Office hours:

Tu $10 - 11^{30}$

Th $13 - 15^{30}$

Lecture 7

Sampling without replacement

$$X_y = \begin{cases} 1 & \text{if } y \in D' \\ 0 & \text{oth.} \end{cases}, \quad \forall y \in D$$

$$X = f(D') = \alpha \sum_{y \in D'} y = \alpha \sum_{y \in D} X_y \cdot y$$

$$E(X) = \alpha \sum_{y \in D} E(X_y) \cdot y = \sum_{y \in D} y = f(D)$$

$$P(y \in D') = \frac{1}{\alpha}$$

$$\mathrm{Var}(X) = E(X^2) - E^2(X)$$

$$E(X^2) = \alpha^2 \sum_{y \in D} \sum_{z \in D} E[X_y X_z] \cdot y \cdot z$$

$$E[X_y X_z] = \begin{cases} \frac{1}{\alpha} & y = z \\ P(y \in D' \wedge z \in D') = P(y \in D') \cdot P(z \in D' \mid y \in D') = \frac{1}{\alpha} \cdot \underbrace{\frac{|D'| - 1}{|D| - 1}}_{\frac{1}{\alpha_1}} & \end{cases}$$

$$= \begin{cases} \frac{1}{\alpha} & y = z \\ \frac{1}{\alpha \alpha_1} & y \neq z \end{cases}$$

$$E[X^2] = \alpha^2 \left[\sum_{y\in D} \frac{1}{\alpha} y^2 + \sum_{y\in D} \sum_{z\in D, z\neq y} \frac{1}{\alpha\alpha_1} \cdot y \cdot z \right]$$

complete the sum

$$- \sum_{y\in D} \frac{1}{\alpha\alpha_1} y^2$$

?

$$= \alpha^2 \left[\frac{1}{\alpha} \sum_{y\in D} \left(1 - \frac{1}{\alpha_1}\right) y^2 + \frac{1}{\alpha\alpha_1} \underbrace{\sum_{y\in D} \sum_{z\in D} y \cdot z}_{\left(\sum_{y\in D} y\right)^2} \right]$$

$$= \alpha \left[\sum_{y\in D} \left(1 - \frac{1}{\alpha_1}\right) y^2 + \frac{1}{\alpha_1} \left(\sum_{y\in D} y\right)^2 \right]$$

$$\text{"}\mathrm{Var}(X)\text{"} = \frac{\alpha}{\alpha_1} \cdot (\alpha_1 - 1) \sum_{y\in D} y^2 + \frac{\alpha - \alpha_1}{\alpha_1} \left(\sum_{y\in D} y\right)$$

Cases makes this proof "complic...
Need algebraic manipulation for "ca...

Kronecker delta symbol

$$\delta_{i,j} = \begin{cases} 1 & \text{if } i = \\ 0 & \text{oth} \end{cases}$$

just care about equality

$$E[X_y X_z] = \{$$

Interaction between $\delta_{y,z}$ and $\sum_{y\in D}$ $= \delta_{y,z}$

$$\sum_{y\in D} \delta_{y,z} \cdot f(y,z) = f(z,$$

$$E[X_y X_z] = \underbrace{\delta_{y,z}\left(\frac{1}{\alpha}\cdot\frac{1}{\alpha\alpha_1}\right) + \frac{1}{\alpha\alpha_1}}$$

$$E[X^2] = \alpha^2 \cdot \sum_{y\in D}\sum_{z\in D} \cdot y \cdot z$$

$$= \alpha^2\left(\underbrace{\sum_{y\in D}\sum_{z\in D} \delta_{yz}\,\frac{1}{\alpha}\left(1-\frac{1}{\alpha_1}\right)}_{\sum_{z\in D}\frac{1}{\alpha}\left(1-\frac{1}{\alpha_1}\right)\cdot z^2}\overset{yz}{} + \underbrace{\sum_{y\in D}\sum_{z\in D}\frac{1}{\alpha\alpha_1}\, yz}_{\frac{1}{\alpha\alpha_1}\left(\sum_{y\in D} y\right)^2}\right)$$