

Homework 4

July 24, 2008

The homework is due in class (BEFORE the lecture begins) on **Wednesday, 30th July**. As mentioned in the course syllabus, there are no electronic submissions and no late homeworks will be accepted unless you have an illness spanning the full period from the time the homework was assigned until it was due (and I shall need to see a medical practitioner's certificate to that effect). Standard academic honesty rules apply. You can discuss problems but the solutions turned in should be entirely your own. Cases of plagiarism will be dealt with strictly. Most of the problems are from the book by Stewart, 6th Edition.

1 Compulsory Problems

1. Section 3.1: Problems 18, 26, 27, 30, 32.
2. Section 3.1: Problems 53, 54, 57.
3. Section 3.2: Prove the quotient rule from first principles. Here is how to start: Let $F(x) = f(x)/g(x)$, then $F'(x) = \lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h}$. Now, we have $F'(x) = \lim_{h \rightarrow 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - g(x+h)f(x)}{hg(x)g(x+h)}$. Use tricks similar to the one we employed for the proof of the product rule in class, and complete the derivation.
4. Section 3.2: Problem 34. [You may use the quotient rule or the logarithm trick].
5. Section 3.3: Problem 31 (You must use the quotient rule here, for practice).
6. Section 3.4: Problems 10, 20, 38, 40, 46. You may use the logarithm trick here if you feel the need for it.
7. Section 3.6: Problem 30.
8. Find the derivative of $f(x) = \sqrt{x}^{\sqrt{x}}$. Use the logarithm trick discussed in class.

2 Some notes/guidelines

1. Make sure you understand how to manipulate logarithms THOROUGHLY, i.e. you should have no doubts or difficulties whatsoever while using them in solving problems.
2. Make sure you understand the two alternative notations for a derivative.
3. Go through the proofs of the product rule, quotient rule and the chain rule.

4. Practice lots of problems on chain rule.

5. Note the following formulae: If $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$. If $f(x) = \log_a x$, then $f'(x) = \frac{1}{x \ln a}$. Here a is considered to be a constant. If $f(x) = x^n$, then $f'(x) = nx^{n-1}$, where n is considered to be a constant. If $f(x) = a^x$, then $f'(x) = a^x \ln a$ (and NOT equal to xa^{x-1}), where a is considered to be a constant. If $f(x) = x^x$, then the derivative is obtained as follows:

Consider $y = f(x) = x^x$, then $\ln y = x \ln x$. Differentiating both sides w.r.t. x , we have $\frac{1}{y} \frac{dy}{dx} = x \frac{d}{dx}(\ln x) + \frac{d}{dx}(x) \ln x$ using the product rule. [If you don't see this, consider that $g(x) = x$ and $k(x) = \ln x$. Then $y = g(x)k(x)$, and hence $\frac{dy}{dx} = g(x)k'(x) + g'(x)k(x)$ using the product rule]. This gives us $\frac{1}{y} \frac{dy}{dx} = \frac{x}{x} + 1 \ln x = 1 + \ln x$. Simplifying further, we have $\frac{dy}{dx} = y(1 + \ln x) = x^x(1 + \ln x)$. As a checkpoint to see whether you have understood this, attempt the derivative of $f(x) = \sqrt{x}^{\sqrt{x}}$.

Note further on that you cannot use the chain rule directly for the derivative of $f(x) = x^x$. In other words, the following procedure is WRONG. I am putting it here below as it's a mistake that students tend to make quite frequently:

$y = x^x = u^x$ where $u = x$. Using the chain rule, we have $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = u^x \ln u \frac{du}{dx} = x^x \ln x \frac{dx}{dx}$, as $u = x$, which further gives us $x^x \ln x$.

Again note, that the above use of the chain rule is INCORRECT. Where's the error? The error is that while obtaining $\frac{dy}{du}$, we used the formula of the derivative of a^x giving us $a^x \ln a$. This is valid IF and ONLY IF a is a constant and independent of x . It is not valid in the case of u , because $u = x$, and hence u is obviously not independent of x .