

Sketching Sampled Data Streams

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Motivation & Goal

Motivation

- Multicore processors — How to use all the processing power?
 - Parallel algorithms
 - Side tasks (analytical, exploratory)

Goal

- Analyze data at wire speed — single pass, small memory
 - *Skew* of a relation read from the disk
 - *Correlation* between flows passing through a high-speed router

Class of Queries

Aggregates over Joins

- Equi-join J between relations F and G with the join constraint $F.a = G.a$
- Queries specified by a join and an aggregate: COUNT, SUM
 - size of join (dot product), self-join size (second frequency moment)

Example

- Stream F :

a	1	1	2	3	1	3
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, frequency vector \mathbf{f} :

i	1	2	3
f_i	3	1	2
- Stream G :

a	3	1	3	1	1
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, frequency vector \mathbf{g} :

i	1	2	3
g_i	3	0	2

$$\begin{aligned}\text{COUNT}(F \bowtie_a G) &= \mathbf{f}\mathbf{g}^T = \sum_i f_i g_i \\ &= [3 \quad 1 \quad 2] \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} = 13\end{aligned}$$

Basic Sketching Technique (AGMS99)

Main Idea:

- Summarize frequency table by projecting it onto a random vector \Rightarrow *sketch*
- Use sketches to recover query result

Random vectors:

- $\xi = [\xi_1 \dots \xi_n]$ random vector of ± 1 values, called ξ **family**

Sketches:

- Sketch of F , $X_F = \mathbf{f}\xi^T$
- Sketch of G , $X_G = \mathbf{g}\xi^T$
- $X = X_F X_G$ estimates $\text{COUNT}(F \bowtie_a G)$ since

$$E[X] = E[\mathbf{f}\xi^T \xi \mathbf{g}^T] = \mathbf{f}E[\xi^T \xi] \mathbf{g}^T = \mathbf{f}I \mathbf{g}^T = \mathbf{f}\mathbf{g}^T$$

if $E[\xi^T \xi] = I$.

\Rightarrow distinct elements of ξ must be pair-wise independent $\forall i \neq i', \xi_i^2 = 1, E[\xi_i \xi_{i'}] = 0$

Basic Sketching Technique

Example: $\xi = [\xi_1 \ \xi_2 \ \xi_3] = [-1 \ +1 \ -1]$

$$X_F = \mathbf{f}\xi^T = [3 \ 1 \ 2] \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix} = -4$$

$$X_G = \mathbf{g}\xi^T = [3 \ 0 \ 2] \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix} = -5$$

$$X = X_F X_G = (-4)(-5) = 20 \approx 13$$

- Error of estimate X due to its variance

$$\text{Var}[X] = \sum_{i \in I} f_i^2 \sum_{j \in I} g_j^2 + \left(\sum_{i \in I} f_i g_i \right)^2 - 2 \sum_{i \in I} f_i^2 g_i^2$$

$$\text{Var}(X) \leq 2\mathbf{f}\mathbf{f}^T \mathbf{g}\mathbf{g}^T = 2 \text{SJ}(F) \text{SJ}(G)$$

- ξ is a family of 4-wise independent random variables

Sketch Maintenance over Streams

- Choose seed s that generates ξ

- Stream F :

a	1	1	2	3	1	3
---	---	---	---	---	---	---

, frequency vector \mathbf{f} :

i	1	2	3
f_i	3	1	2

$$\begin{aligned} X_F &= \mathbf{f}\xi^T = \sum_i f_i \xi_i = \sum_i (\xi_i + \dots + \xi_i) \\ &= \sum_{t \in F} \xi_{t.a} = \xi_1 + \xi_1 + \xi_2 + \xi_3 + \xi_1 + \xi_3 \\ &= h(s, 1) + h(s, 1) + h(s, 2) + h(s, 3) + h(s, 1) + h(s, 3) \end{aligned}$$

