Lecture 12

$$F_m(t_{m+1}, t'_{m+1}, \ldots, t_k, t'_k) = \sum_{t_1 \in R_1} \sum_{t'_1 \in R_1} \cdots \sum_{t_m \in R_m} \sum_{t'_m \in R_m} \prod_{i=1}^{m} (a_i + \delta_{t_i t'_i} \cdot s_i) \cdot g(t_1, \ldots, t_m, t_{m+1}, \ldots, t_k)\, g(t'_1, \ldots, t'_k)$$

$\prod_{i=m+1}^{k} (a_i + \delta_{t_i t'_i} s_i)$

$$F_{m+1}(t_{m+2}, t'_{m+2}, \ldots) = \sum_{t_{m+1} \in R_{m+1}} \sum_{t'_{m+1} \in R_{m+1}} F_m(\ldots)$$

$\prod_{i \in S} a_i \prod_{i \notin S} s_i$

$$\boxed{\begin{array}{l} F_k(\,) = \ell \cdot h \cdot n \\ G_k(\,) = \Delta\, h\, n \end{array}}$$

$$G_m(t_{m+1}, t'_{m+1}, \ldots) = \sum_{S \in \{1, \ldots, m\}} \sum_{\{t_i \in R_i \mid i \in S\}} \left(\sum_{\{t_j \in R_j \mid j \notin S\}} g(t_1, \ldots, t_m, t_{m+1}, \ldots, t_k) \right) \left(\sum_{\{t'_j \in R_j \mid j \notin S\}} g(t_1, \ldots, t_m, t'_{m+1}, \ldots, t'_k) \right)$$

$$G_{m+1}(t_{m+2}, t'_{m+2}, \ldots) = \sum_{t_{m+1} \in R_{m+1}} \sum_{t'_{m+1} \in R_{m+1}} G_m(\ldots) + \sum_{t_{m+1} \in R_{m+1}} G_m(t_{m+1}, t_{m+1}, t_{m+2}, t'_{m+2}, \ldots)$$

Induction:

Sp $T_m(\ldots) = S_m(\ldots)$

$\forall t_{m+1}, t'_{m+1}$

To prove $T_{m+1}(\ldots) = S_{m+1}(\ldots)$

$$T_{m+1}(\ldots) = \sum_{t_{m+1}, t'_{m+1}, \ldots, P_{m+1}} \sum \left(a_{m+1} + \delta_{t_{m+1}, t'_{m+1}, S_{m+1}}\right) T_m(\ldots)$$

$= S_{m+1}(\ldots)$ if we simplify the $\delta_{t, t'}$

Types of graphs we considered:

$$f(\mathbb{I}) = \sum_{t_1 \in \mathbb{R}} \cdots \sum_{t_k \in \mathbb{R}_k} g(t_1, \dots, t_k)$$

$$f(\mathbb{I}) = \mathcal{I}\left(\quad \right)$$

Not possible to compute moments exactly.

Possible approach:

- get conf. int. for inner expansion
- get probabilistic c.i. for $\mathcal{I}(\ )$

Immediate generalization to $\mathcal{I}\left(\quad \right)$

linear extensions