Geometric continuity

The area of geometric design has coined the concept of geometric continuity (GC) to characterize when two point sets join smoothly independent of the parameterization – that is, a measure of continuity that treats parameterizations as tools for describing curves or surfaces, without introducing parametric artifacts. While computer-aided geometric design (CAGD) has relied heavily on mathematical descriptions of point sets based on parametric functions in recent years, geometric continuity for parametric curves and surfaces actually needs a notion different from the direct matching of Taylor expansions, which is used to define the continuity of piecewise functions.

1. motivation

Since the x, y and z components of point sets (curves and surfaces) are functions, we always tend to relate the continuity of functions, C^k continuity, to geometric continuity.

C^k continuity:

Two C^k function pieces join smoothly at a boundary to form a joint C^k function if, at all common points, their kth derivatives agree for k = 0, 1, … , k.

But even if the derivatives of the component functions agree, this criterion is neither sufficient nor necessary for characterizing the smoothness (geometric continuity) of curves or surfaces.

The following example illustrate the inadequacy of the standard notion of smoothness for functions when applied to curves.

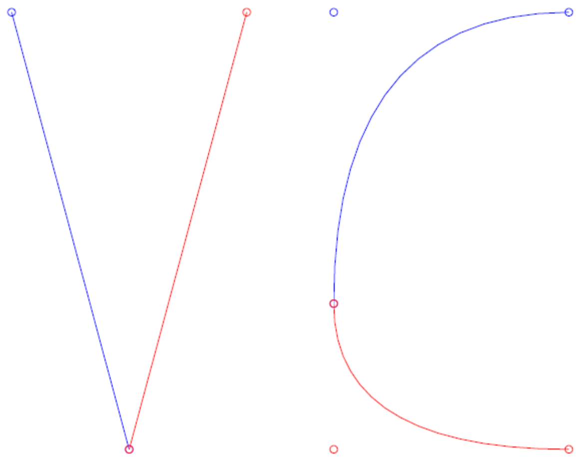

Figure 1. Geometric continuity and Function continuity

Matching derivatives of the component functions and geometric (visual) continuity are not the same: the V of VC is parameterized by two parabolic arcs with equal derivatives at the tip, but the V shape is not geometrically continuous; the C of VC is parameterized by two parabolic arcs with unequal derivatives at their common point, but the C shape is geometrically continuous.

In Figure 1, the V shape of VC is parameterized by these two quadratic pieces, $u, v \in [0,1]$,

2.