- Stream G :

a	3	1	3	1	1
---	---	---	---	---	---

, frequency vector \mathbf{g} :

i	1	2	3
g_i	3	0	2

$$\begin{aligned} X_G &= \mathbf{g}\xi^T = \sum_i g_i \xi_i = \sum_i (\xi_i + \dots + \xi_i) \\ &= \sum_{t \in G} \xi_{t.a} = \xi_3 + \xi_1 + \xi_3 + \xi_1 + \xi_1 \\ &= h(s, 3) + h(s, 1) + h(s, 3) + h(s, 1) + h(s, 1) \end{aligned}$$

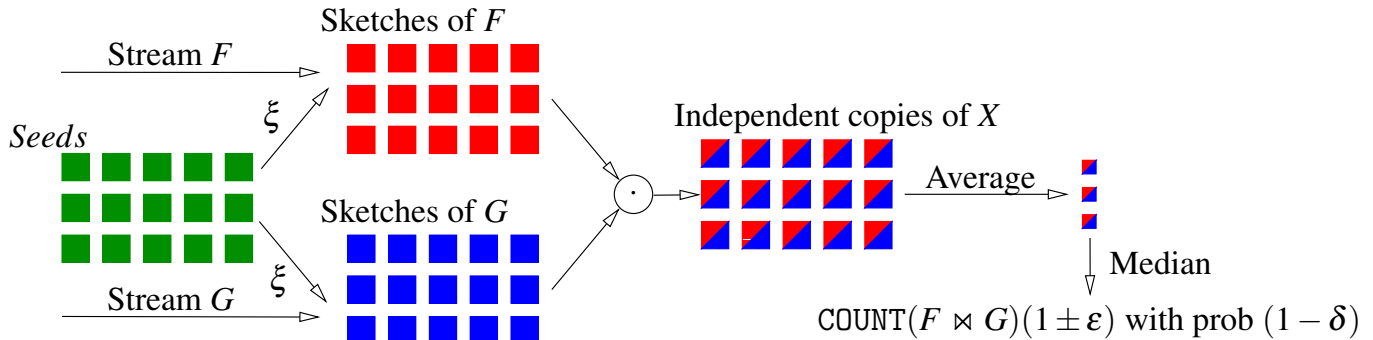
- Counters X_F and X_G need only log space in the size of the stream

Sketch Error Reduction

- Estimation of $\text{COUNT}(F \bowtie_a G)$ from single sketches of F and G is *too noisy*

Solution:

- Average $\frac{8\text{Var}(X)}{\varepsilon^2 E^2[X]}$ independent copies of X to reduce error to ε
- Compute median of $2 \log 1/\delta$ such averages to increase confidence to $1 - \delta$



- Memory required independent of the size of the stream

Speed-Up Methods

- Hashing
 - Fast-AGMS sketches are faster and have better accuracy
- Pseudo-random number generating schemes
 - EH3 is as good as any 4-wise scheme + faster and denser

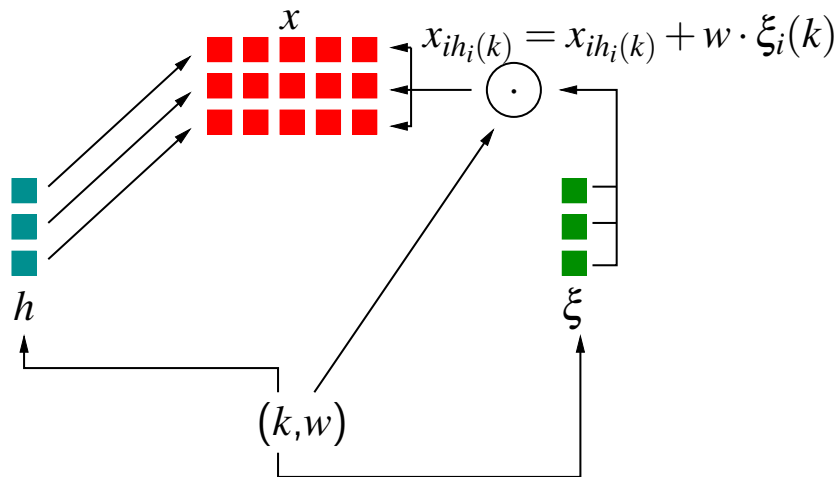
Fast-AGMS Sketches (CG05)

Randomization

- Vector h_i of 2-universal hash functions, $h_i : I \rightarrow B$
- Vector ξ_i of 4-wise independent ± 1 random variables, $\xi_i : I \rightarrow \{-1, +1\}$

