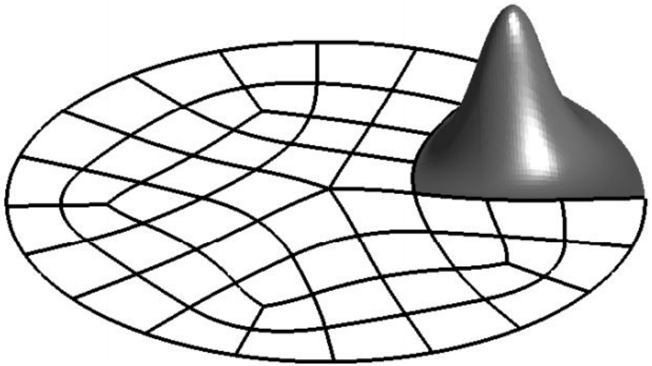
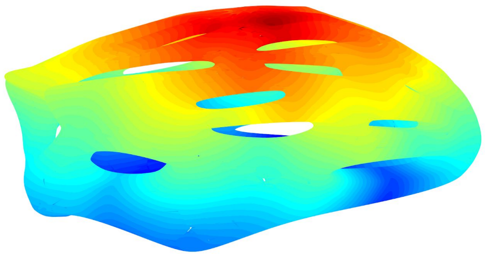
Semi- structured splines for design & analysis





Jorg Peters (joint work with Kestutis Karciauskas, Thien Nguyen)

INdAM 2020, Rome, Italy

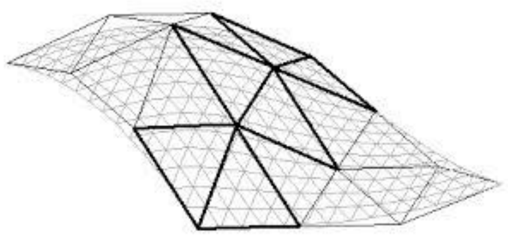
Support: INdAM, DARPA

Overview

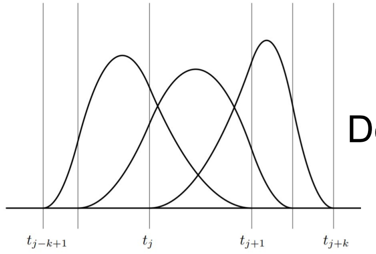
- Irregularities
- Classification of splines on meshes with irregularities
    -
    - Semi-structured G-splines
    - Subdivision Surfaces
    -

⟶ hybrid for IGA

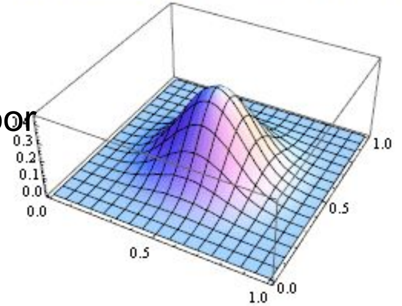
Assumes you are familiar with



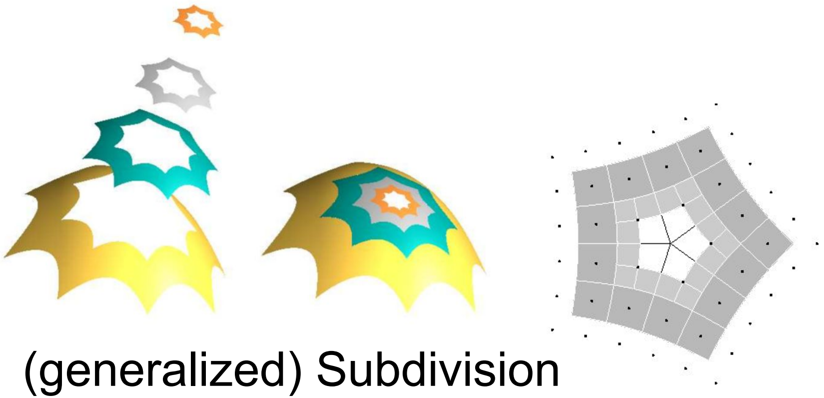
Bernstein-Bezier



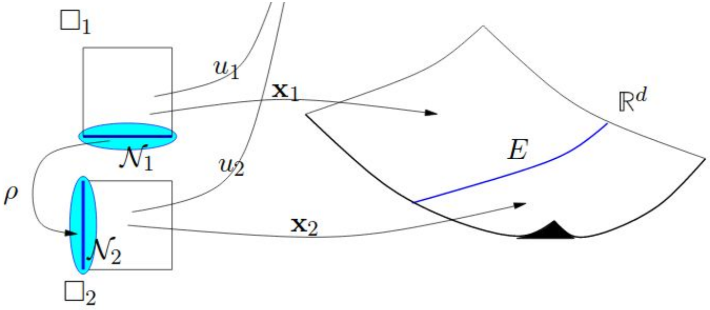
De Boor



B-spline



(generalized) Subdivision Surfaces



G^k continuity of surfaces

Surface Quality vs C^k

Farin

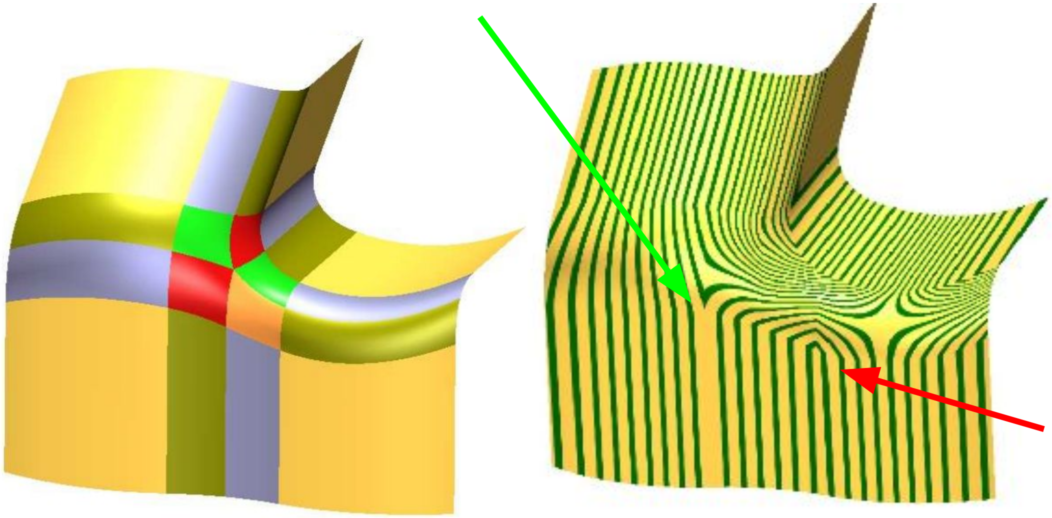
(a) reflection lines

uniform, parallel = good (unless feature)

