In the limit as *a* approaches infinity, what happens to this function?

2. Let $\phi(v)=tanh\left(\frac{av}{2}\right)$.

   Show that $\frac{d\phi}{dv}=\frac{a}{2}\left[1-\phi^2(v)\right]$.|

   What is the value of this expression at the origin?
   In the limit as *a* approaches infinity, what happens to this function?

3. Let $\phi(v)=\frac{v}{\sqrt{1+v^2}}$.

   Show that $\frac{d\phi}{dv}=\frac{\phi^3(v)}{v^3}$.

4. Which of the functions in problems 1 through 3 can be treated as a cumulative probability distribution function. Why?

5. Show that the binomial expansion of $\frac{1}{(1-w\,z^{-1})}=\sum_{i=0}^{\infty} w^i z^{-i}$.