

Real-time DRR generation Using Cylindrical Harmonics

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Abstract. *In this paper, we present a very fast algorithm for generating Digitally Reconstructed Radiographs (DRRs) using cylindrical harmonics. Real-time generation of DRRs is crucial in intra-operative applications requiring matching of pre-operative 3D data to 2D X-ray images acquired intra-operatively. Our algorithm involves representing the pre-operative 3D data set in a cylindrical harmonic representation and then projecting each of these harmonics from the chosen projection point to construct a set of 2D projections whose superposition is the DRR of the data set in its reference orientation. The key advantage of our algorithm over existing algorithms such as the ray-casting or the voxel projection or the hybrid schemes is that in our method, once the projection set is generated from an arbitrarily chosen point of projection, DRRs of the underlying object at arbitrary rotations are simply obtained via a complete exponentially weighted superposition of the set. This leads to tremendous computational savings over and above the basic computational advantages of the algorithm involving the use of truncated cylindrical harmonic representation of the data. We present examples of DRR synthesis with fanbeam projection geometry for synthetic and real data. As an indicator of the speed of computation of one DRR from an arbitrary projection point, only 2-3 CPU seconds are required on a DELL Precision420 using MATLAB as the program development environment.*

1 Introduction

The registration of pre-operative volumetric datasets (CT data) to intra-operative two-dimensional projections (x-rays) of the represented object is a common problem in a variety of medical applications, including image guided surgery, medical image analysis, etc. With 2D/3D registration methods, the position and orientation of a 3D image can be determined with respect to the projection geometry used to acquire the 2D x-ray image. One of the key challenges for solving the 2D-3D registration problem is the need for an appropriate way to compare input images that are of different dimensions. Because similarity information is difficult to extract, it is very hard, to attack the registration problem directly based on the 2D and 3D images. The most common approach is to simulate the 2-D images given the 3-D volume dataset and estimate their relative spatial

relationship, so that the images can be compared within the same dimension. Thus simulating x-ray images of the volumetric dataset is essential to the registration process between 3D CT data and 2D x-ray. Simulated projection images, which represent the x-ray projection acquisitions from the volumetric CT data, are called Digitally Reconstructed Radiographs (DRRs). DRRs and the methods used to generate them are critically important in the 2D/3D registration problem, because they largely determine both the end result registration accuracy and the computation effort needed to achieve it. In the following section we will briefly review some of the methods typically used to generate DRRs.

1.1 Existing DRR Computation Algorithms

The most popular method used to generate DRRs is the ray-casting algorithm, which simulates ideal radiographic image formation by modeling the attenuation that x-rays experience as they pass through a dense object. Rays are constructed between points of the imaging plane and the imaging source. Thus each ray corresponds to an individual image plane point and each intensity value in the image plane is computed by integrating (summing) the attenuation coefficient along the corresponding ray. Because the projection rays usually do not coincide with the 3D data set coordinate system, interpolation is required to implement the projection and the chosen interpolation scheme determines the resulting accuracy of the generated projection. If a sufficiently large sampling rate along the rays is used, this algorithm tends to give accurate gray-value DRRs but at an enormous computational cost. This method is frequently also called volume rendering. As it must visit every voxel in the 3D dataset when computing the projection image, it tends to be extremely computationally intensive. The accuracy of the DRR produced by the ray-casting scheme is limited by the chosen interpolation scheme. The interpolation method is crucial to the resolution of ray-casting DRRs. Because several DRRs from different projection geometries must be generated in the typical 2D/3D registration problem, the computational complexity of the DRR calculation algorithm is critical. Several research studies have focussed on searching for more practical ways to improve on the the ray-casting algorithm. One effective computation reduction technique is the hierarchical approach, wherein the image is downsampled and smoothed before the registration [1][2]. In [3][4], the region of interest in the pre-operative CT image is segmented out prior to the generating the DRR, so that the alignment algorithm is applied only to that sub-image, thus reducing the computational burden. Larose et.al.,[11] presented a novel intermediate data representation called Transgraph. The basic idea is to precompute some DRRs for sampled viewpoints, and then the DRRs for arbitrary viewpoints can be generated by interpolating the existing DRRs. This method can greatly speed up the computation of DRRs by selecting only coarse samples. However, the accuracy is limited by the the density of samples and there is a tradeoff between the speedup and accuracy.

The voxel-projection DRR algorithm generates the DRR by accumulating the image plane projections for each voxel in the volume data set. The voxels are processed in storage order, and each is projected onto the image plane according

to the projection geometry and its position in the volume data set coordinate system. Processing the voxels in storage order (without interpolating the volume data set into a new coordinate system) allows faster traversal of the data set. The reconstruction filter controls the distribution of the voxel's projected data value or its corresponding visual parameter values to the pixels of the projection image, thereby taking advantage of the coherence if a voxel projects onto more than one pixel,[6][7]. In the volume rendering literature, this approach is called an object space method because the DRRs are generated by traversing 3D image voxel by voxel in storage order. These methods are faster than the ray-casting methods because each voxel is visited sequentially in the memory order and no interpolation process is required. Therefore, the resolution of the Voxel-Projection algorithm is better than that of the Ray-Casting method.