Surface Quality vs C^k

Farin

(a) reflection lines



(b) G-spline surface + highlight lines on the surface

Zebra C^1

Uniform, parallel → good

Surface Quality vs C^k in automobile styling

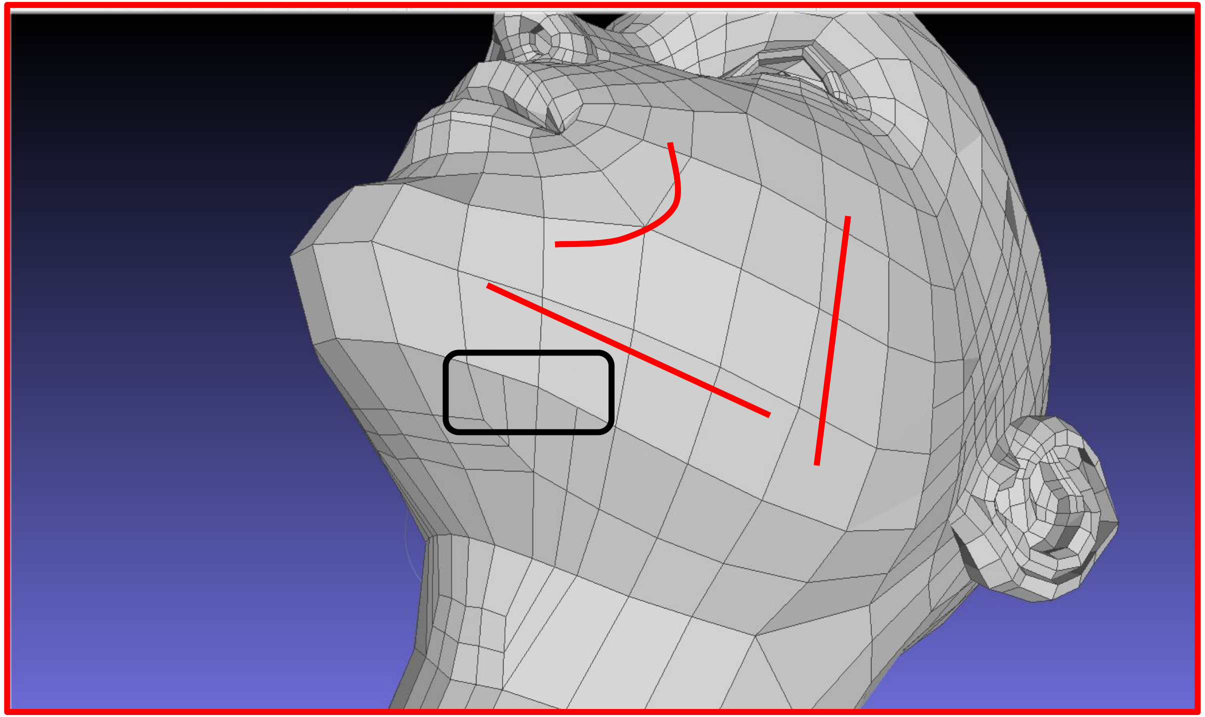
Farin

(a) reflection lines

Sven Havemann

Uniform, parallel → good

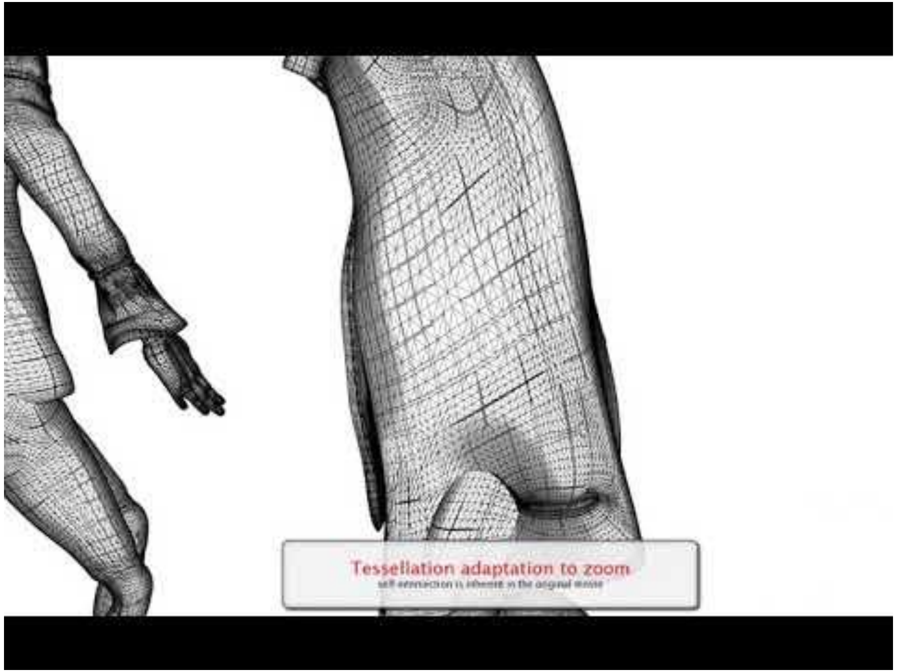
Irregularities

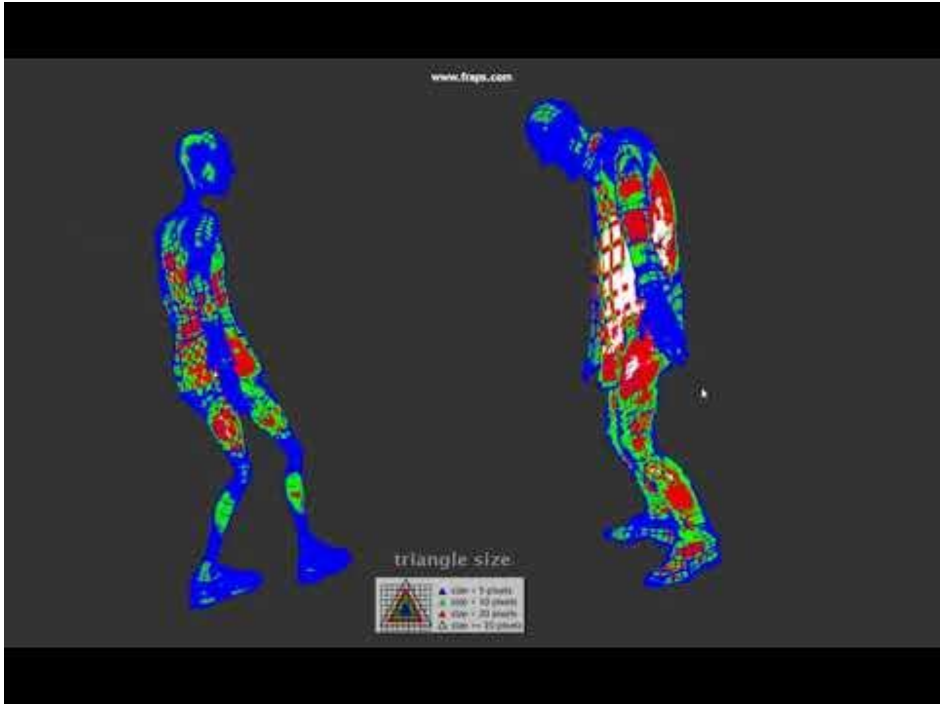


merge parameter directions

transition between coarse and fine meshes

Infinite Resolution

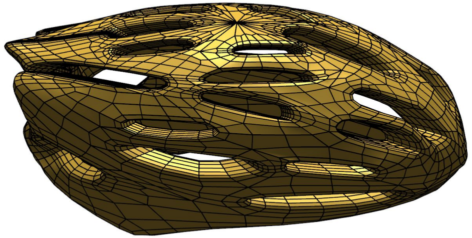




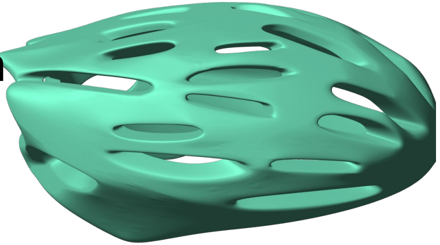
Efficient Pixel-accurate Rendering of Animated Curved Surfaces [YBP12]

Elephant's Dream Blender

Thin shell analysis



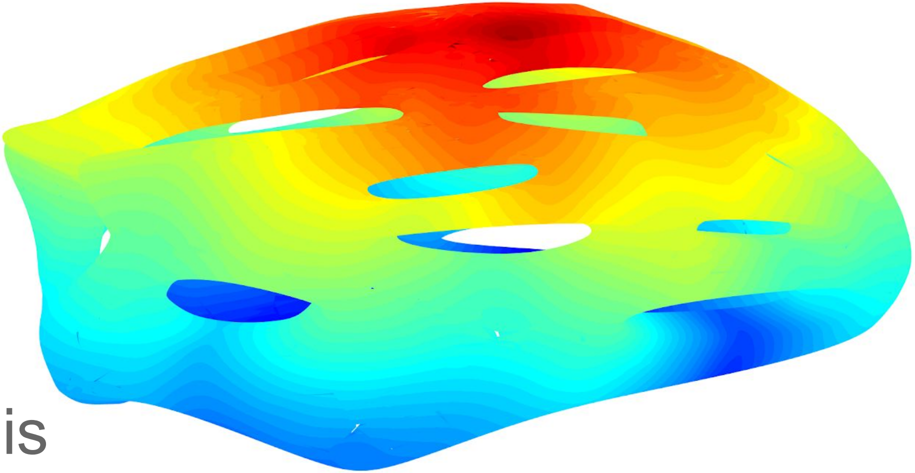
Quad mesh



Spline manifold

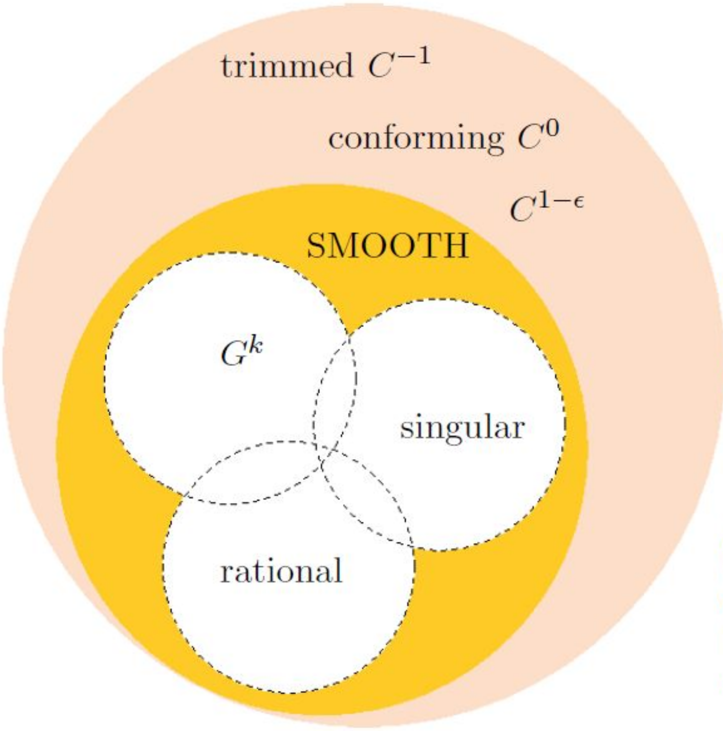
heat dissipation on thin shell

14K bi-3 pieces, 27.5K spline-dof, 3.6 secs



Design and Analysis

Classification of Splines for Irregular Layout



G^k:
geom smooth

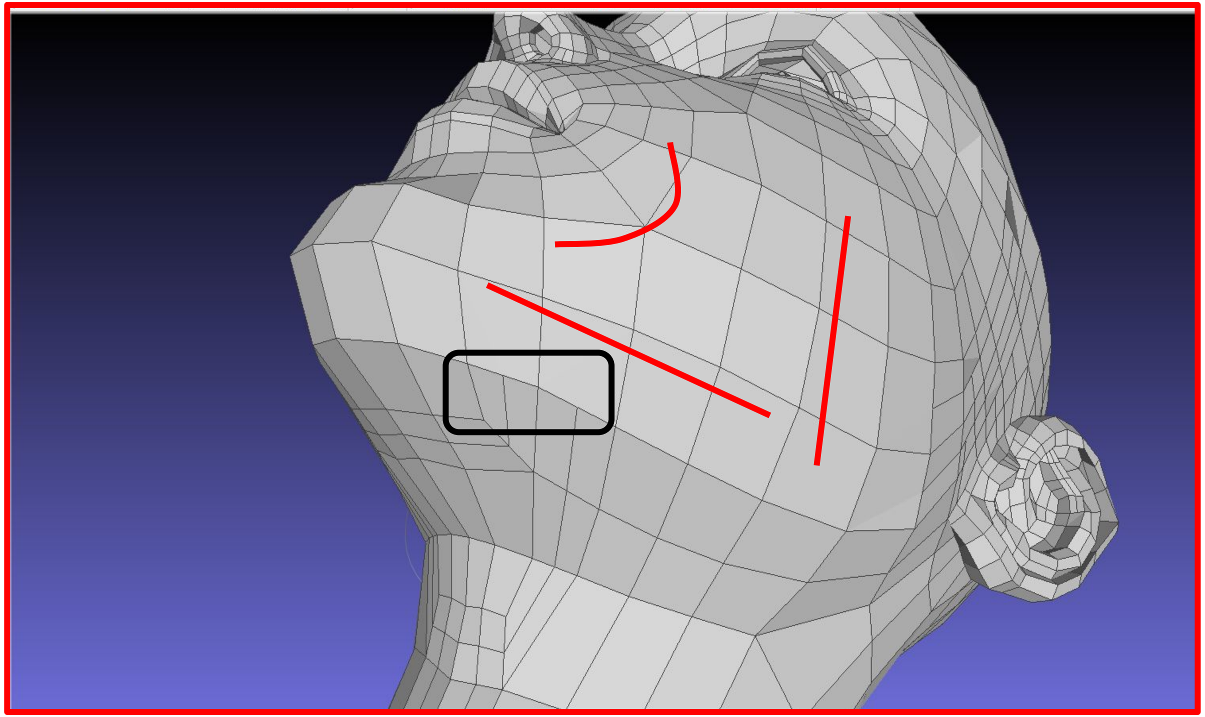
singular:
polar
subdivision
singular jet

rational:
transfinite
Gregory
barycentric
orbifold

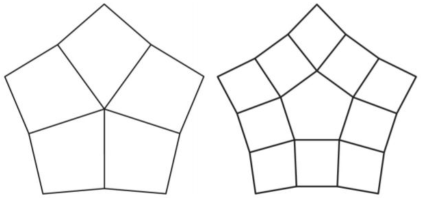
Overview

- Motivation
- Classification of splines on meshes with irregularities
    -
    - **Semi-structured G-splines**
    - Subdivision Surfaces
    -

