

Superman does not exist: A valid argument from Kalish/Montague

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Abstract

An argument is *valid* if its conclusion must follow from its premises. However, even a valid argument may have a false premise, in which case we are not compelled to accept its conclusion. In contrast, an argument containing a logical fallacy is *invalid*, and its conclusion does not follow from its premises, even if all its premises are accepted as true. Two related arguments are considered: an invalid one from the COT 3100 textbook (*Discrete Mathematics and Its Applications*, Rosen, 7th Edition), and the valid argument on which it is based (from *Logic: Techniques of Formal Reasoning*, by Kalish and Montague).

1 Overview

Section 2 provides the definition of a *valid argument*. An argument from Rosen is shown to contain a logical fallacy (Section 3), although a “proof” is given if we ignore this fallacy. Rosen’s argument is a simplification of a more interesting valid argument by Kalish and Montague (Section 4). Section 5 provides a rigorous proof of the Kalish/Montague argument, and shows that their argument is valid. However, Section 6 presents the first of two radically different worldviews that would discard one or more of its premises as false. The second worldview is presented in Section 7 which discusses the reasons why I believe the Kalish/Montague argument contains a false premise.

2 What is a valid argument?

An argument is a sequence of statements that starts with a set of premises, or assumptions, and ends with a conclusion. If the argument is *valid*, then the conclusion must follow from the preceding statements. That is, an argument is valid if and only if it is impossible for the premises to be true and the conclusion to be false.

In other words, when faced with a valid argument, **IF** you believe the premises to be true, **THEN** you must accept the conclusion. Otherwise, the rules of logic themselves must be discarded.

With a valid argument, however, if you do not accept the truth of one or more premises, then the conclusion may or may not be true. You are not compelled to believe the conclusion is true or false (assuming your objections to the premise(s) are reasonable). The valid argument does not apply, and the conclusion has not been shown to be either true or false.

3 A supposedly valid argument from the textbook, but with a logical fallacy

Problem 35 in Section 1.6 of the COT 3100 textbook (Rosen, 7th Edition), is given below. It appears as problem 35 in Section 1.5 in the 6th edition as well. I have enumerated the statements in the argument to refer to them later.

***35.** Determine if whether this argument, taken from Kalish and Montague (1964) is valid:

1. If Superman were able and willing to prevent evil, he would do so.
2. If Superman were unable to prevent evil, he would be impotent;
3. if he were unwilling to prevent evil, he would be malevolent.
4. Superman does not prevent evil.
5. If Superman exists, he is neither impotent nor malevolent.
6. Therefore Superman does not exist.

Except for the *fallacy of the excluded middle* (in premise (2)), the above argument would otherwise be valid. I will elaborate on this further, but for now, let's ignore this fallacy. Statements 1 through 5 are premises, and statement 6 is the conclusion. To show that this argument is "valid," we need a proof. First, let's define some basic propositions:

P	Superman is able to prevent evil
W	Superman is willing to prevent evil
D	Superman prevents evil
I	Superman is impotent
M	Superman is malevolent
E	Superman exists

Our goal is to show that if the 5 premises are all true, then we must conclude $\neg E$, that Superman does not exist.

Let's rewrite each English statement into a mathematical logic statement:

1. If Superman were able and willing to prevent evil, he would do so.

$$(P \wedge W) \rightarrow D$$

2. If Superman were unable to prevent evil, he would be impotent;

$$\neg P \rightarrow I$$

3. if he were unwilling to prevent evil, he would be malevolent.

$$\neg W \rightarrow M$$

4. Superman does not prevent evil.

$$\neg D$$

5. If Superman exists, he is neither impotent nor malevolent.

$$E \rightarrow \neg(I \vee M)$$

(or equivalently)

$$E \rightarrow (\neg I \wedge \neg M)$$

6. Therefore Superman does not exist.

$\neg E$

In a proof by contradiction, you assume your premises are true but your conclusion is false, and derive a contradiction.

More precisely, the theorem S above is of the form

$$S \equiv \{(p_1 \wedge p_2 \wedge p_3 \wedge p_4 \wedge p_5) \rightarrow q\}$$

where p_1 through p_5 are the 5 premises, and q is the conclusion (that Superman does not exist).

