Iso-geometric analysis:
what works and what needs work

Jorg Peters with T Nguyen, Kestutis Karciauskas

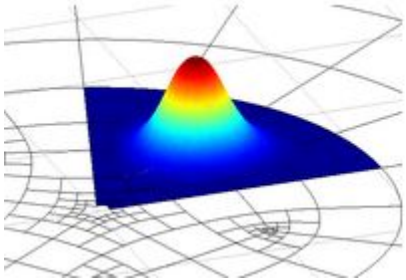
USACM 16

NSF CCF-0728797 (NIH EB005765)

Irregularities in manifolds

(a) input (b) surface pieces (c) capped bi-2

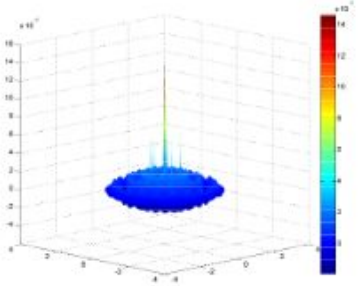
IGA -- what works: 1. Many constructions for irregularities



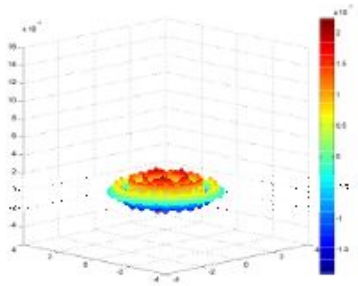
" A comparative study of several classical, discrete differential and isogeometric methods for solving Poisson's equation on the disk "

Thien Nguyen, Kestutis Karciauskas and Jörg Peters
Axioms, **2014** talk (2013)

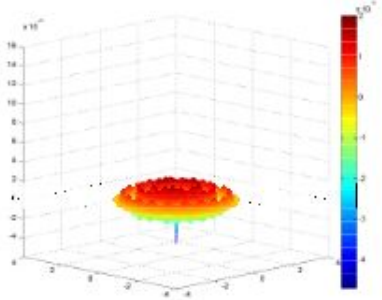
This paper outlines and qualitatively compares implementations of seven different



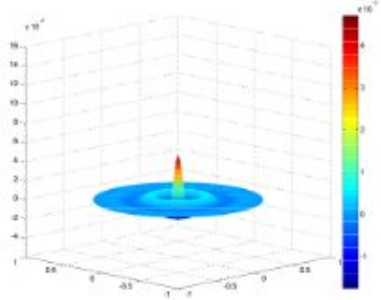
C^0, bi-3



G^1 bi-3/bi-5

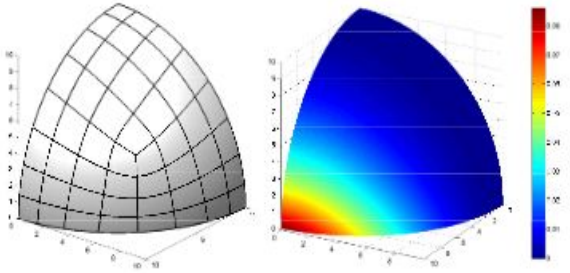


Catmull-Clark

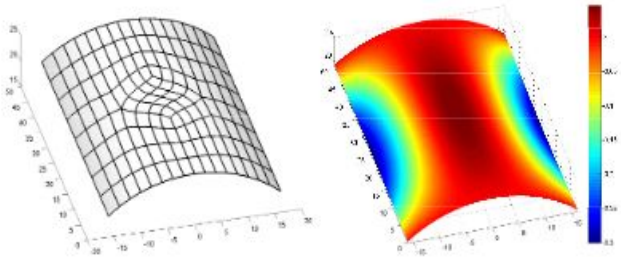


C^1 polar, bi-3

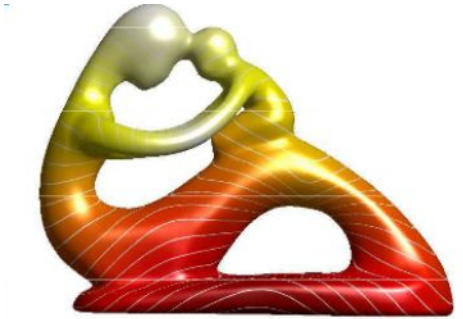
IGA -- what works: 1. irregularities on manifolds