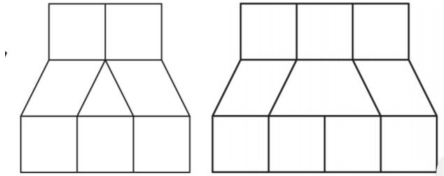
Irregularities



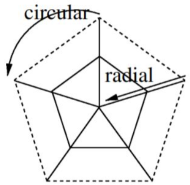
merge parameter directions



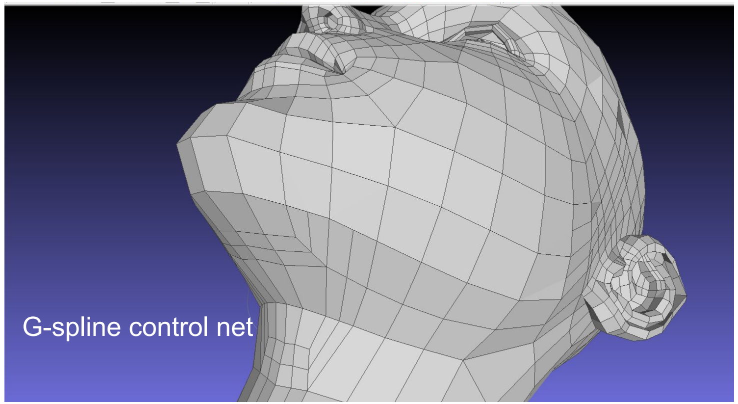
transition between coarse and fine meshes



polar

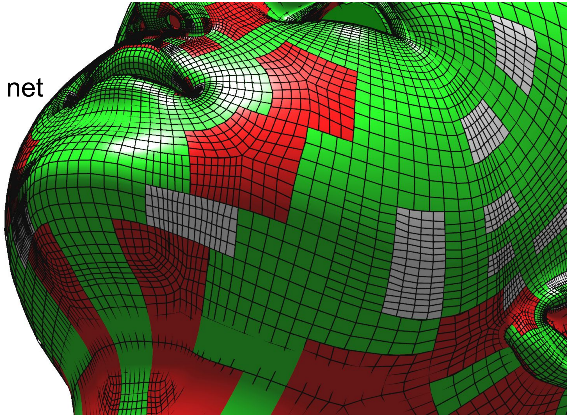


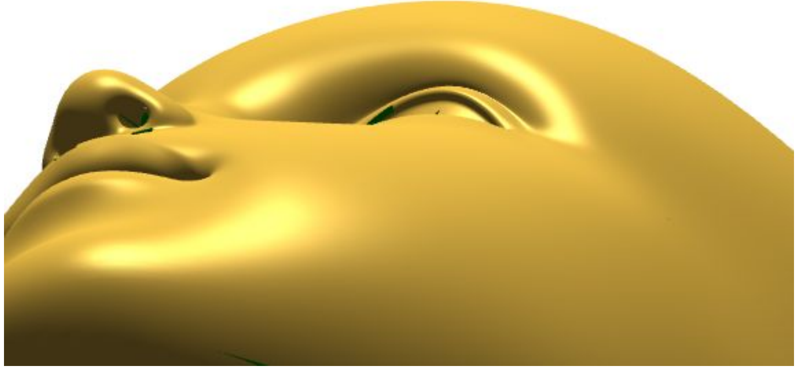
Irregularities → semi-structured G-splines