In a proof by contradiction, you show that $\neg S \rightarrow F$ is a tautology (always true). This works because $\neg S \rightarrow F$ is logically equivalent to S itself.

$$(\neg S \rightarrow F) \equiv (\neg\neg S \vee F) \equiv S$$

The negation of the theorem S is

$$\neg\{(p_1 \wedge p_2 \wedge p_3 \wedge p_4 \wedge p_5) \rightarrow q\}$$

or equivalently,

$$p_1 \wedge p_2 \wedge p_3 \wedge p_4 \wedge p_5 \wedge \neg q$$

That is, if you assume $\neg S$, you get to assume all your premises are true and your conclusion q is false. You must use these assumptions to derive a contradiction.

The proof-by-contradiction of this particular theorem is below. The proof shows that the argument is valid (ignoring the fallacy in premise (2)). I have listed it in two columns, with logic statements and English. Numbers in parentheses refer to statements in the argument, above.

step 1	Suppose E . This is the negation of our conclusion (6).	Suppose Superman exists.
step 2	From premise (5) $E \rightarrow (\neg I \wedge \neg M)$ and E , we conclude $\neg I \wedge \neg M$.	Since Superman exists, he is not impotent and not malevolent.
step 3	From step 2, $\neg I$, and premise (2) $\neg P \rightarrow I$, we conclude P .	Since Superman is not impotent, he is able to prevent evil.
step 4	From step 2, $\neg M$, and premise (3) $\neg W \rightarrow M$, we conclude W .	Since Superman is not malevolent, he is willing to prevent evil.
step 5	From premise (1) $(P \wedge W) \rightarrow D$, we conclude D .	Since Superman is able and willing to prevent evil, he does so.
step 6	From step 5, we have D . However, from premise (4), we have $\neg D$. This is a contradiction.	Superman cannot both prevent and not prevent evil. This is a contradiction.

This argument is an over-simplification. It assumes in premise (2),

$$\neg P \rightarrow I$$

that Superman is either completely impotent, or he is fully capable of preventing all evil. From the comic books, Superman is said to be very powerful (not impotent), but not all-powerful (he is weakened by Kryptonite, for example). This is an example of the “fallacy of the excluded middle.”

So is this argument truly valid? It uses a premise which is not merely false, but based on a logical fallacy. We must conclude that the argument is not valid, as a result of its reliance on this fallacy.

Alternatively, if we accept the dichotomy (Superman is either impotent or all-powerful, and nothing in between), then this argument is valid. The solution in the book makes this assumption. However, the book's answer is incorrect, since this truly is a false dichotomy. The argument is thus invalid.

I would of course agree with the conclusion, since Superman is just a comic book character. But my assertion that Superman does not exist is not due to this flawed argument.

To consider a well-formed valid argument that does not make use of the fallacy of the excluded middle, we must reject Rosen's argument and consider the original Kalish/Montague argument instead.

4 The Kalish/Montague argument

The original argument is on page 35 of a mathematical textbook by Kalish and Montague entitled *Logic: Techniques of Formal Reasoning*, published in 1964 by Harcourt, Brace, and Jovanovich (New York).

The original is more complex and more interesting than the simplified version in the 6th Edition of Rosen. The argument in Kalish and Montague refers to God, not Superman, but this switch (made by Rosen) has no relevance to Rosen's use of the fallacy of the excluded middle. Rosen's argument could be repaired while maintaining the use of Superman instead of God, although it sounds odd to consider Superman as omnipotent.

Below is the original argument. It is valid, as will be shown in Section 5, and it does not rely on the fallacy of the excluded middle. I have enumerated the statements to refer to them later in a proof.

1. If God exists, then He is omnipotent.
2. If God exists, then He is omniscient.
3. If God exists, then He is benevolent.
4. If God can prevent evil, then if He knows that evil exists, then He is not benevolent if He does not prevent it.
5. If God is omnipotent, then He can prevent evil.
6. If God is omniscient, then He knows that evil exists if it does indeed exist.
7. Evil does not exist if God prevents it.
8. Evil exists.
9. Therefore, God does not exist.

This valid argument contains 8 assumptions, or premises (statements 1 through 8) and one conclusion (statement 9). Since it is valid, **IF** you accept **all** 8 premises, then you must accept the conclusion, that God does not exist.

This valid argument does not suffer from the fallacy of the excluded middle, because anyone (God, Superman, or anyone else) is either all-powerful, or they are not. In other words the negation of "omnipotent" is "not all-powerful" rather than "impotent," which is the source of the error in Rosen's flawed argument.

