

Lecture 20

Count-Min

$$q_i, i \in I, \quad M$$

~~$q_i, i \in I$~~

$$q_i \in \mathbb{I}$$

$$q_{i_1}, q_{i_2}, \dots, q_{i_n}$$

Problem 1 - Point Query

Given i estimate q_i

Problem 2 - Inner Product (size of join)

$$a, b$$

$$a \cdot b = \sum q_i b_i$$

Problem 3 - Range Query

$$b_i = \begin{cases} 1 & i \in R \\ 0 & \text{oth} \end{cases} \quad \sum_{i \in R} q_i$$

Stream model.

1° items q_i arrive in arbitrary order

$$(i, v)$$

$$(10, 13), (11, 12), (11, 2)$$

2° Partial values of q_i on the stream

$$(10, 13), (11, 12), (11, 2) \rightarrow \text{if all input keys hit}$$

$$(10, 14)$$

$$(10, \underline{1}), (11, \underline{1})$$

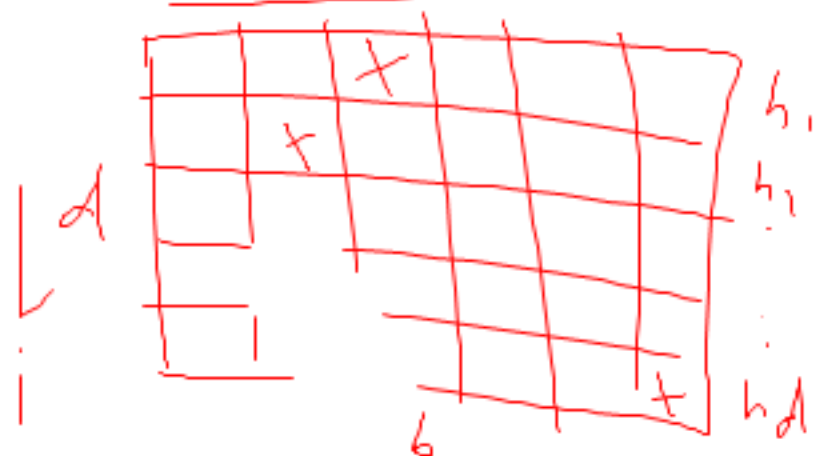
est. freq moments.

AMS Size of sketch $\sim \frac{1}{\epsilon^2}$

$\epsilon = 0.1 \sim 100$

If sketch $\sim \frac{1}{\epsilon} \sim 10$

COUNT-MIN



(i, v_i)

For every $j=1..d$

Compute $h_j(i) = \leftarrow$

$A_{jk} += v_i$

Initially $A_{jk} = 0$

Pair (i, v_i)
 i is given
 v_i

For bucket k

$$A_{jk} = \sum_{i: h_j(i)=k} v_i$$

$A_{jk} = \sum_{i: h_j(i)=k} v_i$
 (if v_i is positive)
 or 0 otherwise

$$A_{jk} = \sum_{i: h_j(i)=k} v_i$$

$$\hat{a} = \min_{j=1..d} \sum_{k: h_j(i)=k} A_{jk}$$

$$\mathbb{E} \sum_i \mathbb{1}_{\{x_i \geq \epsilon\}}$$

$$\leq \sum_i \mathbb{P}(x_i \geq \epsilon)$$

$$\leq \sum_i \mathbb{P}(\sum_{j=1}^n |x_{ij}| \geq \epsilon)$$

$$\leq \sum_i \mathbb{P}(\sum_{j=1}^n |x_{ij}|^2 \geq \frac{\epsilon^2}{6}) \leq \frac{1}{6}$$

$$\mathbb{E} \sum_i |x_{ik}|^2 = \sum_i \mathbb{E} |x_{ik}|^2 \leq \sum_i \frac{1}{n} = \frac{1}{n}$$

$$\mathbb{E} \sum_i |x_{ik}|^2 \leq \frac{1}{n}$$

$$\mathbb{E} \sum_i |x_{ik}|^2 \leq \frac{1}{n}$$

$$\sum_i |x_{ik}|^2 \geq \frac{\epsilon^2}{6}$$

$$\mathbb{P}(\sum_i |x_{ik}|^2 \geq \frac{\epsilon^2}{6}) \leq \frac{1}{6}$$

$$\sum_i |x_{ik}|^2 \geq \frac{\epsilon^2}{6}$$

$$\frac{\epsilon^2}{6} \leq \frac{1}{n}$$

$$\epsilon \leq \frac{1}{\sqrt{n}}$$

$$\sum_i |x_{ik}|^2 \leq \frac{1}{n}$$

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$$\mathbb{P}(\sum_i |x_{ik}|^2 \geq \frac{\epsilon^2}{6}) \leq \frac{1}{6}$$

$$\leq \frac{1}{6} = \frac{1}{6}$$

$$\epsilon \leq \frac{1}{\sqrt{n}}$$

Q:

11

smaller

1. $\frac{1}{\sqrt{2}}$

Answer

$$\sqrt{\frac{1}{2}} < \frac{1}{\sqrt{2}}, \quad \frac{1}{\sqrt{2}} < \sqrt{\frac{1}{2}}$$

↑ ↓
smaller larger