(a) Octant of a spherical shell



(b) Scordelis-Lo roof thin shell

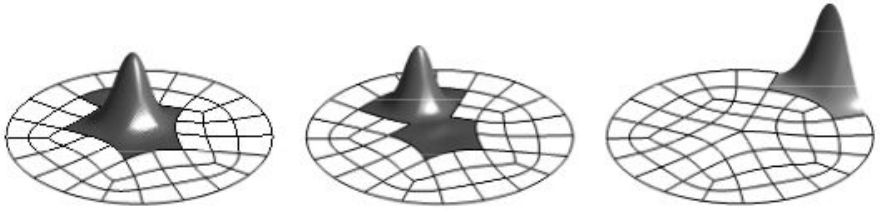


(b) Geodesics on a generalized spline surface via gIGA elements

Finite Element Obstacle course: meshingless analysis

IGA -- what works: 1. irregularities on manifolds

C^1 Basis functions for splines over irregular meshes



(d) $n = 3, 5$ (e) $n = 5, 3$ (f) $n = 4$, boundary

"C^1 *finite elements on non-tensor-product 2d and 3d manifolds*"

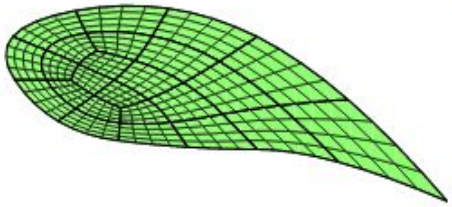
Thien Nguyen and Kestutis Karciauskas and Jörg Peters
AMC **2015** talk

Geometrically continuous (G^k) constructions naturally yield families of finite

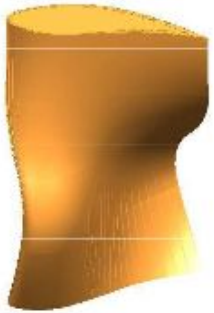
use G-spline for geometry & displacement function

g(eneralized) IGA = use splines over irregular meshes for geometry & displacement

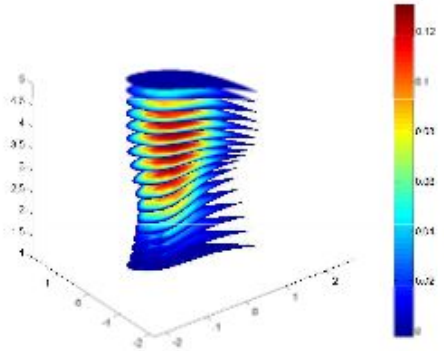
gIGA -- what works: 2. foliated trivariate manifolds



(a) A cross-section



(b) Turbine blade (solid)



(c) Computed solution

"C^1 *finite elements on non-tensor-product 2d and 3d manifolds* "

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Geometrically continuous (G^k) constructions naturally yield families of finite

gIGA -- what works: 3. General Theorem

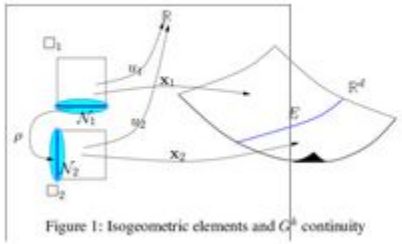


Figure 1: Isogeometric elements and G^k continuity

" Matched G^k-constructions always yield C^k-continuous isogeometric elements "

David Groisser and Jörg Peters

CAGD **2015** talk (2013)

G^k (geometrically continuous surface) constructions were developed to create

In any number of variables, (any degree), for any smoothness,...

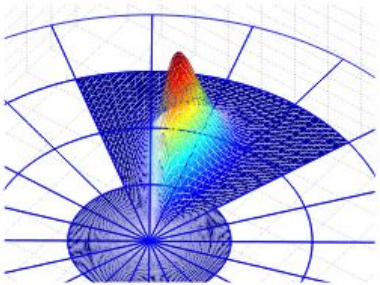
Iso-geometric analysis:
what needs work ?

Jorg Peters with T Nguyen, Kestutis Karciauskas

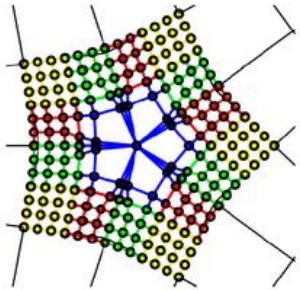
USACM 16

IGA -- what needs work: 1. Elegant nested G-refinement

Subdivision, polar and singular parameterization are nestedly refinable:



A C^1 polar basis function



" Refinable C^1 spline elements for irregular quad layout "