Update

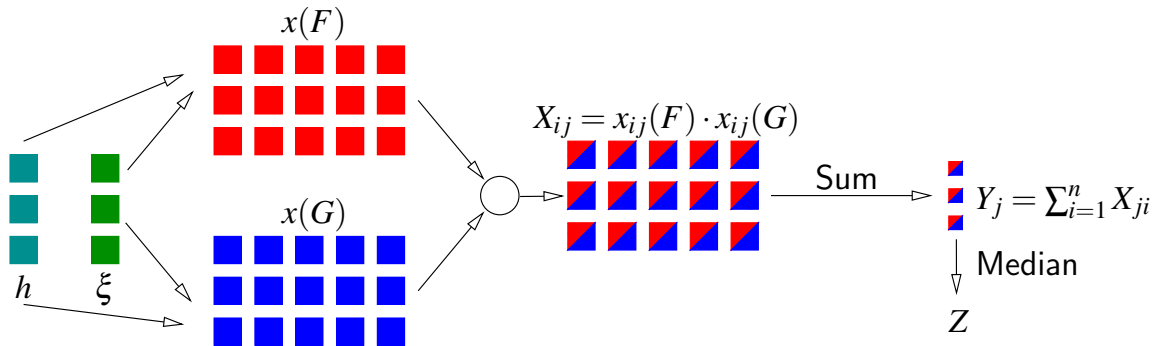
- Time \approx no. of rows m



Fast-AGMS Sketches (CG05)

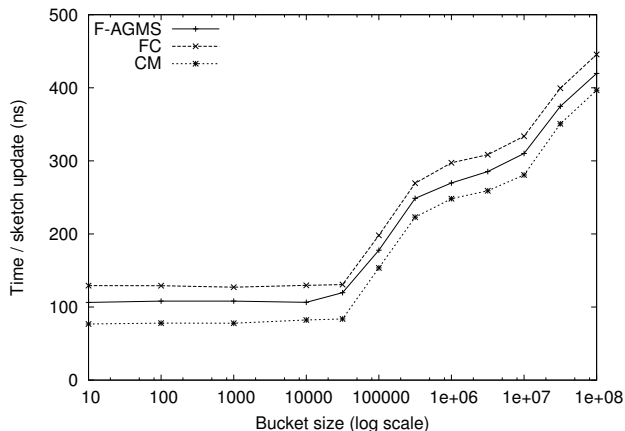
Size of Join Estimator

- $E[Z] = \bar{f} \odot \bar{g} = |F \bowtie G|$, $Z \in (\bar{f} \odot \bar{g} \pm \epsilon \|\bar{f}\|_2 \|\bar{g}\|_2)$ with probability at least $1 - \delta$
- $\|\bar{f}\|_2 = \sqrt{\bar{f} \odot \bar{f}} = \sqrt{\sum_{i \in I} f_i^2}$, $\|\bar{g}\|_2 = \sqrt{\bar{g} \odot \bar{g}} = \sqrt{\sum_{i \in I} g_i^2}$
- Sketch size: $B = n = \mathcal{O}(\frac{1}{\epsilon^2})$ and $m = \mathcal{O}(\log \frac{1}{\delta})$



Update Time

Setup: sketch size (row=1), Xeon 2.8 GHz, 512 KB cache, 4 GB main memory



- $100 \text{ ns} / \text{sketch} \times 10 \text{ sketches} \approx 1 \mu\text{s}$
- $1 \text{ million integers} / \text{second} \Rightarrow 4 \text{ MB} / \text{second}$

Desired rate

- $100 \text{ MB} / \text{second} \Rightarrow 25X$

Sampling

• Stream F :

a	1	1	2	3	1	3
---	---	---	---	---	---	---

,

frequency vector \mathbf{f} :

i	1	2	3
f_i	3	1	2

• Sample F' :

a	1	3	1
---	---	---	---

,

sampled frequency vector \mathbf{f}' :

i	1	2	3
f'_i	2	0	1

• Stream G :

a	3	1	3	1	1
---	---	---	---	---	---

,

frequency vector \mathbf{g} :

i	1	2	3
g_i	3	0	2

• Sample G' :

a	3	1
---	---	---

,

sampled frequency vector \mathbf{g}' :

i	1	2	3
g'_i	1	0	1

$$X = C \cdot \mathbf{f}' \mathbf{g}'^T = C \cdot [2 \ 0 \ 1] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = C \cdot 3 \approx 13$$