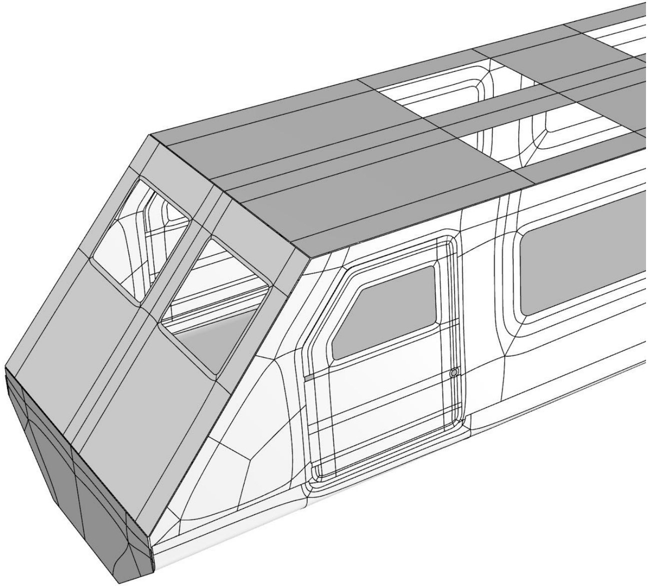
Bezier control net

Bi-3 spline
T-junction
multi-sided

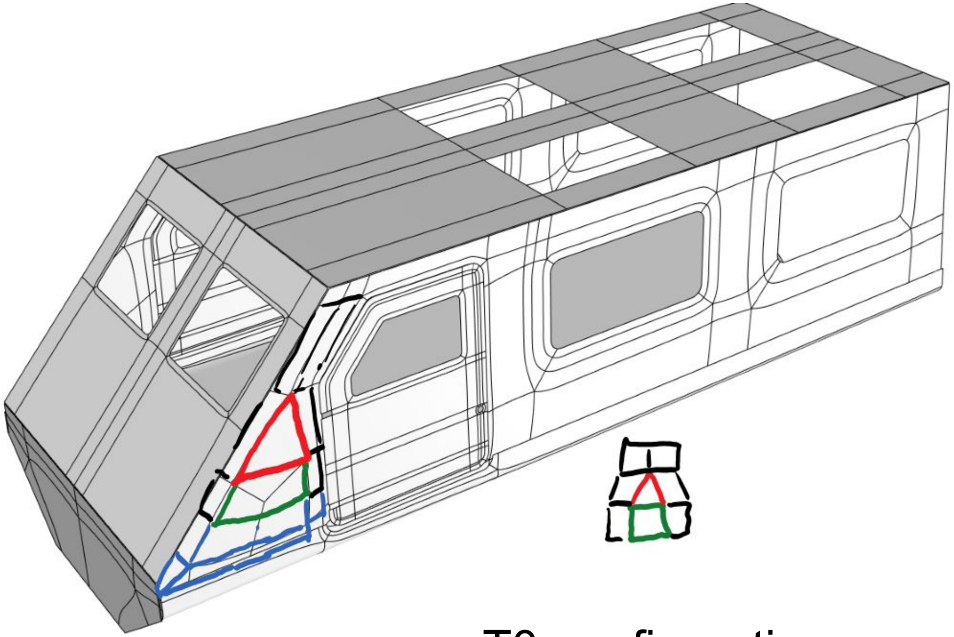




Irregularities

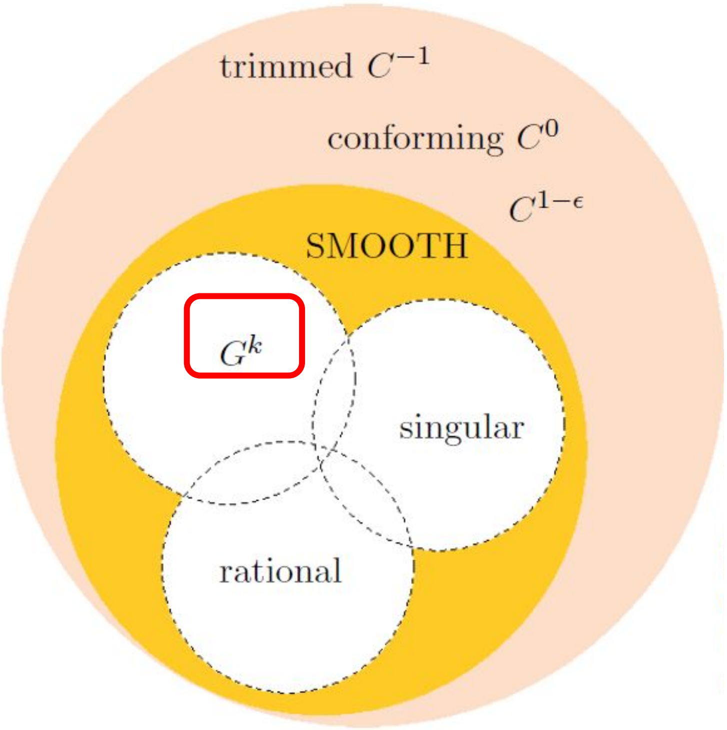


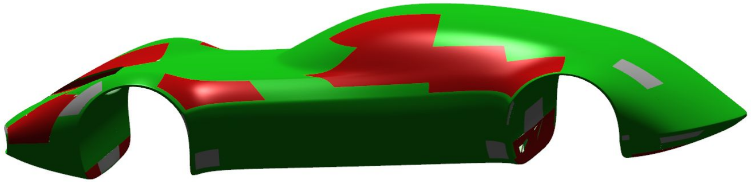
TARDEC Rene Hiemstra

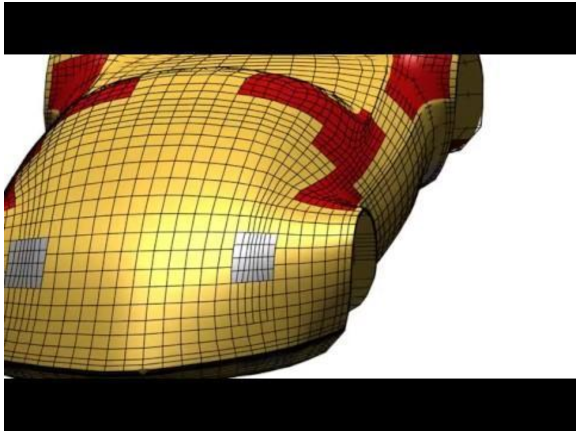


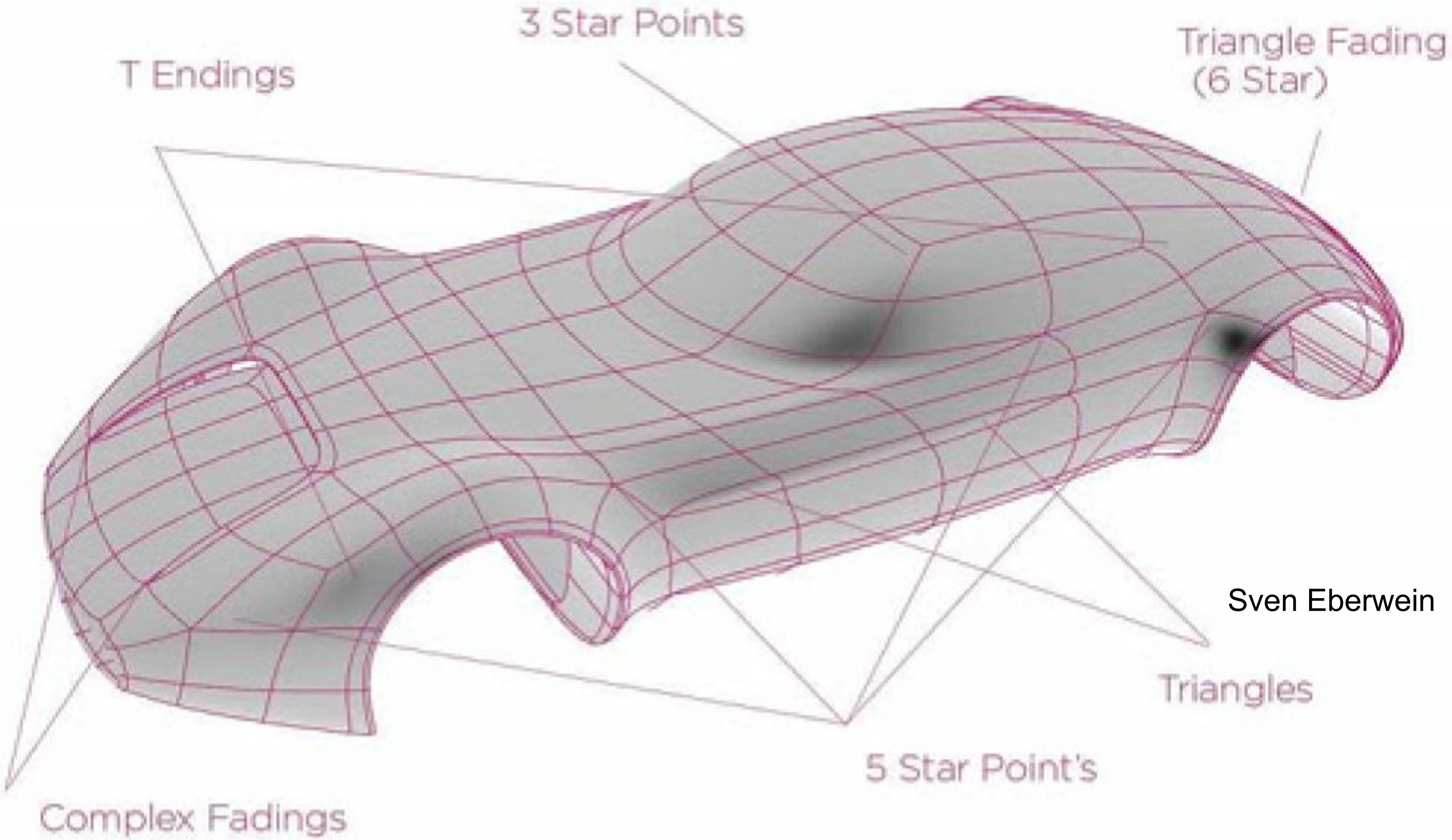
T0-configuration

Semi-structured G splines



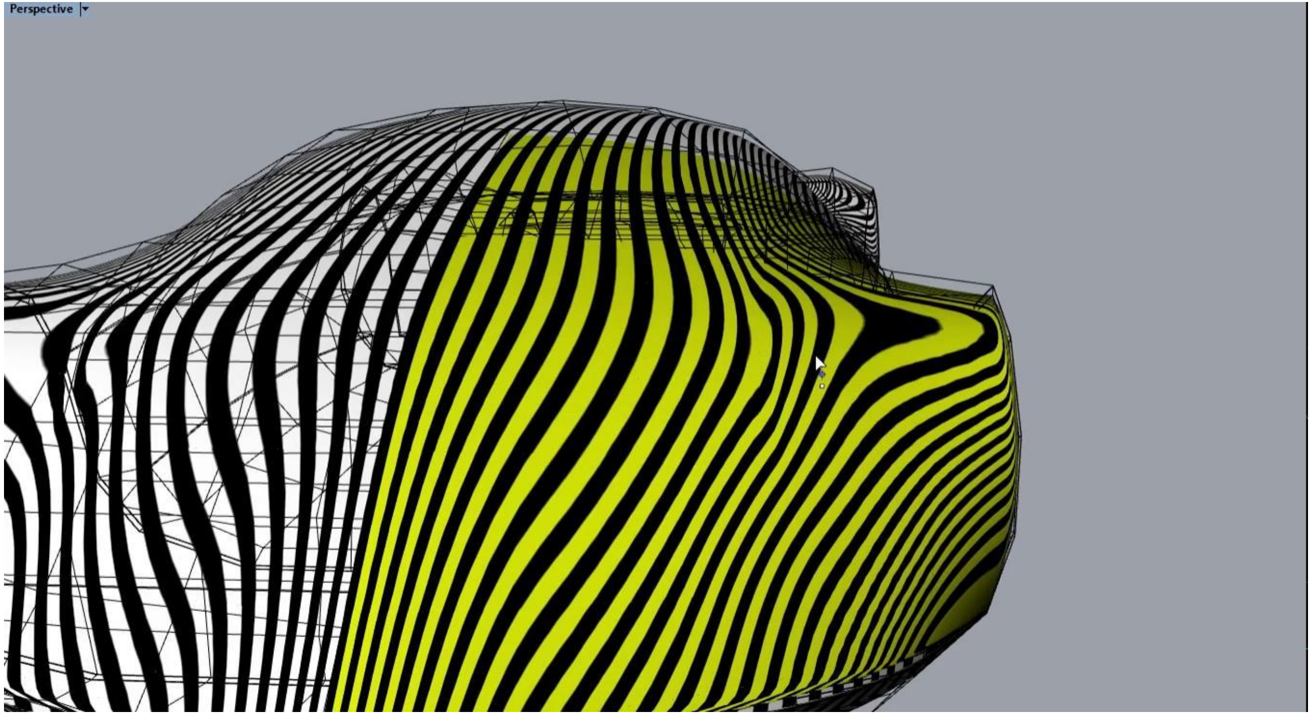






Sven Eberwein

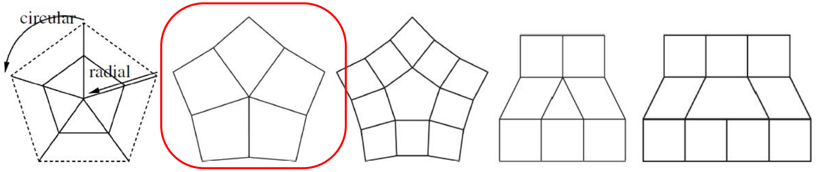
Semi-structured G-Splines: Interactive Rhino plugin



Semi-structured G-splines

Constructions with good shape =
smooth highlight lines, curvature distribution

smooth	regular	irregular	valence	split	reference
C0	bi-3	bi-3	any	2x2	SGP 15
G1	bi-2	bi-3	3,5		
		bi-4	6,7,. . .		SMI 14
	bi-3	bi-4	3,5,6		
		bi-5	7,8,. . .		SPM 15
		bi-4	any	2x2	
	bi-3	bi-5	any		GMOD 15
G2	bi-3	bi-6	any		CAGD 16
	bi-3	bi-5		2x2	CAGD 15
		bi-4	MVS	2x2	
		bi-5	MVS		SPM 16



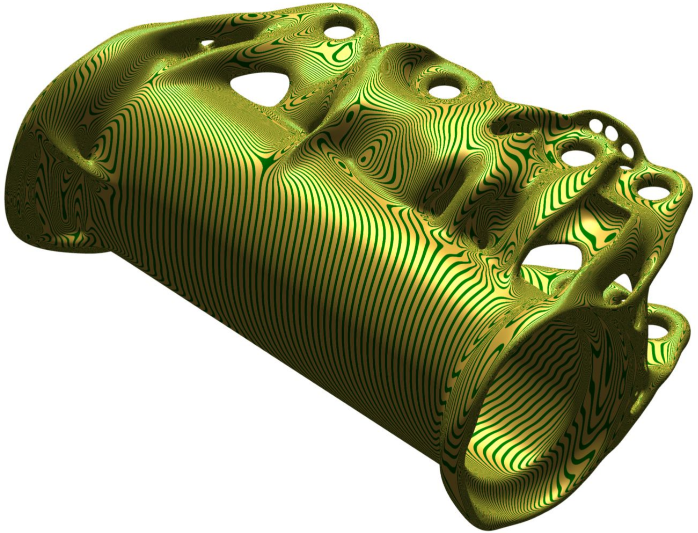
(a) polar-net $m = 5$ (b) star-net $n = 5$ (c) n-gon-net, $n = 5$ (d) τ_0-net (e) τ_1-net

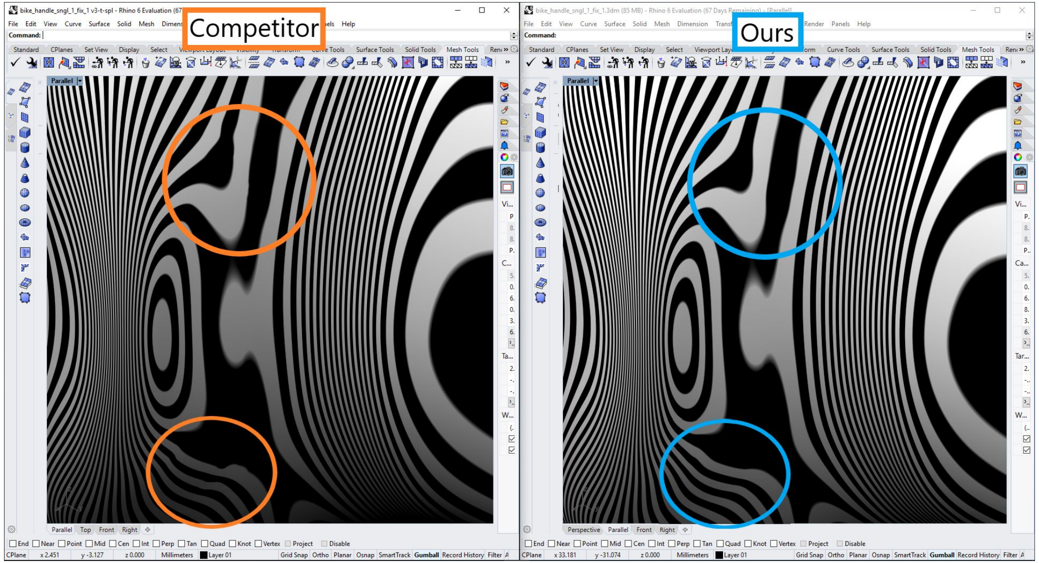
irregularity surrounded by quads

← as many as spline families

each is subtly different
to achieve special properties

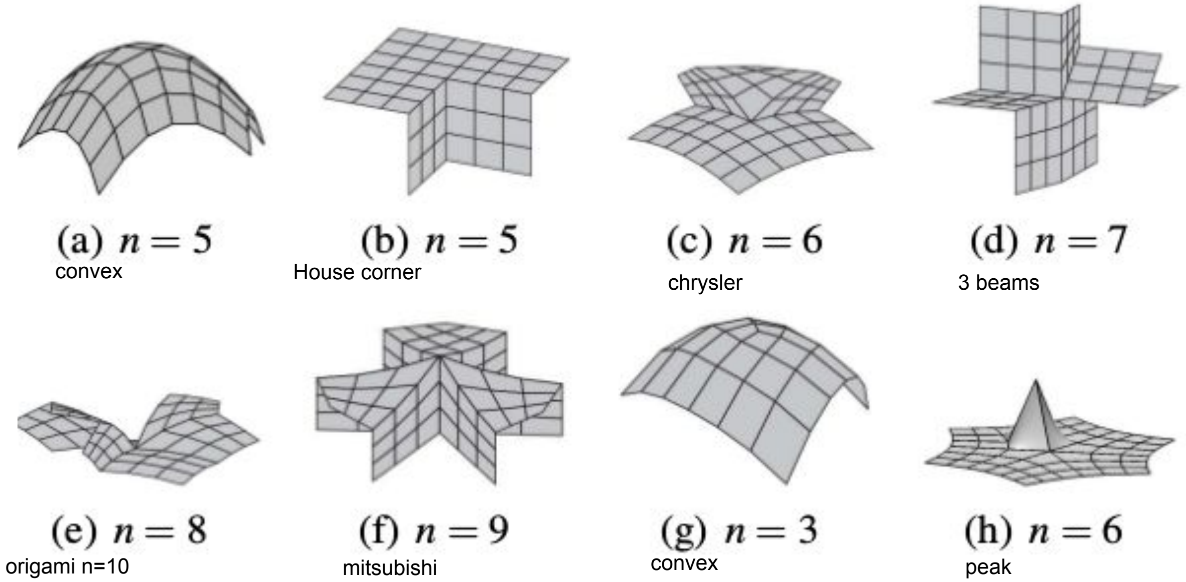
Good highlight lines? Large industrial data sets



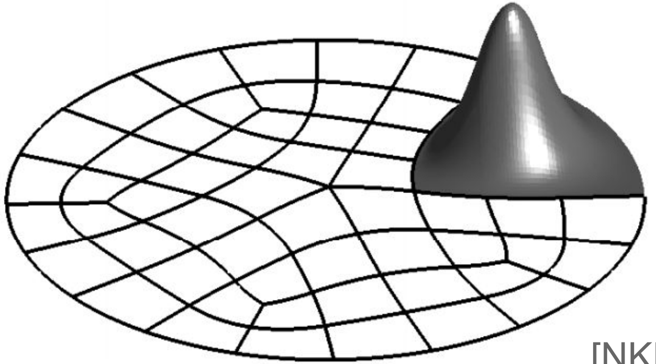


Thanks to Martin Marinov

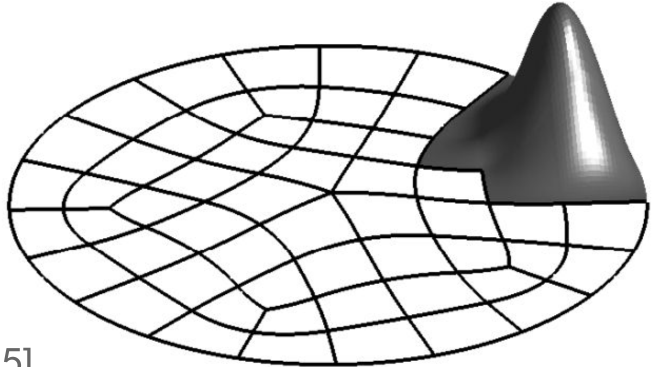
Good highlight lines? Shape obstacle course



