Properties of surface splines

- *free-form modeling capability.* There are no restrictions on the number of cells meeting at a mesh point or the number of edges to a mesh cell. Mesh cells need not be planar.
- *built-in smoothness (unless explicitly reduced) and local smoothness preserving editability.* For given connectivity and shape parameters, the surface spline form a vector space of geometrically smooth surface parametrizations. In order to manipulate the surface spline, it suffices to average or move the mesh points locally.
- *low degree parametrization.* The surface is parametrized by low degree polynomial patches. The representation can be extended to rational patches by using a fourth coordinate.
- *simple interpolation.* Mesh points and normals can be interpolated without solving a system of constraints.
- *evaluation by averaging.* The coefficients of the parametrization in Bernstein-Bézierform can be obtained by applying averaging masks to the input mesh. (The Bernstein-Bézierform in turn is evaluated by averaging.) Thus the algorithm is local and can be interpreted as a rule for cutting an input polytope such that the limit polytope is the spline surface.
- *convex hull property.* The surface lies locally and globally in the convex hull of the input mesh. In other words, every point on the surface can be computed as an average of the mesh points with coefficients that are positive and sum to one.
- *intuitive shape parameters.* The averaging process is geometrically intuitive. Parameters analogous to knot distances govern the depth of the cuts into the polytope outlined by the control mesh. (In concave regions the complement of the polytope is cut.) Smaller cuts result in a surface that follows the input mesh more closely and changes the normal direction more rapidly across the boundary. In the limit this allows adjusting the built-in smoothness, e.g. reducing it to continuity for zero cuts. Discontinuity can be achieved by a change of mesh connectivity.
- *taut interpolation of the control mesh for zero blend ratios.* Cuts of zero depth result in a singular parametrization at the mesh points analogous to singularities of a spline with repeated knots. The continuity of the surface is reduced, but in return the edges of the input mesh are interpolated and the surface is taut, e.g. planar when the mesh cell is planar.

In general any two of the following three properties can be achieved for any input mesh using C^1-surface splines. This is optimal in the sense that no surface construction (in fact not even a curve construction) can achieve all three properties simultaneously for all data sets.

1. The convex hull property
2. Tangent continuity
3. Interpolation of the input mesh points