Sampling

- Sample at the tuple level
- Analyze in the frequency domain
- Random frequency vector (moment generating function)
 - Bernoulli → Binomial
 - WR → Multinomial
 - WOR → Multivariate hypergeometric

$$X = C \sum_{i \in I} f'_i g'_i$$

$$E[X] = C \sum_{i \in I} E[f'_i] E[g'_i]$$

$$\text{Var}[X] = C^2 \left[\sum_{i \in I} \sum_{j \in I} E[f'_i f'_j] E[g'_i g'_j] - \left(\sum_{i \in I} E[f'_i] E[g'_i] \right)^2 \right]$$

Sketches over Sampled Streams

• Stream F :

a		1	1	2	3	1	3
---	--	---	---	---	---	---	---

,

frequency vector \mathbf{f} :

i		1	2	3
f_i		3	1	2

• Sample F' :

a		1	3	1
---	--	---	---	---

,

sampled frequency vector \mathbf{f}' :

i		1	2	3
f'_i		2	0	1

• Stream G :

a		3	1	3	1	1
---	--	---	---	---	---	---

,

frequency vector \mathbf{g} :

i		1	2	3
g_i		3	0	2

• Sample G' :

a		3	1
---	--	---	---

,

sampled frequency vector \mathbf{g}' :

i		1	2	3
g'_i		1	0	1

• $\xi = [\xi_1 \ \xi_2 \ \xi_3] = [-1 \ +1 \ -1]$

$$X'_F = \mathbf{f}' \xi^T = [2 \ 0 \ 1] \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix} = -3$$

$$X'_G = \mathbf{g}' \xi^T = [1 \ 0 \ 1] \begin{bmatrix} -1 \\ +1 \\ -1 \end{bmatrix} = -2$$

$$X = C \cdot X'_F X'_G = C \cdot (-3)(-2) = C \cdot 6 \approx 13$$

Sketching a Sample

- Build the sketch over the non-materialized sample

$$X = C \cdot \sum_{i \in I} f'_i \xi_i \cdot \sum_{j \in I} g'_j \xi_j$$

$$E[X] = C \cdot \sum_{i \in I} E[f'_i] E[g'_i]$$

$$\begin{aligned} \text{Var}[X] = & C^2 \cdot \left[\sum_{i \in I} E[f_i'^2] \sum_{j \in I} E[g_j'^2] + 2 \cdot \sum_{i \in I} \sum_{j \in I} E[f'_i f'_j] E[g'_i g'_j] \right. \\ & \left. - 2 \cdot \sum_{i \in I} E[f_i'^2] E[g_i'^2] - \left(\sum_{i \in I} E[f'_i] E[g'_i] \right)^2 \right] \end{aligned}$$

Averaging Multiple Sketches

- Sketches share the same sample \rightarrow correlation

$$\text{Var} \left[\frac{1}{n} \cdot \sum_{k=1}^n X_k \right] = \frac{1}{n} [\text{Var} [X_k] + (n-1) \cdot \text{Cov}_{k \neq l} [X_k, X_l]]$$

$$\begin{aligned} \text{Var} \left[\frac{1}{n} \cdot \sum_{k=1}^n X_k \right] &= C^2 \cdot \left[\sum_{i \in I} \sum_{j \in I} E [f'_i f'_j] E [g'_i g'_j] - \left(\sum_{i \in I} E [f'_i] E [g'_i] \right)^2 \right. \\ &\quad \left. + \frac{1}{n} \left(\sum_{i \in I} E [f_i'^2] \sum_{j \in I} E [g_j'^2] + \sum_{i \in I} \sum_{j \in I} E [f'_i f'_j] E [g'_i g'_j] - 2 \cdot \sum_{i \in I} E [f_i'^2] E [g_i'^2] \right) \right] \end{aligned}$$

$$\text{Var}_{\text{sketch over samples}} = \text{Var}_{\text{sketch}} + \text{Var}_{\text{sampling}} + \text{Var}_{\text{interaction}}$$