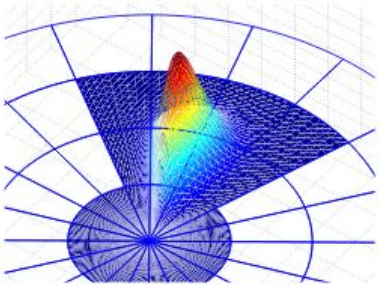
Thien Nguyen and Jörg Peters

CAGD, **2016** talk

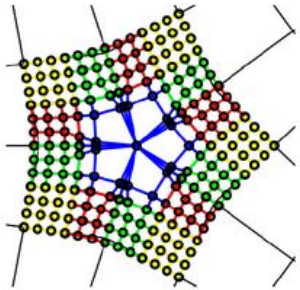
Building on a result of U. Reif on removable singularities, we construct C^1 bi-3 splines that may include irregular points where less or more than four tensor-product patches meet. The resulting space complements PHT splines, is refinable and the refined spaces are nested, preserving for example surfaces constructed from the splines. As in the regular case, each quadrilateral has four degrees of freedom, each associated with one spline and the splines are linearly independent. Examples of use for surface construction and isogeometric analysis are provided.

IGA -- what needs work: 1. Elegant nested G-refinement

Subdivision, polar and singular parameterization are nestedly refinable:



A C^1 polar basis function



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How to nestedly refine G-splines ? (exists, but not yet elegant).

IGA -- what needs work: 2. Speed

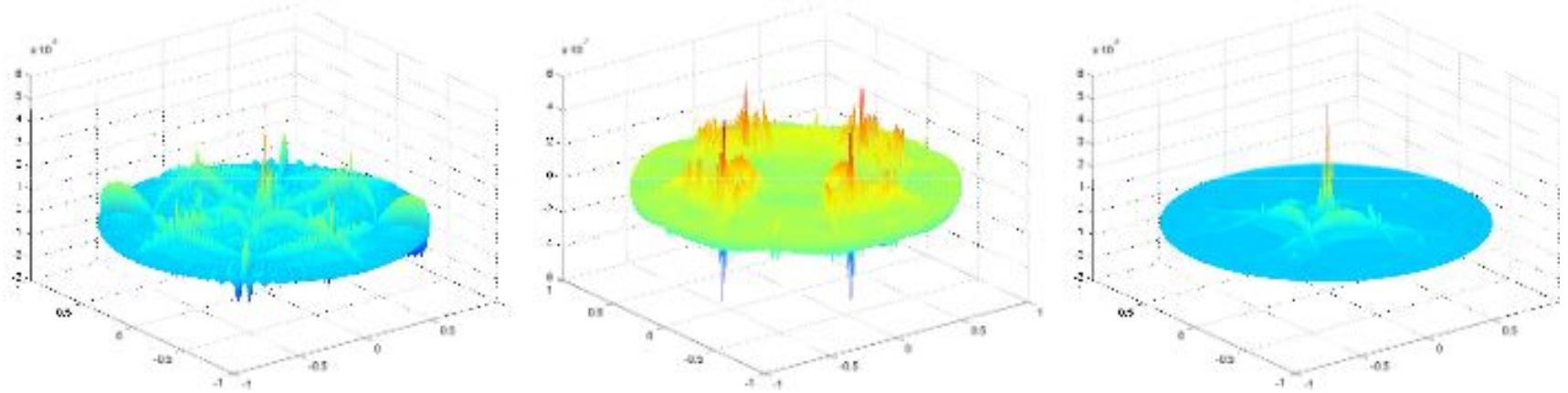
Many good approaches for generating the stiffness matrix [Mantzaflaris, Juettler,.Sangalli, ..] but

Challenge: real-time interactive cloth or volumetric for surgery simulation.

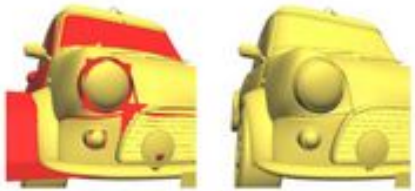
IGA -- what needs work: 3. Max-norm convergence

L2 perfect convergence (unless locking = smoothness too high)

but not max-norm? Approximation theory.



IGA -- what needs work: 4. Trimmed patches



" Correct resolution rendering of trimmed spline surfaces "