Premise (4) is rather convoluted. A simplified version (which I will derive below) would be the statement "*If God is all-good, He would prevent the existence of evil.*"

5 A proof that the Kalish/Montague argument is valid

First, let's define some basic propositions:

E	God exists
P	God is omnipotent
C	God is omniscient
B	God is benevolent
R	God can prevent evil
K	God knows that evil exists
D	God does prevent evil
V	Evil exists

Our goal is to show that if the 8 premises are all true, then we must conclude $\neg E$, that God does not exist.

Let's rewrite each English statement into a mathematical logic statement:

1. If God exists, then He is omnipotent.

$$E \rightarrow P$$

2. If God exists, then He is omniscient.

$$E \rightarrow C$$

3. If God exists, then He is benevolent.

$$E \rightarrow B$$

4. If God can prevent evil, then if He knows that evil exists, then He is not benevolent if He does not prevent it.

$$R \rightarrow (K \rightarrow (\neg D \rightarrow \neg B))$$

or equivalently:

$$R \rightarrow (K \rightarrow (B \rightarrow D))$$

5. If God is omnipotent, then He can prevent evil.

$$P \rightarrow R$$

6. If God is omniscient, then He knows that evil exists if it does indeed exist.

$$C \rightarrow (V \rightarrow K)$$

7. Evil does not exist if God prevents it.

$$D \rightarrow \neg V$$

8. Evil exists.

$$V$$

9. Therefore, God does not exist.

$$\neg E$$

The method of proof is a proof by contradiction. That is, we assume the premises are all true but the conclusion is false, and show that this leads to a contradiction. The conclusion is $\neg E$, so to negate it, we assume E , that God exists. In the proof below, I have used both logic and English. In the English column, I have just shown the statement shown to be true by that step, not the reasons for which it is true. Numbers in parentheses refer to the 9 statements in the argument above.

step 1	Suppose E .	Suppose God exists.
step 2	Since (1) $E \rightarrow P$, we have P true.	God is omnipotent.
step 3	Since (2) $E \rightarrow C$, we have C true.	God is omniscient.
step 4	Since (3) $E \rightarrow B$, we have B true.	God is benevolent.
step 5	C is true from step 2, and from (6) $C \rightarrow (V \rightarrow K)$. Thus $V \rightarrow K$.	If evil exists, then God knows about it.
step 6	From (8), V is true. From step 5, $V \rightarrow K$, and thus K is true.	God knows that evil exists.
step 7	From (7), $D \rightarrow \neg V$. The logically equivalent contrapositive of this statement is $V \rightarrow \neg D$.	If evil exists, then God does not prevent its existence.
step 8	From (8), V is true, and from step 7, $V \rightarrow \neg D$, we conclude $\neg D$.	God does not prevent evil.
step 9	From step 2, P is true. From (5), $P \rightarrow R$, we conclude R .	God can prevent evil.
step 10	From (4) $R \rightarrow (K \rightarrow (B \rightarrow D))$. From step 9, we know R is true, thus $K \rightarrow (B \rightarrow D)$ is true.	If God knows that evil exists, then if He is good he would prevent it.
step 11	From step 6, K is true. From step 10, $K \rightarrow (B \rightarrow D)$, and thus $B \rightarrow D$ is true.	If God is good, he would prevent evil.
step 12	From step 4, B is true. From step 11, $B \rightarrow D$. Thus D is true.	God prevents evil.
step 13	From step 8, D is false. From step 12, D is true. This is a contradiction, since D cannot be both true and false.	God cannot both prevent and not prevent evil.
step 14	Our logic has led to a contradiction, so our initial hypothesis, E , in step 1, must be false.	God does not exist.

No logical fallacies are used (unlike Rosen's argument), and thus the proof shows that the Kalish/Montague argument is valid.

So, does God not exist? If you agree to all 8 premises of the Kalish/Montague argument, then you are compelled to agree that God does not exist.

If, however, you believe that one or more of these 8 premises are false, then you are not compelled either way. God may not exist after all, but this (valid) argument cannot be used to reach such a conclusion. Or God may indeed exist, but this would require a different argument not considered here.

6 Evil is a problem, whether God exists or not

The problem of evil is not trivial, and this short note can hardly scratch the surface. Given that caveat, there are two radically different worldviews considered here, both of which must deal with the problem of evil.

For those who believe God exists, the existence of evil seems to be incompatible with a God who is all-good, all-knowing, and all-powerful. However, this need not be the case, as I discuss in the next section.