Bernoulli Sampling

- p, q are sampling probabilities in F, G

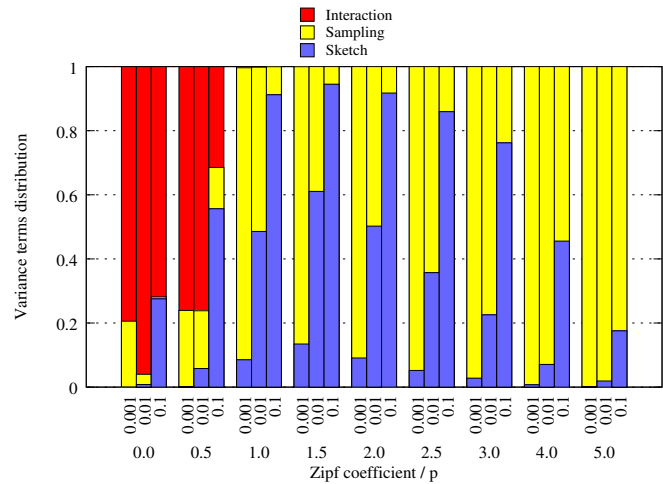
$$\begin{aligned} \text{Var} \left[\frac{1}{n} \cdot \sum_{k=1}^n X_k \right] = & \\ & \frac{1}{n} \left[\sum_{i \in I} f_i^2 \sum_{j \in I} g_j^2 + \left(\sum_{i \in I} f_i g_i \right)^2 - 2 \sum_{i \in I} f_i^2 g_i^2 \right] \\ & + \frac{1-p}{p} \sum_{i \in I} f_i g_i^2 + \frac{1-q}{q} \sum_{i \in I} f_i^2 g_i + \frac{(1-p)(1-q)}{pq} \sum_{i \in I} f_i g_i \\ & + \frac{1}{n} \left[\frac{1-p}{p} \sum_{i \in I} \sum_{j \in I, j \neq i} f_i g_j^2 + \frac{1-q}{q} \sum_{i \in I} \sum_{j \in I, j \neq i} f_i^2 g_j + \frac{(1-p)(1-q)}{pq} \sum_{i \in I} \sum_{j \in I, j \neq i} f_i g_j \right] \end{aligned}$$