Recently, a novel hybrid DRR generation method has been presented in [8]. The method introduces shear-warp factorization [9], a new method proposed by Lacroute et.al.[9] in volume rendering, into the DRR generation methodology. It decomposes the viewing transformation into a shear matrix and a warp matrix, the viewing transformation matrix being their composition. The shear matrix transforms the volume data into sheared 3D volume voxels so that all viewing rays are parallel to each other. A 2D intermediate image, which is dependent on the viewing direction, is generated by summing the sheared volume voxels along the projection axis. The DRR is then obtained by warping the intermediate image to the projection plane. This algorithm exploits the merit of scanline-order volume rendering in both the pre-operative CT volume data and the projection image, which is more efficient than ray casting algorithms because the latter must perform analytic geometry calculations before sampling the data along each ray.

Accuracy of the Shear-Warp algorithm is limited by the number of times an interpolation technique is applied - as well as the accuracy of the interpolation process - which in turn effect the resolution of the resulting DRR. Finally, all the existing methods for generating DRRs suffer from the fact that whenever the underlying object pose is changed to obtain a new DRR, the entire process of DRR construction must be restarted, which makes these methods unsuitable for real-time applications. In contrast, the algorithm we present makes use of the cylindrical harmonic projections (DRR set) generated from any given point of projection in generating DRRs from any other points of projection. This key and unique feature of our algorithm leads to tremendous computational advantages over existing algorithms in literature.

2 Generating DRRs using Cylindrical Harmonics

Let $f(x, y, z)$ be a CT scan volume density function and let

$$f(x, y, z) = \sum_n a_n \phi_n(x, y, z) \quad (1)$$

be its expansion in cylindrical harmonics about the Z-axis. Thus, $\{\phi(x, y, z)\}$ are the cylindrical harmonics basis functions, resampled back into the Cartesian

coordinate system, and we assume that the expansion coefficients ($\{a_n\}$) are selected so that $\langle \phi_m, \phi_n \rangle = \delta_{m,n}$, i.e. $\{\phi_n\}$ are orthonormal. Then, if $f_\theta(x, y, z)$ denotes $f(x, y, z)$ rotated through angle θ about the expansion axis, it follows that

$$f_\theta(x, y, z) = \sum_n e^{-in\theta} (a_n \phi_n(x, y, z)) \quad (2)$$

i.e., that $f_\theta(x, y, z)$ is the superposition of linearly phase shifted versions of the given cylindrical harmonics. If θ is expressed on a discrete rotation grid with K uniformly distributed points in the interval $[0, 2\pi)$, then Equation (2) becomes

$$f_k(x, y, z) = \sum_n e^{-ink2\pi/K} (a_n \phi_n(x, y, z)) \quad (3)$$

Now, let $g(r, s)$ be the projection of $f(x, y, z)$ for some specified projection geometry and let P denote the projection operator. Then since P is linear,

$$g(r, s) = Pf(x, y, z) = P \sum_n a_n \phi_n(x, y, z) = \sum_n a_n \psi_n(r, s) \quad (4)$$

That is, the projection $g(r, s)$ is simply the superposition of the projections of the cylindrical harmonics of $f(x, y, z)$, i.e. $P\phi_n(x, y, z) = \psi_n(r, s)$. The significance of this result is that the projection of $f_k(x, y, z)$, can be expressed as the superposition of linearly phase shifted versions of the cylindrical harmonic projections, as follows

$$g_k(r, s) = Pf_k(x, y, z) = \sum_n e^{-ink2\pi/K} a_n \psi_n(r, s) \quad (5)$$

Thus, the algorithm for computing DRRs via cylindrical harmonics can be summarized as follows:

- (1) Compute cylindrical harmonics of the given 3D CT scan data set
- (2) Compute projections(DRRs) of the cylindrical(spherical) harmonics(via the desired arbitrary projection geometry)
- (3) Compute DRRs as weighted(phase shifted) superpositions of the harmonic DRRs

Note that one need not compute all the harmonics when using the harmonic expansion, instead, the harmonic expansion may be truncated in accordance with the desired accuracy of the DRR, as discussed later.

3 Experiments and Discussion

In this section, we presents two sets of experiments. In the the first set, we use the well-known 3D shepp-logan model to quantify the DRR fidelity. The second experiments is carried out with a 3D CT data set. Reconstructions are compared with those obtained from the use of the ray casting algorithm [5][10]. Finally,

we comment on some additional features of our algorithm that can be exploited to further reduce the computation time per DRR generation making it an ideal candidate for real-time applications.