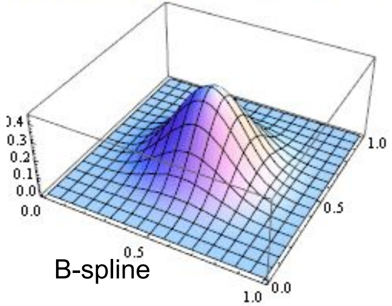
Basic Functions: Polynomial pieces joined with matching derivatives after change of variables (geometric continuity)



valence n=3

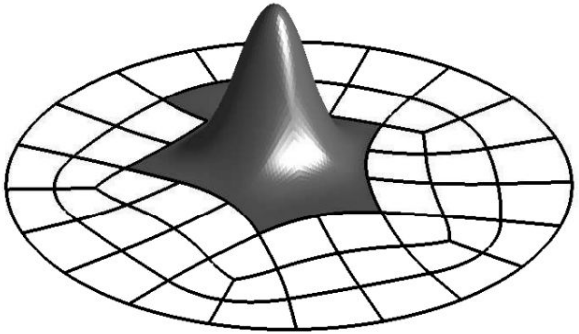


near n=3

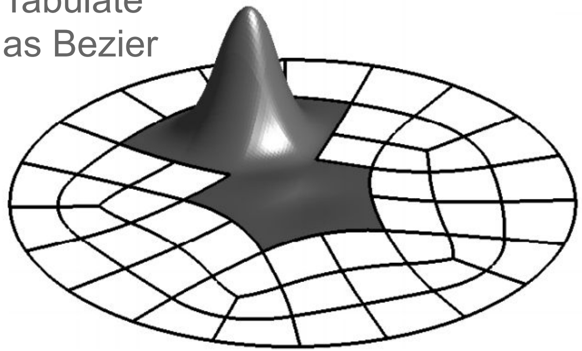


B-spline

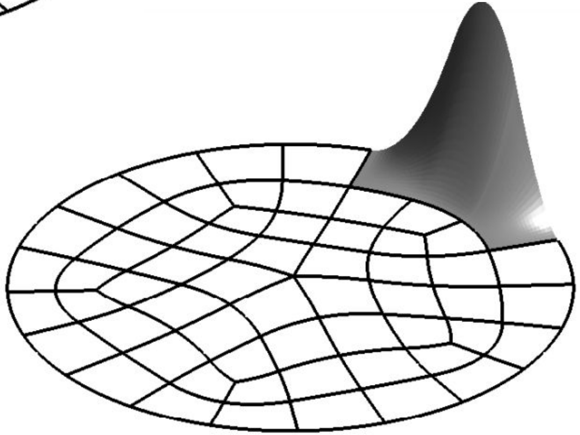
[NKP15]
Tabulate
as Bezier



near n=5



near n=5 and n=3



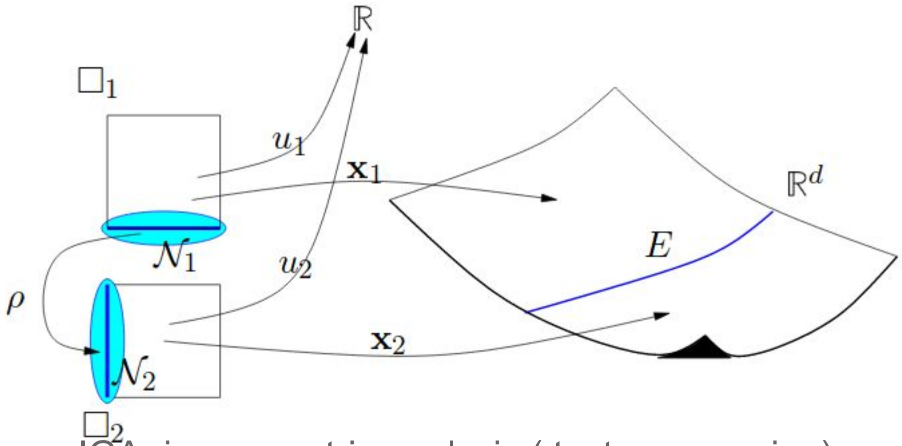
Boundary
G-spline

General Theorem for gIGA

Matched G^k-constructions always
yield C^k-continuous (isogeometric) finite elements [GP15]

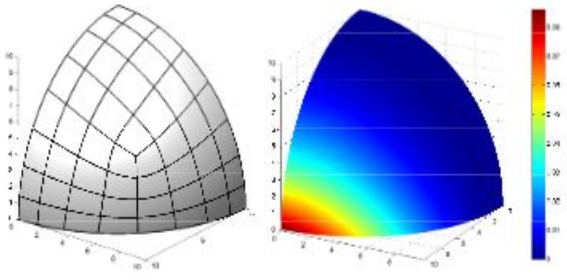
- in any number of variables,
- for any smoothness k,
- any manifold

(including, of course, planar ones)

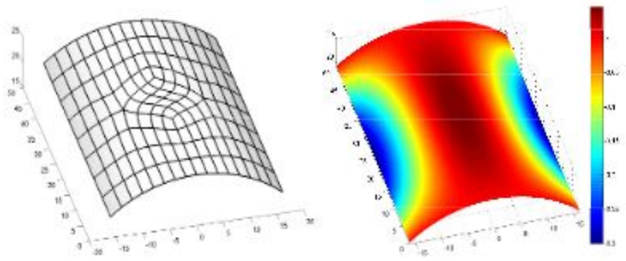


IGA=isogeometric analysis (texture mapping)

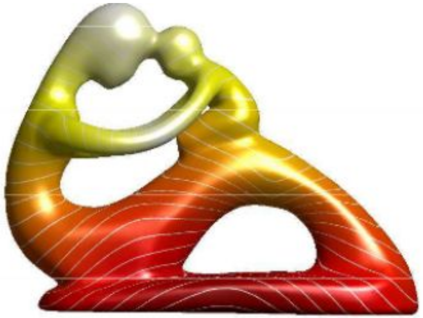
G-splines gIGA (generalized IGA) on manifolds [NKP13,15]



(a) Octant of a spherical shell



(b) Scordelis-Lo roof thin shell



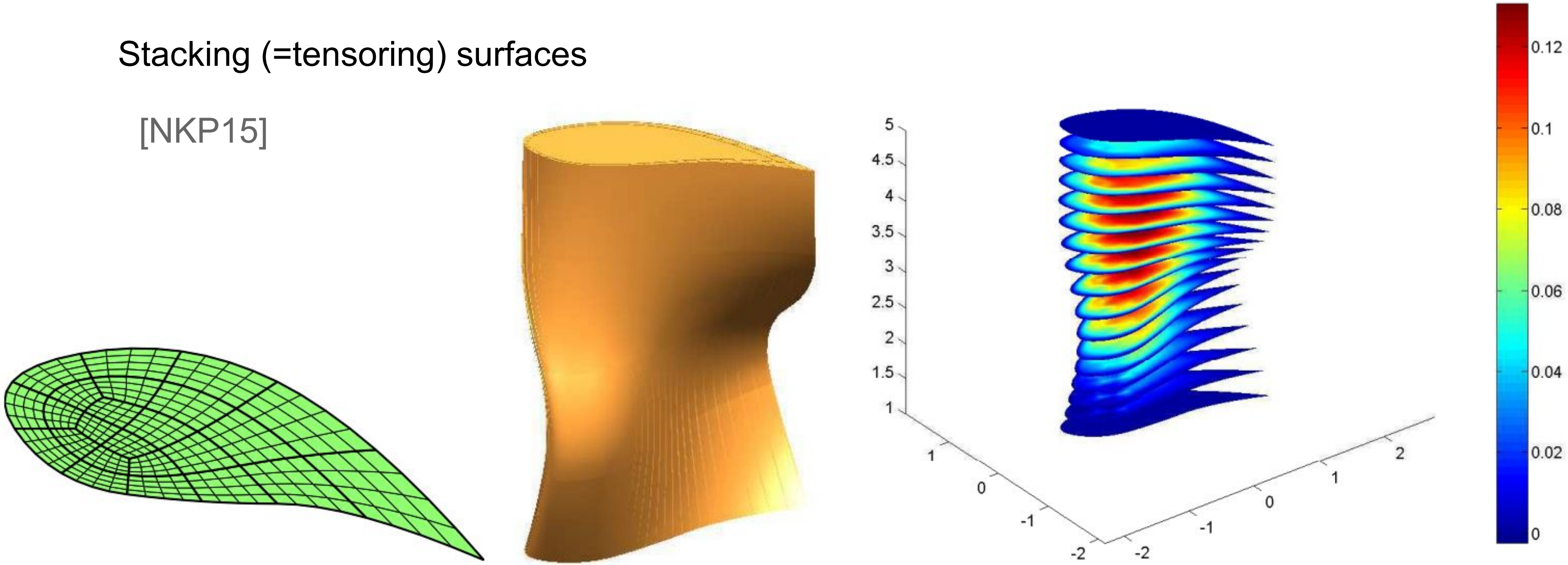
Finite Element Obstacle course: meshing-less analysis

use spline for geometry and displacement function

G-splines Layered tri-variate manifolds

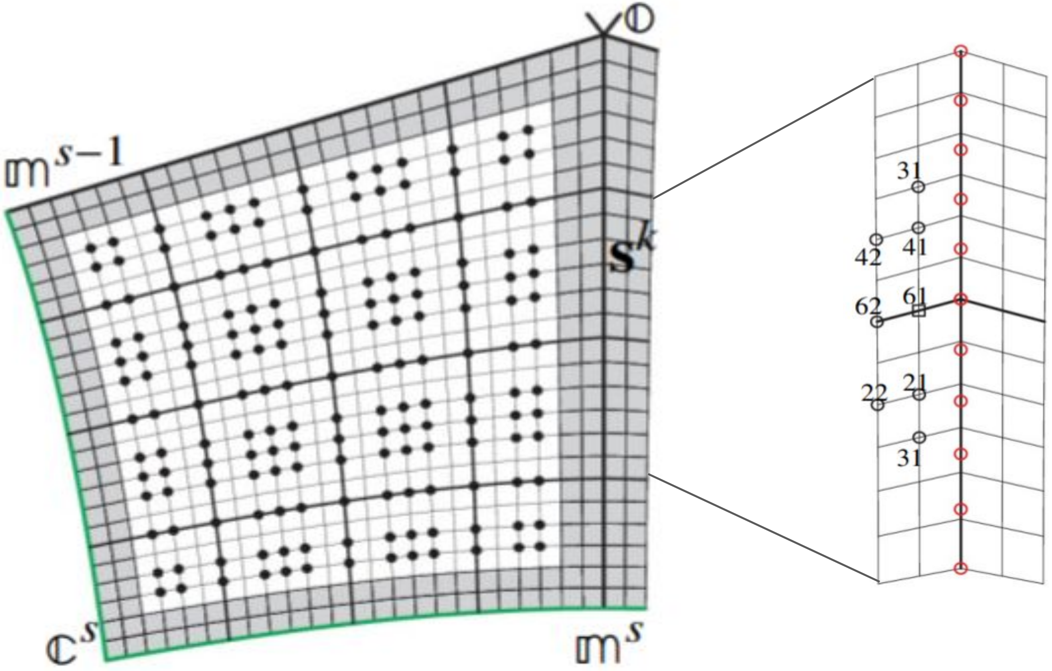
Stacking (=tensoring) surfaces

[NKP15]



