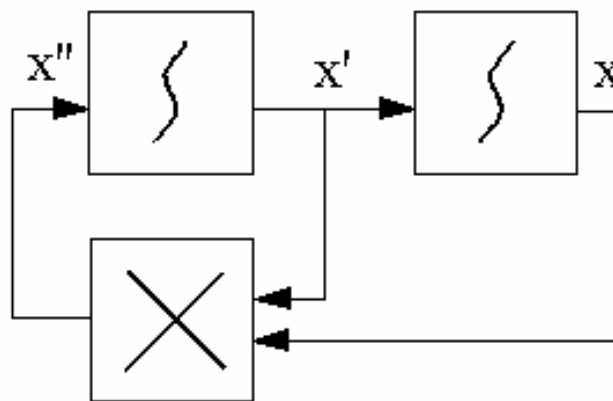


**CAP 4800 and CAP 5805**  
**EXAM 2, Computer Simulation, closed book & notes**  
**ANSWER KEY**  
**DATE: Friday, November 19, 2004**

The exam lasts for **50** minutes and is **100** points total. Each question is weighted the same.  
**CAP 4800: Answer any 3 out of 4 questions. CAP 5805: Answer all 4 questions.**

1. Consider the functional block model below. The top two blocks are integrators, and the bottom block is a multiplier. What is the differential equation for this model? Given the initial conditions  $x(0) = x'(0) = 2$ , and a **delta\_time=0.1**, simulate this system for a total length of time equal to **0.3**. Show the value of  $x(t)$  at the following times: **0, 0.1, 0.2, and 0.3**. Rounding to the first decimal place to the right of the period is acceptable.



**ANSWER**

The equation for this model is:  $x'' = x' * x$

Let's create a small table that we need to fill, rounding to the nearest 1/10, using forward Euler's for integration, and integrating in parallel:

<u>TIME</u>	<u>X''</u>	<u>X'</u>	<u>X</u>
0.0	4.0	2.0	<b>2.0</b>
0.1	5.3	2.4	<b>2.2</b>
0.2	7.0	2.9	<b>2.4</b>
0.3	9.7	3.6	<b>2.7</b>

If you integrated in sequence (i.e., using the output of the first integrator as input to the second integrator at the same time), the values for X are similar.

2. Consider the following table, defining 5 customers who enter a system with one server, with their time since last arrival (TSLA) and their service time amounts (ST) being specified for you. Fill in the rest of the table. Here are the remaining acronym definitions: ACT = Arrival Clock Time (absolute time); SB (time that service begins); SE (time that service ends); CW (amount of time that the customer waits, which includes the time being served); SI (amount of time that the server is idle).

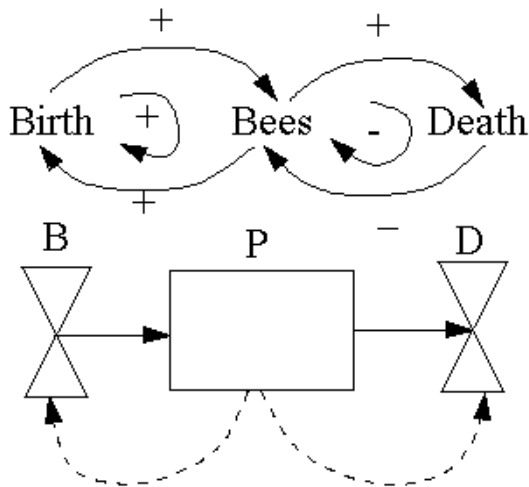
**ANSWER**

These have been filled in below:

C#	TSLA	ACT	ST	SB	SE	CW	SI
1	0	0	4	0	4	4	0
2	3	3	2	4	6	3	0
3	1	4	1	6	7	3	0
4	1	5	4	7	11	6	0
5	1	6	2	11	13	7	0

3. You are asked to model a population of bees over time, and you are given a birth rate and a death rate for the population. Draw and label a 1) signed causal graph, 2) flow graph, and 3) resulting equations representing this System Dynamics model.

**ANSWER**



Equations:

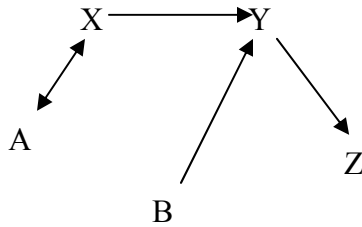
$$dP/dt = k_1 * B - k_2 * D$$

$$B = f(P)$$

$$D = g(P)$$

where  $f$  and  $g$  are arbitrary functions. The minimum causal graph would have birth and death rates affecting bee population (i.e., the dashed causal arrows are not strictly necessary since we don't know whether the rates are constant or dependent on the population).

4. Specify the equations for the following kinetic graph. Assume a separate rate constant ( $k_i$ ) for each arrow. The double-headed arrow represents two different arrows in opposite directions, each using the same rate constant.



### ANSWER

$$dA/dt = k_1 * X - k_1 * A$$

$$dB/dt = -k_3 * B$$

$$dX/dt = k_1 * A - k_2 * X - k_1 * X$$

$$dY/dt = k_2 * X + k_3 * B - k_4 * Y$$

$$dZ/dt = k_4 * Y$$