Variance for Bernoulli Sampling

- Term significance as a function of the frequency distribution



Size of Join

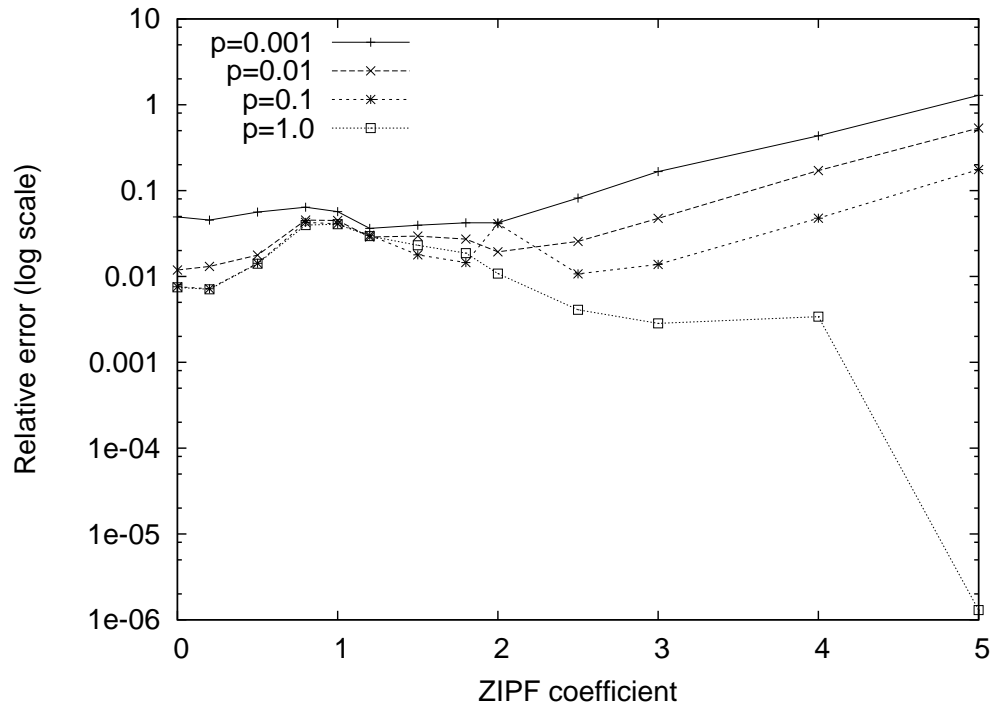


Self-Join Size

Error for Bernoulli Sampling

Settings

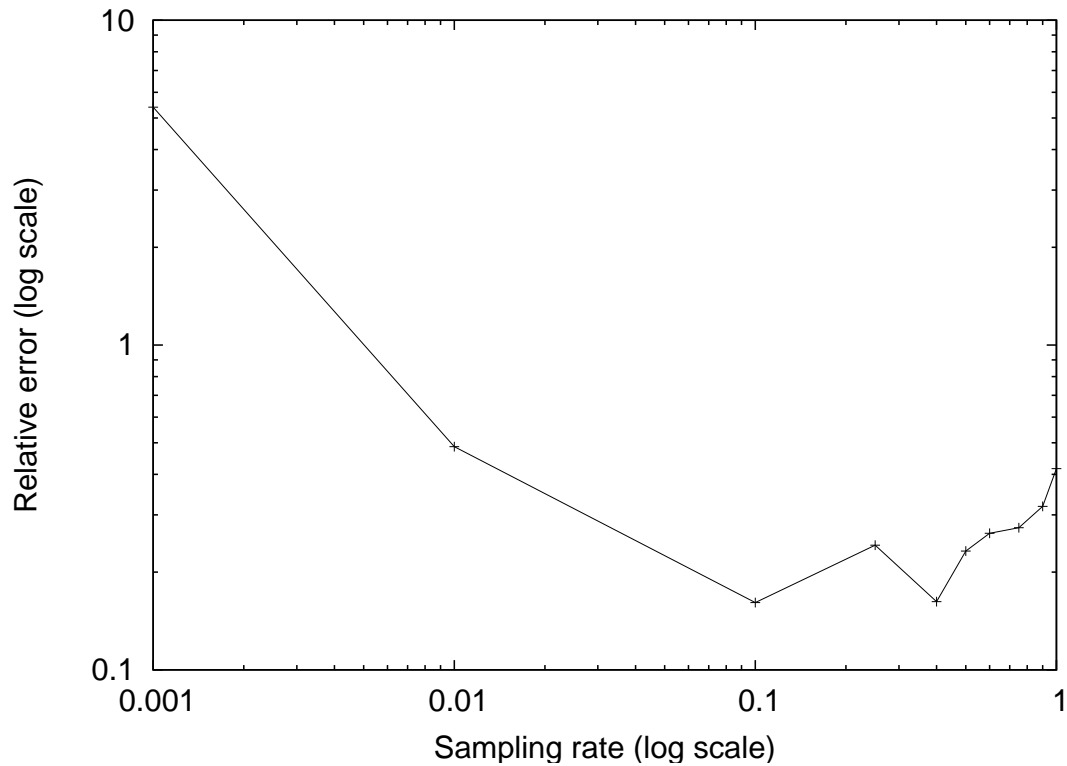
- 100 million tuples
- F-AGMS sketches with 5,000 buckets



Error for WOR Sampling

Settings

- TPC-H scale 1, $|lineitem \bowtie_{l_orderkey = o_orderkey} orders|$



Conclusions

Sketches over sampled data

- Generic moment analysis
 - Sampling in frequency domain
 - Combined estimator
- Three types of sampling
 - Bernoulli
 - With replacement
 - Without replacement
- Experimental evaluation
 - 2 orders of magnitude speed-up without significant error degradation

⇒ Fast-AGMS sketches with EH3 random variables over a sample

Questions