Solutions to Homework 2

March 8, 2006

Fisher geodesic: An egregiously bad mistake was made in setting up the Fisher geodesic problem. The Fisher geodesic for a single variable Gaussian (normal) model is NOT a straight line (or at least it is not a straight line under the metric connection.) It is a straight line in a non-metric connection but that's besides the point. I'm sorry for misleading y'all. Anyway, here's a parametrization which makes Fisher calculations much more easy for the normal model [than the traditional (μ, σ) parametrization].

$$p(x|\theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right). \tag{1}$$

This implies that

$$\begin{aligned} -\log p(x|\theta) &= \frac{(x-\mu)^2}{2\sigma^2} + \frac{1}{2}\log(2\pi\sigma^2) \\ &= \frac{x^2}{2\sigma^2} - \frac{x\mu}{\sigma^2} + \frac{\mu^2}{2\sigma^2} + \frac{1}{2}\log(2\pi\sigma^2). \end{aligned} \tag{2}$$

Assuming a parametrization in which $\theta^{(1)} = \frac{\mu}{\sigma^2}$ and $\theta^{(2)} = -\frac{1}{2\sigma^2}$, we get

$$-\log p(x|\theta) = -x\theta^{(1)} - x^2\theta^{(2)} + \frac{(\theta^{(1)})^2}{4\theta^{(2)}} - \frac{1}{2}\log\left(-\frac{\pi}{\theta^{(2)}}\right). \tag{3}$$

The useful aspect of this parametrization is that second derivatives of $-\log p(x|\theta)$ do not involve x at all and can easily be computed. Unfortunately, this parametrization does not give a "flat" metric tensor since $\theta^{(1)}$ and $\theta^{(2)}$ are coupled.