Homework 2 (due October 3rd, 2005)

23rd September 2005

1. The famous XOR problem is a limited version of the moderate-extreme political classification problem that we've been discussing in class. In the XOR problem, we have four patterns each in $\mathbb{R}^2$; $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$. The first and fourth patterns are in class 1 since 0 XOR 0 and 1 XOR 1 equals 0. The second and third patterns are in class 2 since 1 XOR 0 and 0 XOR 1 equals 1. First, construct a hidden layer using a linear transformation of the input layer ($a_1 = w_{11}x_1 + w_{12}x_2 + w_{10}$ and $a_2 = w_{21}x_1 + w_{22}x_2 + w_{20}$) followed by a tanh nonlinearlity with β set to infinity (which is equivalent to thresholding—$\tanh(\beta a) = 1$ for $a > 0$ and $\tanh(\beta a) = -1$ for $a < 0$). Show that the four patterns are linearly separable at the hidden layer for a carefully chosen set of weights by drawing a suitable diagram. Then, construct an output layer perceptron by designing the perceptron weights and demonstrate that your perceptron linearly separates the hidden layer patterns. You do not need to code anything for this problem.
2. Bishop 4.3, 4.15, 5.5