FALL 2007 Q2.

$$Af(x+ah) = Af(x) + A(ah)f'(x) + A\frac{a^2h^2}{2!}f''(x) + O(h^3)$$

$$Bf(x+bh) = Bf(x) + B(bh)f'(x) + B\frac{b^2h^2}{2!}f''(x) + O(h^3)$$

~~$f'(x) = f(x+ah)$~~

Add both of these:

$$Af(x+ah) + Bf(x+bh) = (A+B)f(x) + (Aa+Bb)hf'(x) + (Aa^2+Bb^2)\frac{h^2}{2!}f''(x) \quad \text{——— (iv)}$$

Now we want

$$\begin{cases} A+B = 0 & \text{——— (i)} \\ Aa+Bb = 1 & \text{——— (ii)} \\ Aa^2+Bb^2 = 0 & \text{——— (iii)} \end{cases}$$

So, we have to "pick" a set of values for (A, B, a, b) which satisfy (i), (ii) & (iii) simultaneously.

So choose $A = 1$, Then $B = -1$ (from (i))

Now, (ii) gives: $a - b = 1$ ~~$\Rightarrow a = b+1$~~

& (iii) gives: $a^2 - b^2 = 0$

$$(a+b)(a-b) = 0$$

$\therefore a+b = 0$ as $a-b = 1$

$$a = -b$$

So $a = \frac{1}{2}$ $b = -\frac{1}{2}$ $A = 1$, $B = -1$

and $f'(x) =$ ~~$f(x+a)$~~ $\dfrac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{h}$

↓ I just put the values of a, b, A & B in (iv)