as an identical object with all z-components collapsed to zero.

6 Modeling to match moments

The ability to measure moments efficiently as demonstrated in the previous section is the key to solving the inverse problem of designing geometry to match a prescribed (set of) moments. In general this problem is underconstrained since the geometry offers many degrees of freedom. The standard approach to regularizing the problem is to add variational constraints. Rather than grazing on this fertile field, we look at some practically relevant problems where the geometry is restricted to change only in one parameter.

6.1 Volume matching

Consider filling water into a jug held in fixed position. Until overflow, the geometry of the 'water object' is constrained by the geometry of the container: the only degree of freedom is the height of the water level. A well-posed problem is therefore to ask for the *fill height* $\bar{h}$ of the container $\mathbf{x}$ given a fixed water volume V_0. The problem is non-trivial, since the top surface of the water at rest is a plane that cuts the container patches in algebraic curves resulting in challenging boundary integrals when an exact solution is sought. Rotating the container if necessary so that the negative z-direction agrees with the direction of gravity the problem can be formally written as

$$\text{find } \bar{h}, \text{such that} \quad \text{volume } \mathbf{x}(\bar{h}, \mathbf{u}) = V_0.$$

The problem has the nice property that $\bar{h}$ is a monotone function of the volume so that bisection will not only succeed if the fluid volume fits into the container but also yield a sequence of upper and lower bounds in the process. Newton's method guarantees no such bound and has to cope with the fact that $\bar{h}$ is generally not a differentiable function of V_0: consider the rate of change at the branch point when filling a T-shape.

Each bisection step requires computing a volume bounded by the intersection of the free-form surface with a plane. To avoid the exact intersection computation, we subdivide the patches in the vicinity of

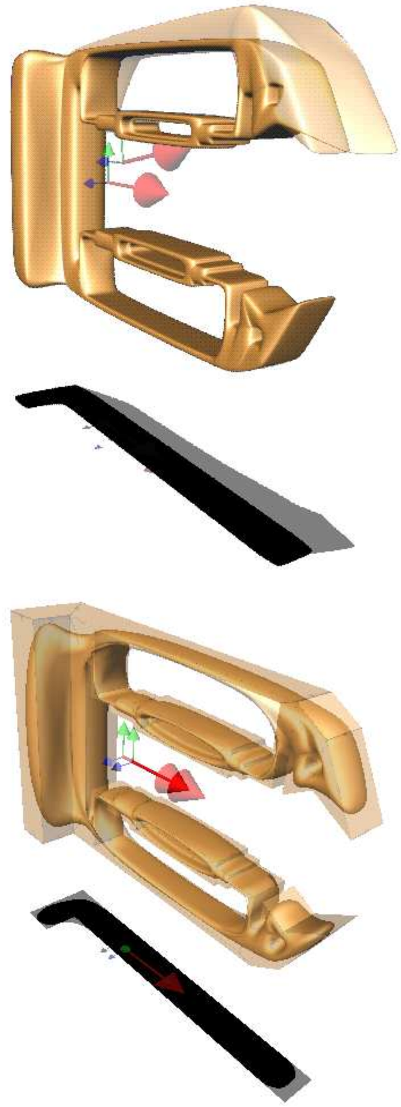

Figure 5: Change of the inertia frame under deformation and smoothing.