(a) A cross-section (b) Turbine blade (solid) (c) Computed solution

d.o.f. for analysis under refinement



Along G-edges:

- Irregularly distributed
- Asymmetrically distributed

Semi-structured G-splines

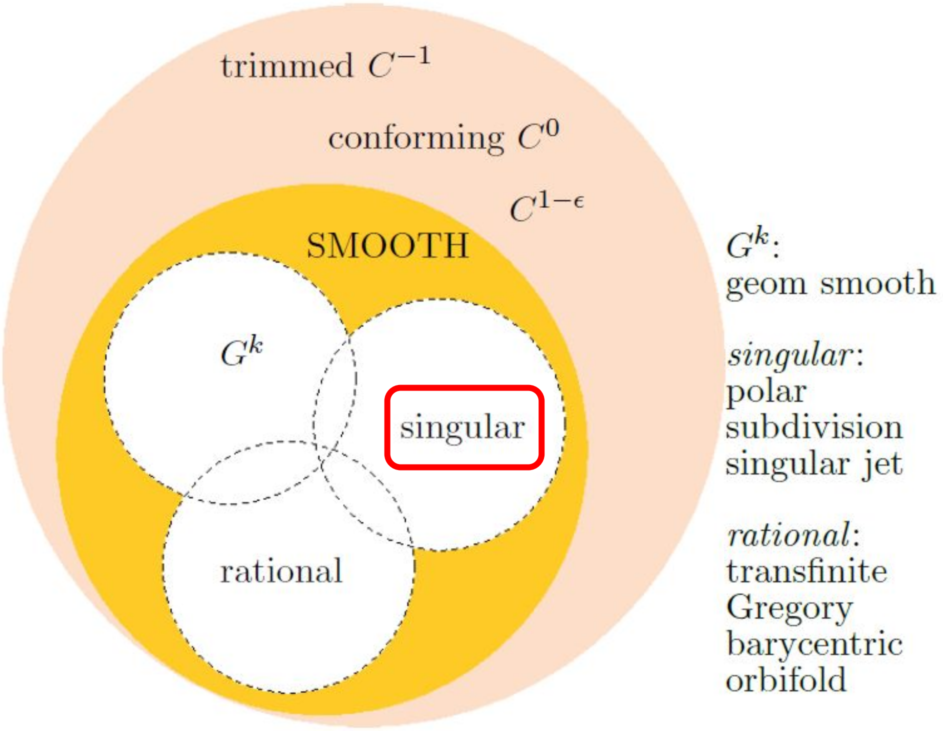
Pro:
Semi-structured control nets
Piecewise polynomial
Good shape

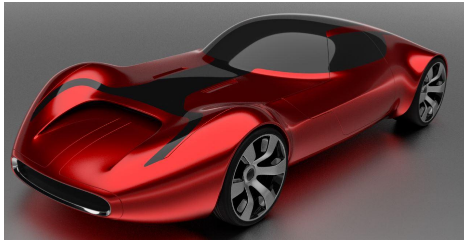
Con:
Refinement not uniform

Overview

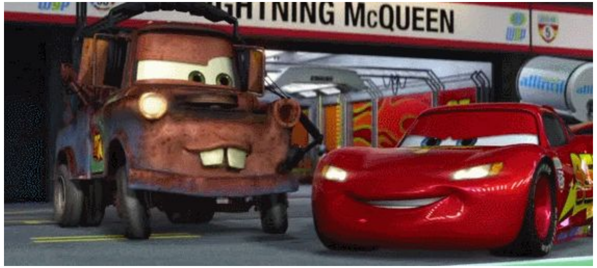
- ➢ Motivation
- ➢ Classification of splines on meshes with irregularities
  - ○
  - ○ Semi-structured G-splines
  - ○ **Subdivision Surfaces**
  - ○

Subdivision Surfaces





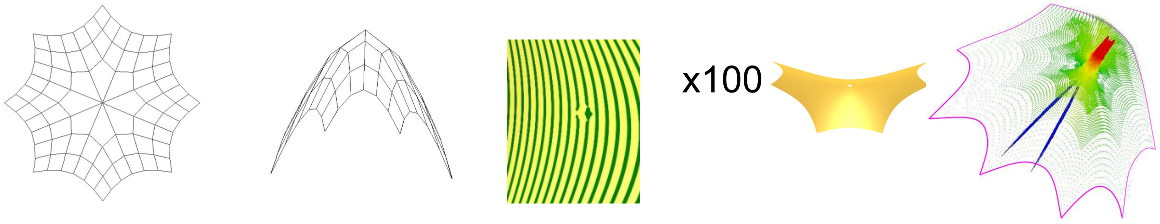
Sven Eberwein



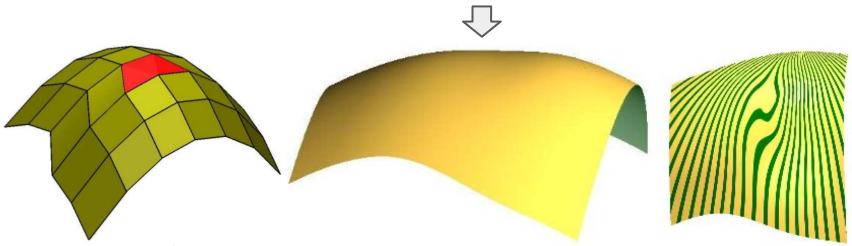
Pixar

Catmull-Clark Subdivision [CC78] is not class A

[KPR04] n>4:
convex input → saddle

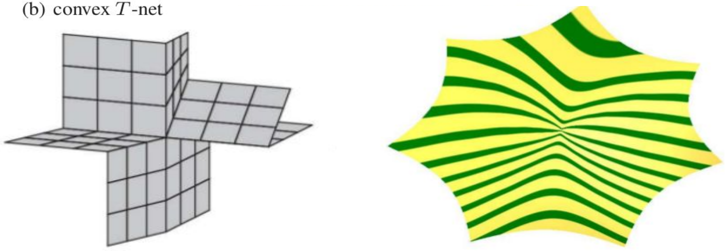


[KPP17] T-junction
convex input → flat

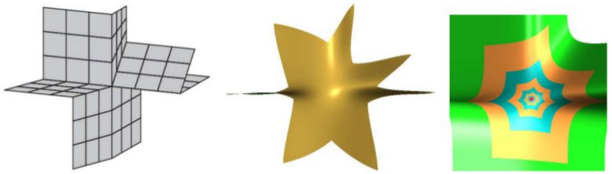


(b) convex $\dot{T}$-net

[KP14] n>5
saddle → pinched highlight lines

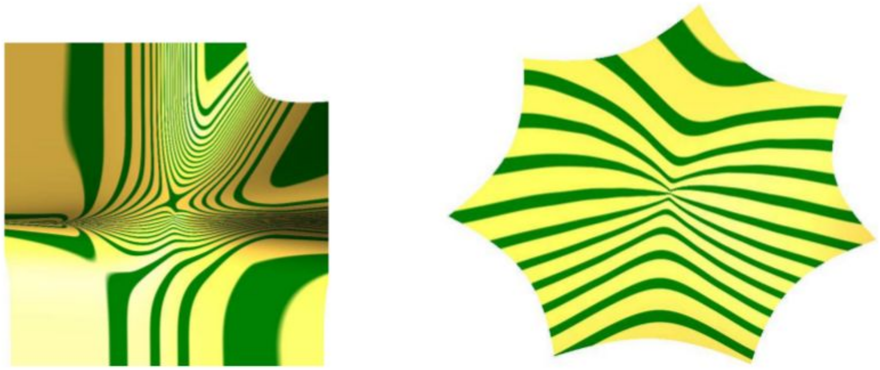


New idea: Guided Subdivision

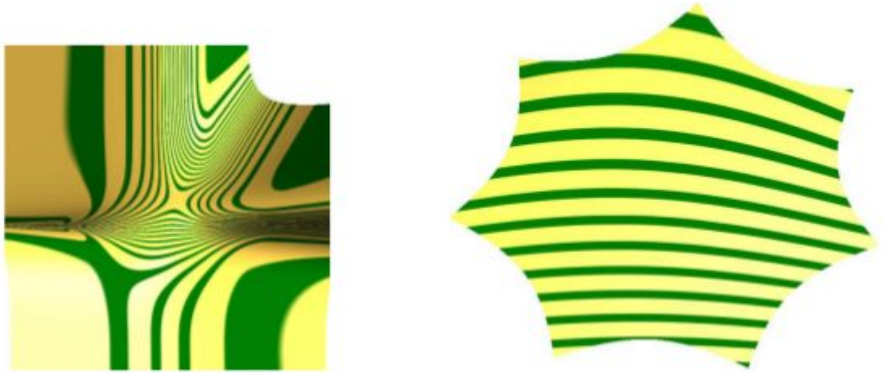


(a) control net (central valence $n = 7$)   (b) G^1 guide surface   (c) C^2 subdivision rings

Separate shape finding from mathematical constraints of final output surface



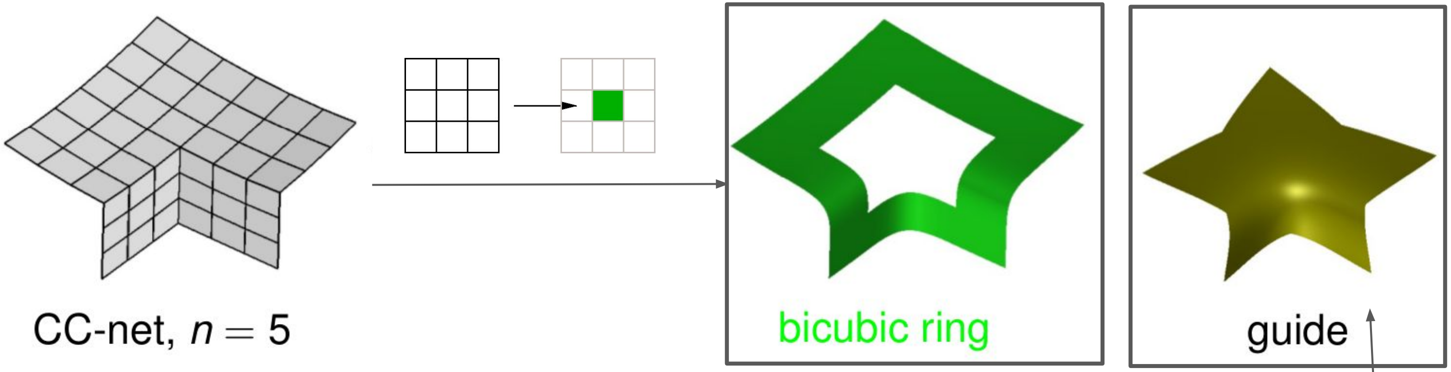
(e) Catmull-Clark: highlight line distribution and zoom



(f) Guided Subdivision: highlight line distribution and zoom

Evolution: Subdivision → Guided Subdivision

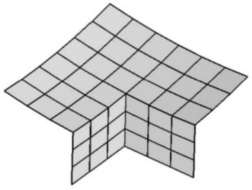
Separate shape-finding from mathematical constraints of final output surface