First, we present the results on the accuracy of our DRR computation algorithm with the aid of a synthetic data example. A rotated and translated version of the 3D Shepp-Logan Phantom was used in our experiments. The slice residing in the xy -plane at $z=0$ is identical to the familiar 2D Shepp-Logan phantom. The reason we use the Shepp-Logan Model is because it is easy to compute the projections analytically for this model and use it for comparison with the DRRs obtained using our algorithm.

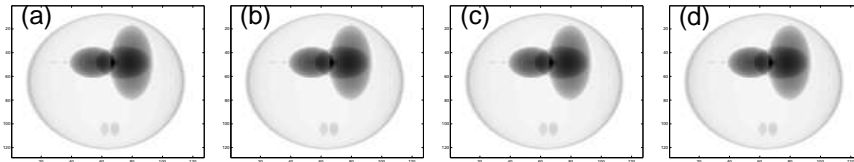


Fig. 1. Experiments using 3D Shepp-Logan Model. (a) Analytically generated Projection with the rotation angle 45° ; (b) DRR generated by cylindrical harmonics using the nearest neighbor interpolation; (c) using linear interpolation and; (d) using cubic spline interpolation with 45° angle rotation.

In figure 1, we show the analytically generated DRR and the DRR generated by our algorithm using cylindrical harmonics. In this example, we take the size of the 3D Shepp-Logan model to be $128 * 128 * 128$. Also we choose $N = 1024$ cylindrical harmonics when computing DRRs using our algorithm.

We employ two measures to quantify the error, namely, (1) the normalized cross-correlation between the analytically generated projection and the DRR obtained using our algorithm. (2) variance of the difference (image) between analytic projection and the DRR obtained using our algorithm.

Interpolation Method	Ncc	Maximum Absolute Value	Variance
Nearest Neighbor	0.9996	0.3349	6.0092e-4
Linear	0.9997	0.1802	2.0530e-4
Piecewise Cubic Spline	0.9998	0.1576	1.8220e-4

Table 1. 3D Shepp-Logan Similarity Measures. N_{cc} denotes normalized cross-correlation of the analytic DRR and the DRR generated using our method. Maximum absolute value and the variance are computed over a specific region of the difference image.

The normalized cross-correlation is a standard measure and is given by

$$N_{cc} = \frac{\sum_{(i,j) \in \Omega} (I_{ana}(i,j) - \bar{I}_{DRR})(I_{DRR}(i,j) - \bar{I}_{ana})}{\sqrt{\sum_{(i,j) \in \Omega} (I_{ana}(i,j) - \bar{I}_{DRR})^2} \sqrt{\sum_{(i,j) \in \Omega} (I_{DRR}(i,j) - \bar{I}_{ana})^2}} \quad (6)$$

Where Ω is image domain, \bar{I}_{ana} and \bar{I}_{DRR} are the mean values of the analytic DRR and our DRR images respectively. The normalized cross-correlation would be 1 when the two images are identical. Another method we used to quantify the

accuracy of the DRR generated by cylindrical harmonics is the variance value and the maximum absolute value of the difference between the two generated images in the region of interest.

From table 1, it is evident that better interpolation methods yield lower variance of the difference image and hence a higher accuracy. Large numerical values of the maximum absolute value in the difference image would be attributed to the size of the sampling interval in the interpolation scheme used.

When dealing with the registration of 3D pre-operative CT with 2D intra-operative fluoro images, real time generation of DRRs from the given CT volume is crucial. The second set contains experiments using 3D CT data set of temporal bones obtained from Virtual Medical Laboratory at University College London [13]. The size of the CT dataset from which we created the simulated projection images is (330x330x83). In order to show that the DRRs generated using cylindrical harmonics can achieve the comparable resolution as the DRRs generated via ray-casting with the same interpolation scheme, we take the DRRs generated using ray-casting as the groundtruth. Figure 2 shows the comparison of DRRs generated using different methods. Fig 2(a) is a DRR generated using ray-casting algorithm, viewing along the sagittal axis; Fig 2(b) is the resulting DRR using our cylindrical harmonics method, viewing in the same direction as in Fig 2(a). Fig 2(c) is the difference between Fig 2(a) and Fig 2(b). Figs 2(d),(e),(f) are another set of DRRs generated using different viewpoints. A bicubic interpolation scheme was used in generating all these DRRs.