The problem of evil is even more of a dilemma for those who do not believe in the existence of God. The existence of evil requires a moral absolute against which moral decisions are tested. If God does not exist, then humans are a mere cosmic accident, there is no moral absolute, and what's true for you might not be true for me. We cannot determine if something is evil if there is no standard to which we can all agree. One person's evil may be another person's good. An accepted practice in one culture may be considered evil in another culture.

Thus, if God does not exist, the concept of "evil" itself is ill-defined and culturally relative ($\neg E \rightarrow \neg V$). From this viewpoint, premise (8) of the Kalish/Montague argument (V) would be discarded as false, causing the entire argument to break down.

7 My personal view

The Kalish/Montague argument is valid (this is not my opinion, but mere fact). However, the argument contains a premise that I consider false, and thus I am not compelled to accept its conclusion. I agree with 7 of the 8 premises, and I disagree with its conclusion. Since I disagree with one of the 8 premises, and have logical reasons to do so, my disagreement with the conclusion is logically sound.

Consider premise (4) of the Kalish/Montague argument:

4. If God can prevent evil, then if He knows that evil exists, then He is not benevolent if He does not prevent it.

$$R \rightarrow (K \rightarrow (\neg D \rightarrow \neg B))$$

The crux of this premise is $\neg D \rightarrow \neg B$, or $B \rightarrow D$ in the simpler contrapositive form, namely:

If God is benevolent, then he prevents evil.

I do not accept $B \rightarrow D$ as being true.

There are logical reasons not to accept this premise. If God exists and desires us to love Him, then He must give us a choice, because love cannot be coerced. However, since we have a choice, we also have the ability to choose evil. Indeed, all of us have at some point chosen evil by causing harm to another. All people are a mixture of good and evil, hatred and love; just consider the past few millennia of human history to see that this is true.

Since we all contain some elements of evil, if God wanted to prevent or destroy all evil, He would either have to destroy us or take away our power of choice. Yet without choice, we would be mere robots, incapable of either good or evil. Evil would be destroyed, but so would any possibility of love.

Thus God must permit evil, temporarily, if He desires us to love Him. However, if He is good and omnipotent, He will ultimately destroy evil.

Premise (4) assumes that being benevolent means preventing evil. Can God be benevolent and still allow evil, not permanently, but for a time, for a higher good? I would answer yes, and I would modify the $B \rightarrow D$ portion of premise (4)

If God is benevolent, then He prevents evil.

to the premise $B \rightarrow \Omega$

If God is benevolent, then He will one day bring a permanent end to evil.

where Ω is the statement:

God will one day bring a permanent end to evil.

The statement Ω does not appear anywhere else in the Kalish/Montague argument. It is not the same as D (*God prevents evil*). The statements Ω and D are unrelated, since it is clear that neither $\Omega \rightarrow D$ nor $D \rightarrow \Omega$ are true.

With this change in premise (4), from $B \rightarrow D$ to $B \rightarrow \Omega$, the Kalish/Montague argument breaks down.

8 Conclusion

Rosen's argument about Superman is an overly simplified version of the Kalish/Montague argument, and thus not as interesting. Its primary flaw is the "fallacy of the excluded middle." In this fallacy, only two alternatives are considered, when there are other options that might be true. If Superman exists, he is neither impotent nor all-powerful, but somewhere in the middle: very powerful, but neither impotent nor omnipotent. Many claim that God exists and is omnipotent, but no one has made those claims of Superman. Rosen's use of Superman instead of God weakens his argument.

The Kalish/Montague argument is more interesting because it is valid and does not make use of any logical fallacies. Thus, you are compelled to accept its conclusion **IF** you accept all 8 of its premises. If you do not accept any one of the 8 premises, then you need not to accept its conclusion, although you might do so for other reasons not related to the Kalish/Montague argument.

What is your view? Do you accept all 8 premises of the Kalish/Montague argument? Or not? In either case, the Kalish/Montague argument provides an intriguing example that a valid argument may have a false premise, and thus its conclusion need not apply. What is even more intriguing about the Kalish/Montague argument is that two radically different worldviews would both question one or more of its premises, although for very different reasons.

What we can all agree on, however, is that no such "god" can exist that is described by all 8 premises of the Kalish / Montague argument. No such entity can exist for which all of these 8 premises are true.

Whatever your worldview, it is always good to examine the basic premises that led you to that particular worldview. Furthermore, the techniques of logic you learn in COT 3100 can be useful in the quest to develop your worldview and place it on a sound logical / rhetorical foundation.