Eg: G-spline

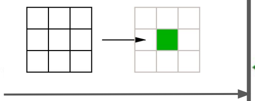
Subdivision → Guided Subdivision

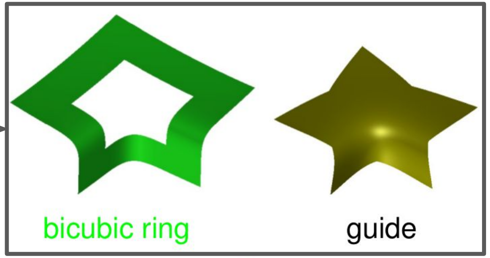
Separate shape-finding from mathematical constraints of final output surface

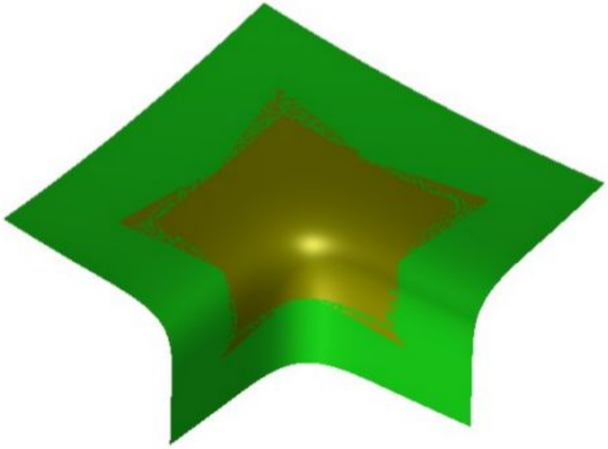


CC-net, $n = 5$

B-spline like



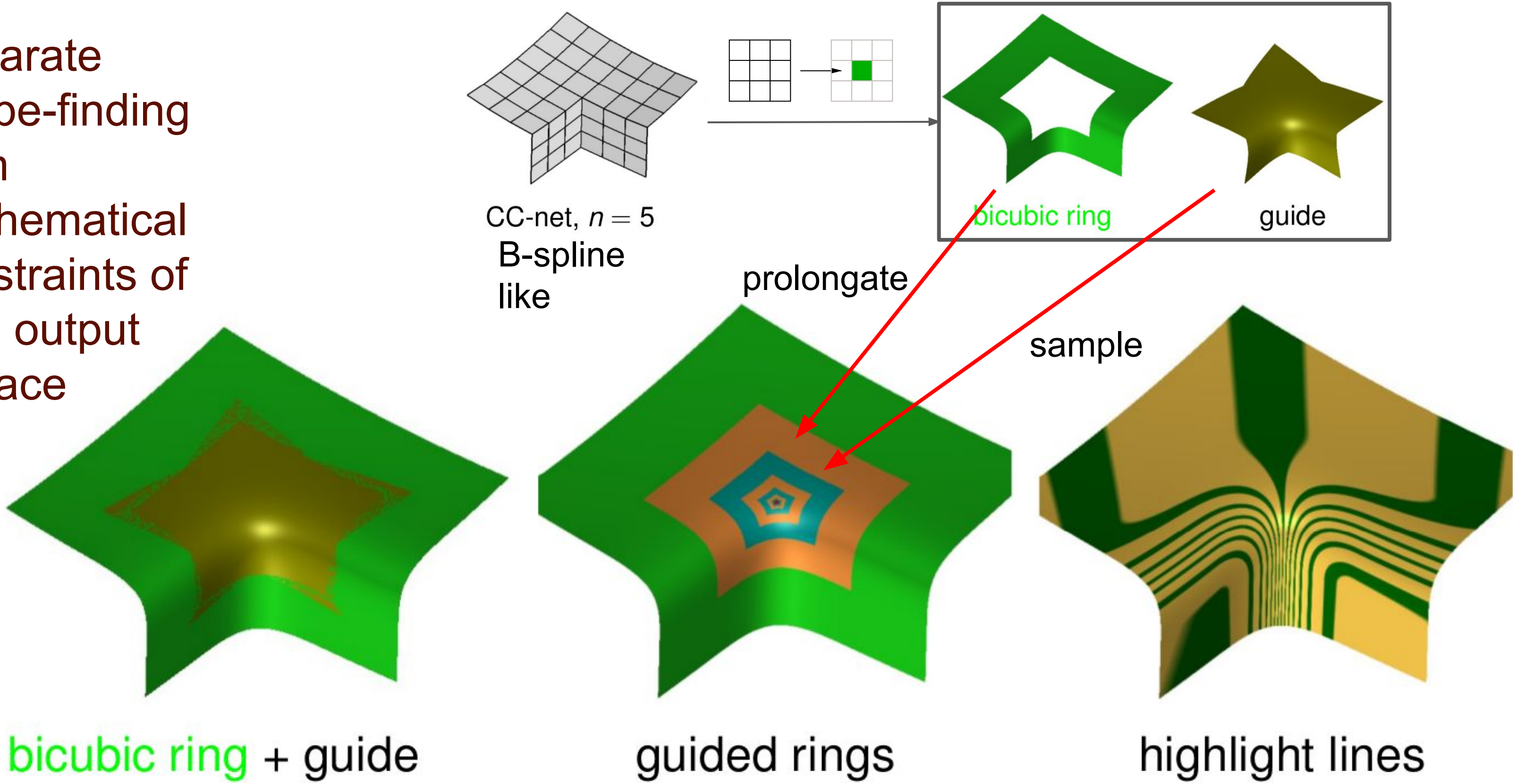




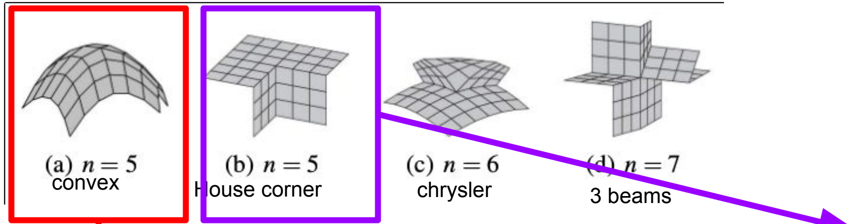
bicubic ring + guide

Subdivision → Guided Subdivision

Separate shape-finding from mathematical constraints of final output surface

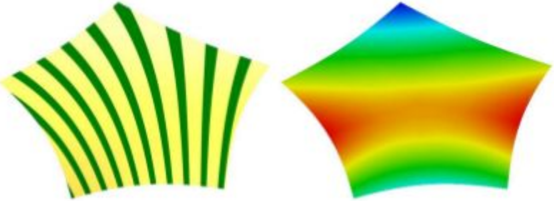


Guided Subdivision Shape obstacle course



(a) 4 bi-5 guided rings (b) highlight lines (c) Gauss curvature

(d) 4 bi-6 guided rings (e) 6 guided rings (f) highlight lines



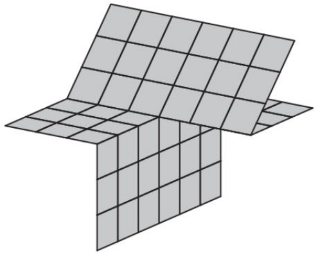
(g) zoom of (d): highlights and mean curvature

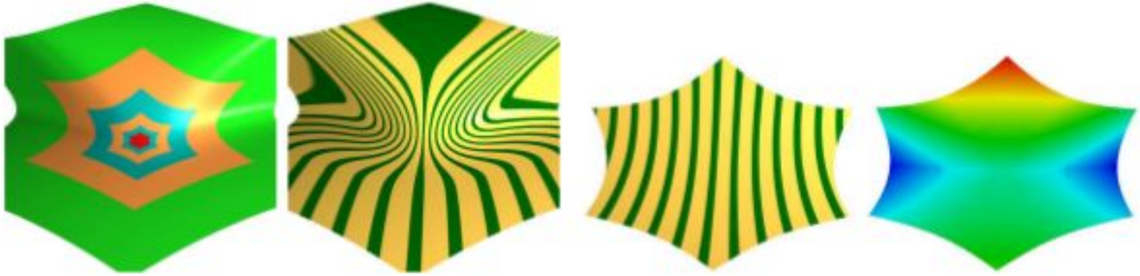
Guided Subdivision   Shape obstacle course

(d) $n = 7$

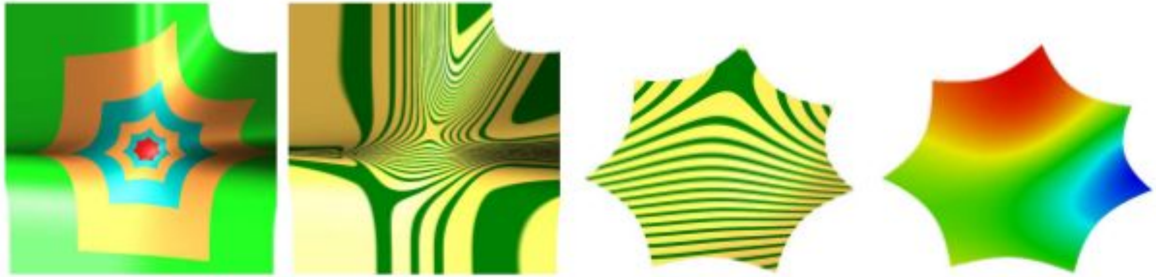
3 beams

2 beams





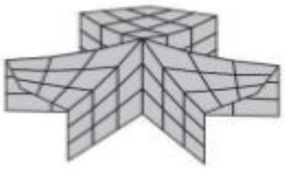
(a) 4 guided rings of degree bi-6   (b) zoom; mean curvature



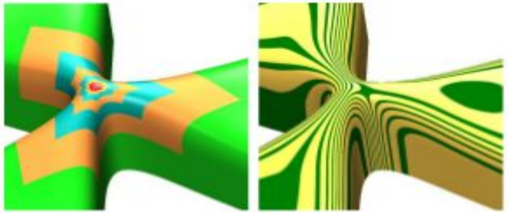
(c) 4 guided rings of degree bi-5   (d) zoom; Gauss curvature

