Homework 1

July 8, 2008

The homework is due in class (BEFORE the lecture begins) on **Tuesday, 8th July**. As mentioned in the course syllabus, there are no electronic submissions and no late homeworks will be accepted unless you have an illness spanning the full period from the time the homework was assigned until it was due (and I shall need to see a medical practitioner's certificate to that effect). Standard academic honesty rules apply. You can discuss problems but the solutions turned in should be entirely your own. Cases of plagiarism will be dealt with strictly.
Most of the problems are from the book by Stewart, 6th Edition.

1 Compulsory Problems

1. Section (1.1), problem 44.

   For the solution open the file:
   `http://www.cise.ufl.edu/~avr/MAC2311/hw1sol_problem1.pdf`.

2. Section (1.2), problem 9. [Hint: Start off by saying that $f(x) = ax^3 + bx^2 + cx + d$. You are given values of $f(x)$ at four values of x. Use them to solve for the four coefficients].

   The cubic function is $f(x) = ax^3 + bx^2 + cx + d$. We have $f(1) = 6$, and hence $a+b+c+d = 6$. Also, we have $f(-1) = 0$ and hence, $-a+b-c+d = 0$. As $f(0) = 0$, we have $d = 0$. As $f(2) = 0$, we have $8a + 4b + 2c + d = 0$, i.e. $4a + 2b + c = 0$ (as $d = 0$). We now have the following 3 simultaneous equations in a, b, c:

   $$a + b + c = 6 \tag{1}$$
   $$-a + b - c = 0 \tag{2}$$
   $$4a + 2b + c = 0 \tag{3}$$

   A quick way to solve these is to observe that $b = a + c$ from the second equation, giving us $a + c = 3$. Also the third equation becomes $2a + c = 0$, i.e. $c = -2a$. Hence we have $a = -3$, $c = 6$ and $b = 3$. We already knew that $d = 0$. Plug in these values of a, b, c, d and verify that you indeed get back the correct values of $f(0), f(1), f(-1)$ and $f(2)$.

3. Section (1.3), problem 36.

   $f(g(x)) = f(\sin 2x) = \frac{\sin 2x}{1+\sin 2x}$. The domain of this consists of all values of x such that $\sin 2x \neq -1$, i.e. $2x \neq -n\pi/2$ (where n is an odd positive integer) and $2x \neq (n\pi + \pi/2)$ (where n is an odd positive integer), i.e. the domain consists of all x such that $x \neq -n\pi/4$ and $x \neq n\pi/2 + pi/4$ (where n is an odd positive integer).

   $g(f(x)) = g(\frac{x}{1+x}) = \sin \frac{2x}{1+x}$. The domain consists of all real values of x such that $x \neq -1$.

   $f(f(x)) = f(\frac{x}{1+x}) = \frac{x}{1+2x}$. The domain consists of all real values of x such that $x \neq -1/2$ and $x \neq -1$.

   $g(g(x)) = \sin(2 \sin 2x)$. The domain consists of all real values of x, i.e. the domain is $(-\infty, \infty)$.

4. Section (1.3), problem 66.

   We are given that g is odd. If f is an odd function, then $h = f \circ g$ is also an odd function. Here is a proof: $h(-x) = f \circ g(-x) = f(g(-x)) = f(-g(x))$. The last equality follows because $g(x)$ is odd. Further on, as f is odd, we have $h(-x) = -f(g(x)) = -h(x)$. Example: $g(x) = x^3$ and $f(x) = \sin x$ (both are odd). Then $h(x) = \sin x^3$ which is definitely an odd function (as $h(-x) = \sin(-x^3) = -\sin x^3 = -h(x)$).

   OTOH, if f is even, then h will be even. Example: $g(x) = x^3$ and $f(x) = x^2$, then $h(-x) = h(x)$ and h is an even function. If f is neither even nor odd, then h will be neither even nor odd. Example: $g(x) = x^3$ but $f(x) = x^3 + x^2$.

5. Section (1.6), problem 58.

   The electric charge at time t is given by $y = Q(t) = Q_0(1 - e^{-\frac{t}{a}})$. The inverse function is obtained by expressing t fully in terms of y, i.e. we have $y/Q_0 = 1 - e^{-\frac{t}{a}}$, i.e. $t = -a \ln(1 - y/Q_0)$. Note that Q_0 is treated as a constant independent of t. Now the inverse function is given as $Q^{-1}(y) = -a \ln(1 - y/Q_0)$. [If you want to express this as a function of t, you need to interchange the positions of t and y.] The meaning of the inverse function is the time it takes for the charge to build up to a certain level, say y.

   We are given $Q(t) = 0.9Q_0$ and $a = 2$, and we need to find t. Now, $t = -a \ln(1 - y/Q_0) = -a \ln(1 - 0.9) = -2 \ln(0.1) = 2 \ln 10$.

6. Sketch the graph of the function $f(x) = 1 + 2ln|x|$ where ln stands for the natural logarithm.
   For the solution open the files:

http://www.cise.ufl.edu/~avr/MAC2311/hw1sol_problem6_page1.pdf
http://www.cise.ufl.edu/~avr/MAC2311/hw1sol_problem6_page2.pdf.