# Homework 5
# (due Wednesday, April 5, 2006)

March 31, 2006

1. **MCMC implementation**: Consider the objective function

$$E(s) = -\left[\frac{1}{2}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} J_{mn}s_m s_n + \sum_{n=0}^{N-1} H_n s_n\right] \tag{1}$$

with $s_m \in \{-1, 1\}, \forall m$. In (1), $J_{mn}, n = m \oplus 1 \,\text{or}\, n = m \ominus 1$ is drawn from a normal distribution with mean zero and standard deviation 1. Similarly, $H_n$ is also drawn from a normal distribution with mean zero and standard deviation 1. (In the above, $m \oplus 1 = 0$ when $m = N - 1$ and $m \ominus 1 = N - 1$ when $m = 0$.) $J_{mn} = 0$ when $n \neq m \oplus 1$ or $n \neq m \ominus 1$. The corresponding probability distribution is

$$\Pr(S = s) = \frac{\exp\{-\beta E(s)\}}{Z(\beta)}. \tag{2}$$

- Construct the probability table over the 1024 possibilities of $s$ corresponding to $N = 10$ for $\beta = 0.1$, 0.3, 1, 3, 10.
- Implement a Gibbs sampler by successively sampling from $\Pr(s_k|s_l, l \neq k)$ and beginning with a suitably chosen random initial condition. Show the approach to convergence for all values of $\beta$ above using histograms.
- Implement a Metropolis-Hastings sampler of your choice - google for different choices of $Q(s'; s^{(t)})$. Show the approach to convergence for all values of $\beta$ above using histograms.
- In all cases, estimate $\frac{1}{N}\sum_{n=0}^{N-1} < s_n >$ for different values of $\beta$ and compare it to the true value.