Lecture 10

$D = \{R, S\}$

$$f(D) = \sum_{t \in R} \sum_{v \in S} g(t,v)$$

R' copy of R

S' copy of S) independent with

$$Z = f(D) \cdot \alpha\beta \sum_{t \in R} \sum_{v \in S} X_t Y_v \delta(t,v)$$

$$E(Z) = f(D)$$

$$E(Z^2) = \alpha^2\beta^2 \sum_{t \in R} \sum_{t' \in R} \sum_{v \in S} \sum_{v' \in S} E[X_t X_{t'}]\, q(Y_v Y_{v'})$$

$$\underbrace{\alpha^2\beta^2}_{c} \cdot g(t,v) \cdot g(t',v')$$

$$E(X_t X_{t'}) = \frac{1}{\alpha}\left(\delta \cdot \frac{1}{\alpha_1}\right) \delta_{t,t'} + \frac{1}{\alpha_1}$$

$$= a_R \delta_{t,t'} + b_R$$

$$E(Y_v Y_{v'}) = q_1 \delta_{v,v'} + b_S$$

$$E(Z^2) = c \sum_{t \in R} \sum_{t' \in R} \sum_{v \in S} \sum_{v' \in S} \left(a_R a_S \delta_{t,t'} \delta_{v,v'} - a_R b_S \delta_{t,t'} + b_R b_S \right) \delta(t,v)\delta(t',v')$$

$$= c\left(a_R a_S \sum_{t \in R} \sum_{v \in S} \delta^2(t,v) + a_R b_S \sum_{t \in R} \sum_{v \in S} \sum_{j \in S} g(t,v)\delta(t,v') \right.$$

$$\left. + b_R a_S \sum_{t \in R} \sum_{t' \in R} \sum_{v \in S} \delta(t,v)\delta(t',v) \right.$$

$$b_R b_S \sum_t \sum_{t'} \sum_v \sum_{v'} \delta(t,v)\delta(t',v') \underbrace{\left(\sum_t \sum_v \delta(t,v)\right)^2}$$

$$\sum_{t\in R} \sum_v \sum_{v'} \delta(t,v)\delta(t',v')$$

$$\sum_t \left(\left(\sum_v \delta(t,v)\right)\left(\sum_{v'} \delta(t',v')\right)\right)$$

$$\sum_t \left(\sum_v \delta(t,v)\right)^2$$

$$E[Z^2] = C\left\{ a_R a_S \sum_t \sum_v \delta(t,v)^2 + a_R b_S \sum_t \left(\sum_v g(t,v)\right)^2 + b_R a_S \sum_v \left(\sum_t \delta(t,v)\right)^2 + b_R b_S \left(\sum_t \sum_v \delta(t,v)\right)^2 \right\}$$

$$R, S, T$$

$$a_R, b_R \quad a_S, b_S \quad a_T, b_T$$

$$\rho \quad \mu$$

$$Z = \rho\mu \sum_{t\in R} \sum_{v\in S} \sum_{w\in T} X_t Y_v W_w \cdot \delta(t,v,w)$$

$$E(Z^2) = c\Big(a_R a_S a_T \sum_t \sum_v \sum_w s(t,v,w)^2$$

$$+ b_R a_S a_T \sum_v \sum_w \Big(\sum_t s(t,v,w) \Big)^2$$

$$\vdots$$

$$b_R b_S a_T \sum_v \Big(\sum_t \sum_v s(t,v,w) \Big)^2$$

$$b_R b_S b_T \Big(\sum_t \sum_v \sum_w s(t,v,w) \Big)^2$$

$$E(Z^2) = \alpha^2 \sum_t \sum_{t'} E(X_t X_{t'}) s(t) s(t')$$

$$a_R s_{t,t'} + b_R$$

$$E(Z^2) = \alpha^2 \Big(a_R^2 \sum_t s(t)^2 + b_R \Big(\sum_t s(t) \Big)^2 \Big)$$

Lemma:

$$E_{R'S'}(Z^2) = E_{S'}\Big[\underbrace{E_{R'S'}(Z^2)}_{T} \Big]$$

$$c = \alpha^2 \beta^2 \gamma^2 E_{S'}\Big[E_{R'}(Z^2) \Big]$$