Lecture 11

Lemma Given:

- relations $R_1 \ldots R_k$
- an k-variable fn $g : R_1 \times R_2 \times \ldots \times R_k \to \mathbb{R}$ ($g(t_1, \ldots, t_k)$ given "access" for tuples $t_1 \ldots t_k$)
- $\forall i \in \{1..k\}$ we are given constants a_i, b_i (that form expression $a_i + \delta_{t_i} b_i$)

Then:

$$\sum_{t_1 \in R_1} \sum_{t_1' \in R} \cdots \sum_{t_k \in R_k} \sum_{t_k' \in R_k} \prod_{i=1}^{k} (a_i + b_i \delta_{t_i, t_i'}) \cdot g(t_1, \ldots, t_k) \cdot g(t_1', \ldots, t_k') =$$

$$\sum_{S \in \mathcal{P}(\{1..k\})} \left(\sum_{\{t_i \in R_i \mid i \in S\}} \prod_{i \notin S} a_i \prod_{i \in S} b_i \left(\sum_{\{t_i \in R_i \mid i \notin S\}} g(\{t_i \in R_i \mid i \in S\}, \{t_i \in R_i \mid i \notin S\}) \right)^2 \right)$$

$\bar{S}$: complement of S

S: indices of sum outside of square

$\sum_{t_{i_1}} \ldots \sum_{t_{i_{|S|}}}$ for $S = \{i_1, \ldots, i_{|S|}\}$

Proof:

By induction:

Base case: $k =$ ✓

Induction:

Need to generalize the problem:

Suppose we have proven $1 \le m \le k$

$$F_m(t_{m+1}, t'_{m+1}, \dots, t_k, t'_k) = \sum_{t_i \in \ell_i, i \in \ell_i} \sum \cdot \sum_{t_i \in \ell_i} \sum_{t'_i \in \ell_i} \delta(t_1, \dots, t_k) \cdot \delta(t'_1, \dots, t'_k)$$

$\tilde{F}_k$ in $d \cdot L \cdot \eta$ of Lambda to prove

$$G_m(t_{m+1}, t'_{m+1}, \dots, t_k, t'_k) = \sum_{S \in P(1:m)} \prod_{i \in S} a_i \prod_{i \notin S} b_i \sum_{\{t_i \in \ell_i \mid i \in S\}} \left(\sum_{\{t_i \in \ell_i \mid i \in S\}} \delta(t_1, \dots, t_m, t_{m+1}, \dots, t_k) \right) \left(\sum_{\{t_i \in \ell_i \mid i \in S\}} \delta(t'_1, \dots, t'_m, t'_{m+1}, \dots, t'_k) \right)$$

G_k in $\eta \cdot L \cdot$ mid of Lambda

$$F_{m+1}(t_{m+2}, t'_{m+2}, \dots) = \sum_{t_{m+1} \in \ell_{m+1}} \sum_{t'_{m+1} \in \ell_{m+1}} F_m(t_{m+1}, t'_{m+1}, \dots)$$