Guided Subdivision

Shape obstacle course



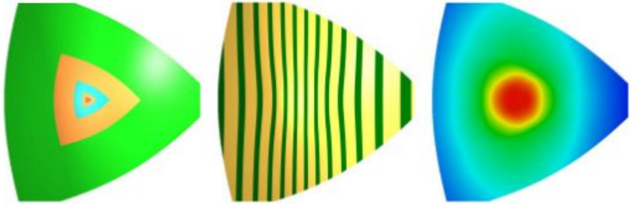
(f) $n = 9$



(b) 5 guided rings, $n = 9$, bi-5

(e) $n = 8$

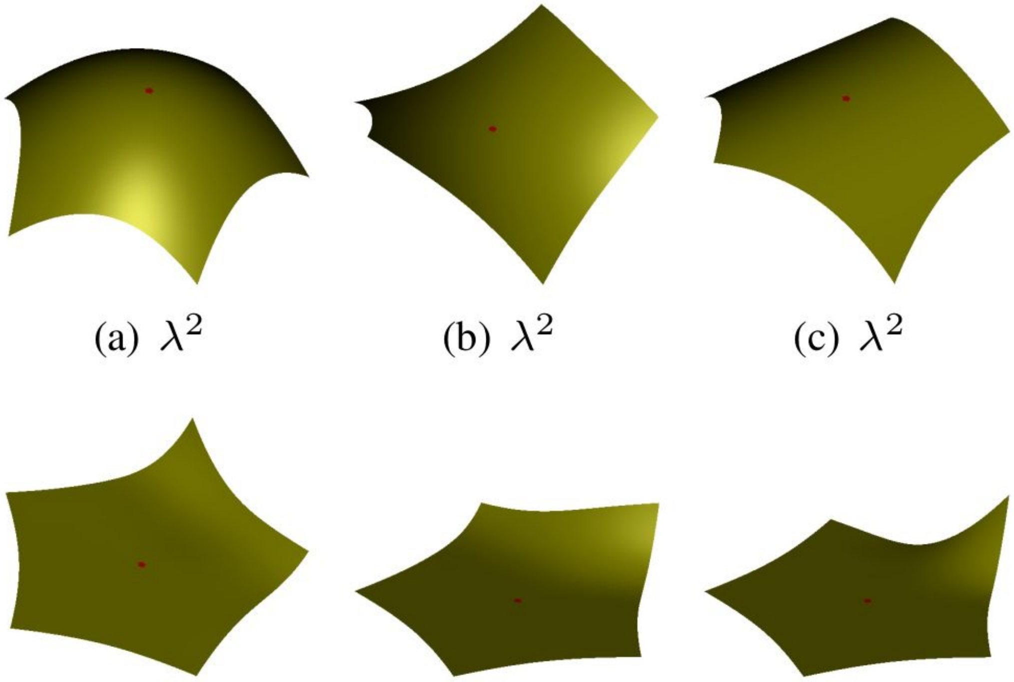
(a) 4 guided rings, $n = 8$, bi-5



(c) 3 guided rings, $n = 3$, bi-5, Gauss curvature

Guided Subdivision Eigenfunctions

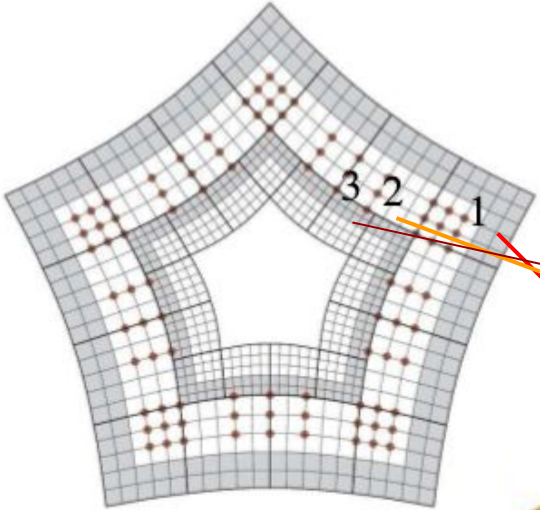
Stationary subdivision
with
larger, denser stencil
than CC



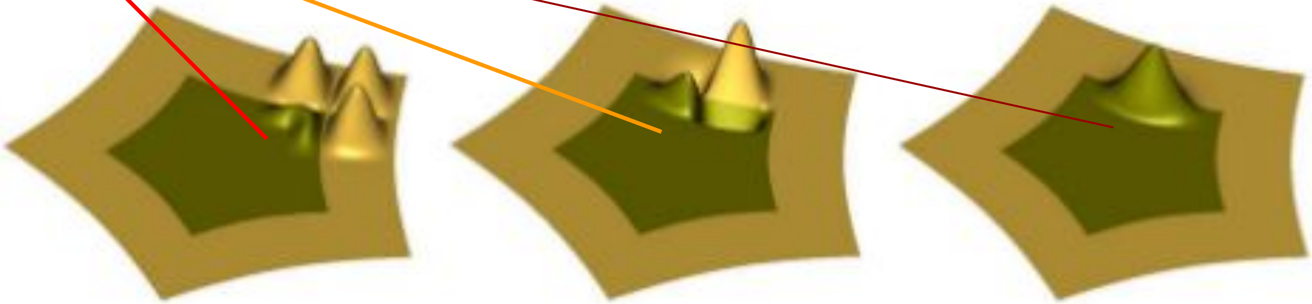
(a) λ^2 (b) λ^2 (c) λ^2

(d) λ^3 (e) λ^4 (f) λ^5

Guided Subdivision: refinable d.o.f. for analysis



(a) d.o.f. of bi-5 C^2 ring



(b) f_1 (c) f_2 (d) f_3

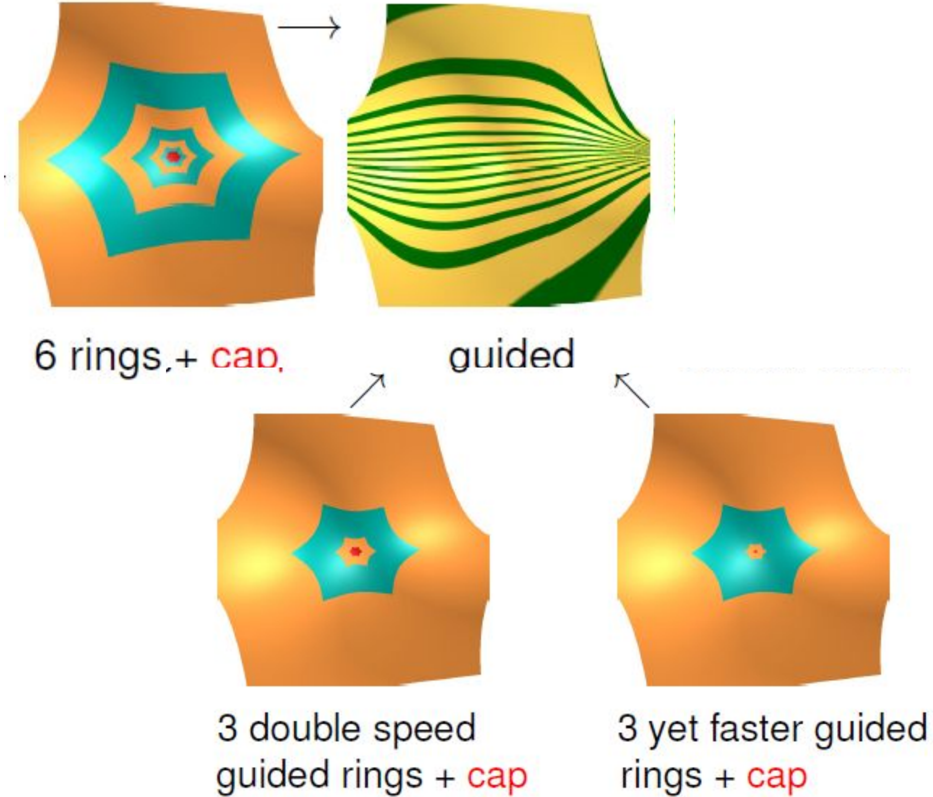
Guided Subdivision

Pro:
Semi-structured control nets
Uniform refinability: all transitions C^k
Good shape (guided)

Con:
Not finite

Accelerated Guided Subdivision

Can choose contraction speed!



Evolution

subdivision $\Rightarrow$ refinable C^2 surfaces;

guided subdivision $\Rightarrow$ good highlight line distribution;

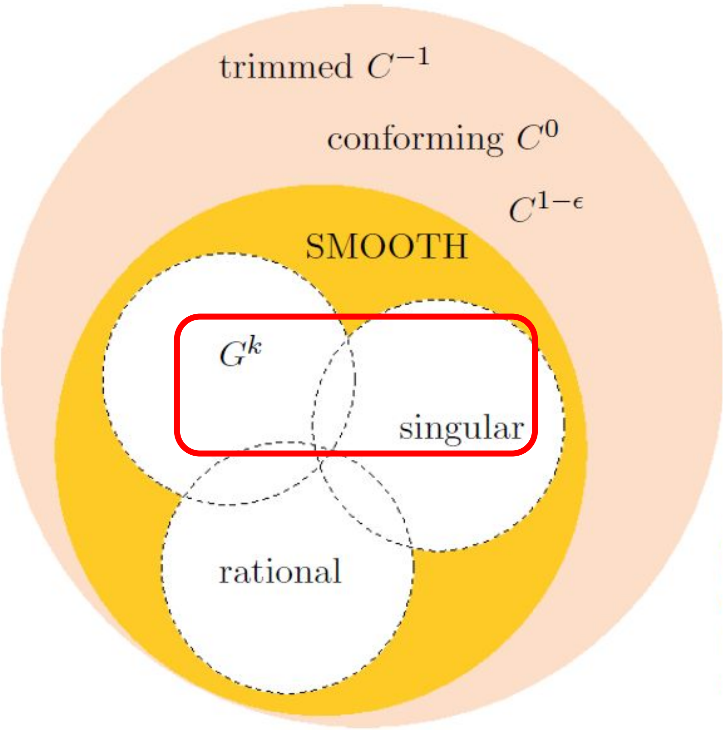
accelerated guided subdivision $\Rightarrow$ essentially finite!

Overview

- Irregularities
- Classification of splines on meshes with irregularities
    -
    - Semi-structured G-splines
    - Subdivision Surfaces
    -

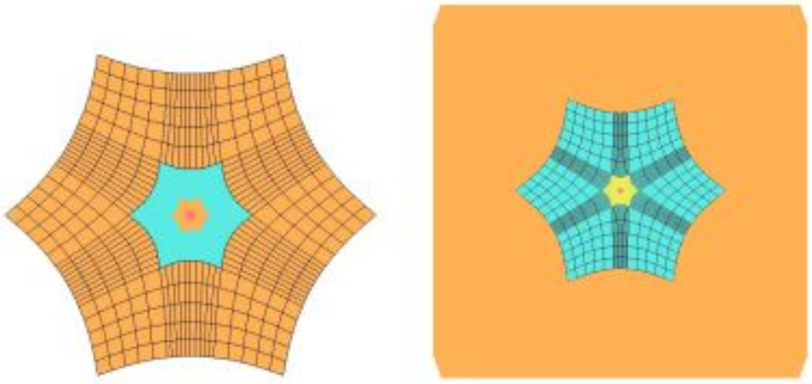
→ **hybrid for IGA**

Recommendation for gIGA:



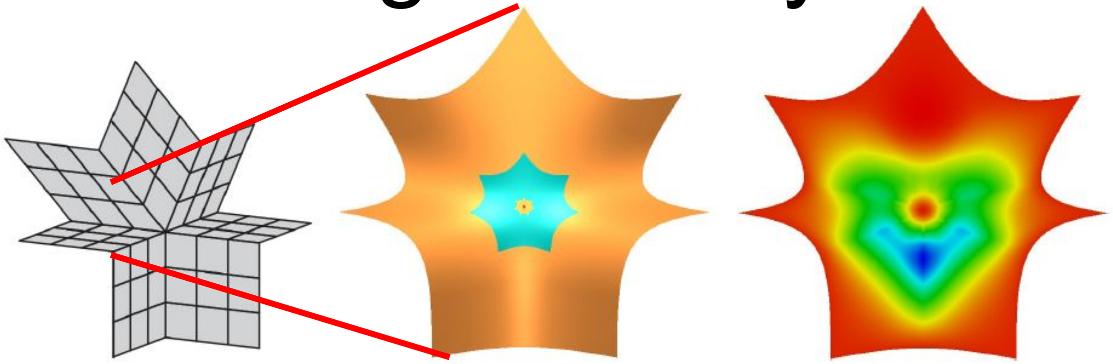
accelerated guided subdivision + G-cap

smaller than max refinement



(b) rapid contraction (*right*: zoom)

Semi- structured splines for design & analysis



Gaussian smile

Smooth polynomial spaces with semi- structured layout for design & analysis Jorg Peters

https://www.cise.ufl.edu/research/SurfLab/pubs.shtml

Fine