We observed that the ratio of the maximal absolute value of the difference image and the DRRs generated by ray-casting is 0.0045. That's why the difference images as shown in Fig 2(c & f), are almost everywhere of very low magnitude in intensity and had to be rescaled by a factor of 10 for display purposes. This indicates the accuracy of our DRR generation algorithm in comparison to the ray-casting method. The typical running time to generate one DRR using our algorithm for a CT image of size 330x330x83 is 2.21 seconds on a DELL Precision420. While ray-casting algorithm takes about 96.6 seconds (when using the same linear interpolation in both techniques). Generating new DRRs in our method is quite easily accomplished via the accumulation of complex exponentially weighted basis projections. Note that there is no need to recompute the new DRR from ground zero as in other existing methods.

In addition to the already mentioned computational advantages of our algorithm, we can exploit the following additional features to accrue further computational savings.

- Symmetry of the rays in the fan-beam projection with respect to the y axis. The rays on the right half of the plane and rays on the left half are symmetric, exploiting this property can reduce the interpolation time by a factor of 2.
- Conjugate characteristics for cylindrical harmonics.

$$a_k = a_{N-k}^* \quad \text{for } k = 1 \rightarrow 511 \quad (7)$$

where N=1024 represents the total number of the cylindrical harmonics.

- Spectrum Truncation: With the increase of the order of cylindrical harmonics, the magnitude of the cylindrical harmonic coefficients decrease sharply.

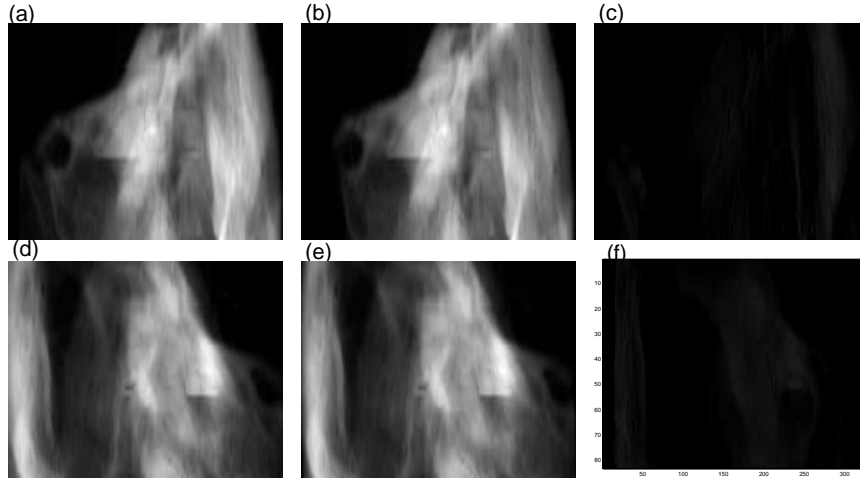


Fig. 2. Comparison of DRRs generated via different methods and viewpoints from temporal bone CT data. (a) DRR generated using ray-casting algorithm, viewing from the sagittal axis; (b) using cylindrical harmonics viewing from the same direction as in (a); (c) difference between (a) and (b). (d) DRR generated using ray-casting viewing from a 135° rotated sagittal axis; (e) using cylindrical harmonics viewing from the same viewpoint as in (d); (f) difference between (d) and (e). For display purpose, we have rescaled the difference image (c) and (f) by a factor of 10

We can use a truncated set of cylindrical harmonic instead of the whole set and thus cutting the processing time for generating the DRRs further. Fig.3 depicts the spectrum truncation error when using a truncated set of the cylindrical harmonics. From this figure, it is evident that to ensure a high fidelity DRR, 500 harmonic coefficients suffice. Note that the figure shows only half the number of harmonics due to the conjugate symmetry property.

4 Conclusions

In this paper, we presented a very fast algorithm for synthesis of DRRs from a given CT scan. The algorithm was based on constructing projections of each harmonic in a cylindrical harmonic expansion of the given data. A key advantage of the algorithm over the existing algorithms is that after computing a DRR from a single projection point, DRRs from other projection points can be computed very efficiently without having to restart the application of the algorithm from scratch. Instead, all that is needed is the weighted superposition of the basis projections. Examples involving DRR generation from synthetic and real data was shown and as evident, the accuracy achieved in the synthesis is quite high. Future work will involve matching the synthesized DRR with the given X-ray data.

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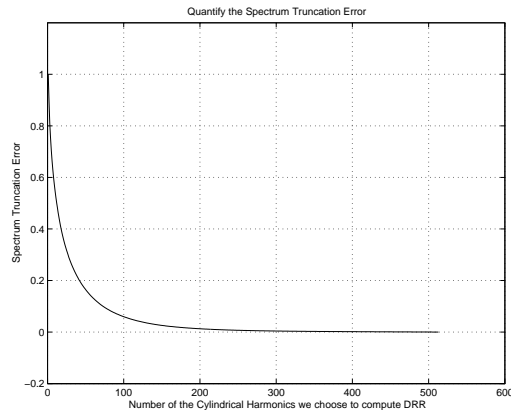


Fig. 3. Quantify the Spectrum Truncation Error

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