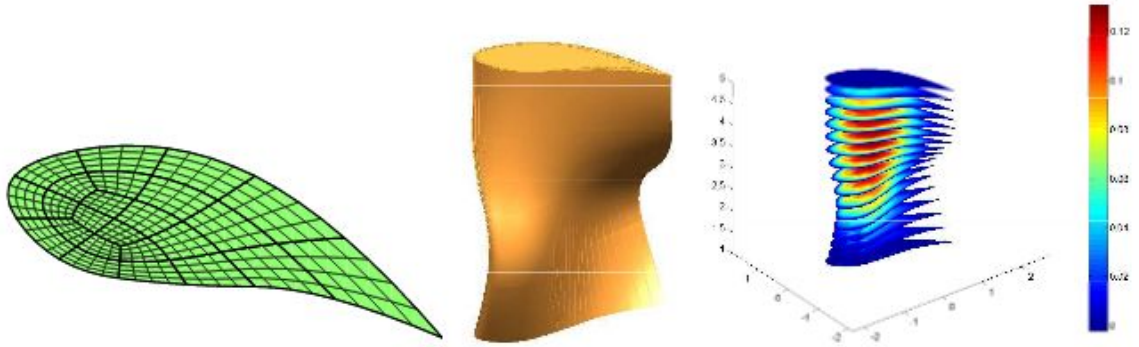
Ruijin Wu and Jörg Peters

Computer-Aided Design, (SPM 14: 1st prize, Best paper award) **2014** talk

Current strategies for real-time rendering of trimmed spline surfaces re-approxim

IGA -- what needs work: 5. C1 for all trivariate manifolds

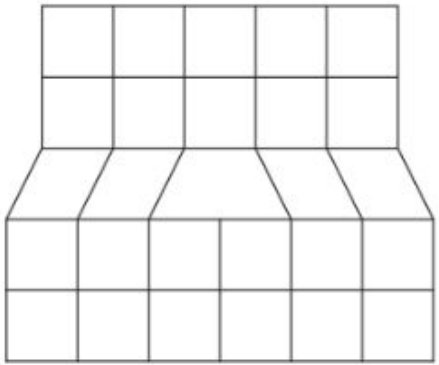
Not just foliated….



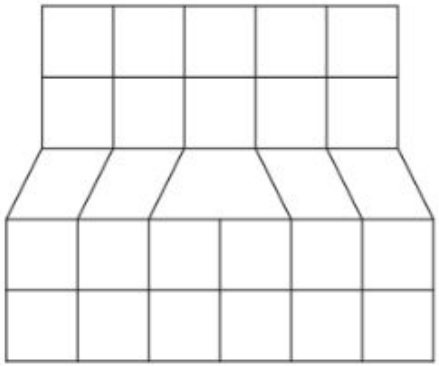
(a) A cross-section (b) Turbine blade (solid) (c) Computed solution

4 cubes meeting, … (few cases)

IGA -- what needs work: 6. T-junctions



IGA -- what needs work: 6. T-junctions



T-splines (in fact all hierarchical splines) fail to produce a smooth parameterization for simple meshes (no irregularities) with isolated single T-junctions [KPP 16] !

IGA -- what needs work: 7. non-NURBS approaches

Do we gain from box-splines & co?

ELSEVIER

Computer-Aided Design

Available online 1 September 2016

In Press, Corrected Proof — Note to users

Splines over regular triangulations in numerical simulation ☆

Francesca Pelosi[a], Carlotta Giannelli[b], Carla Manni[a], Maria Lucia Sampoli[c], Hendrik Speleers[a]

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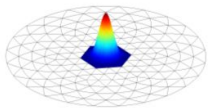
Isogeometric analysis on Generalized 3-Direction Triangulations

Thien Nguyen Jörg Peters[1]

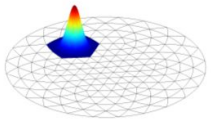
[1]Department of Computer and Information Science and Engineering
University of Florida

2015 SIAM Conference on Geometric and Physical Modeling

$$b_i(\mathbf{x}_\alpha) := N_{\alpha,i} \circ \mathbf{x}_\alpha^{-1}$$

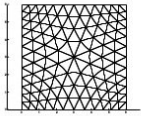


(a) at ordinary point

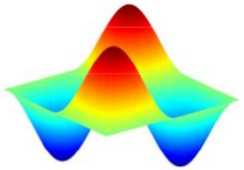


(b) at extraordinary point, $n = 5$

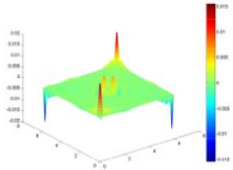
Figure: Isogeometric elements.



(e) The domain



(f) The computed solution



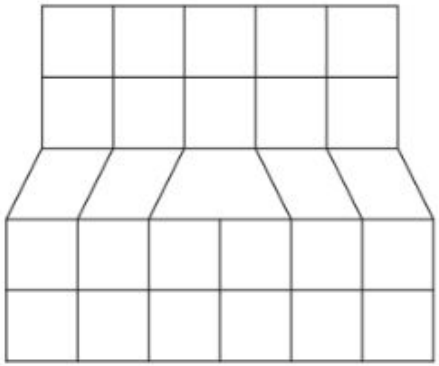
(g) Error plot

IGA -- what needs work

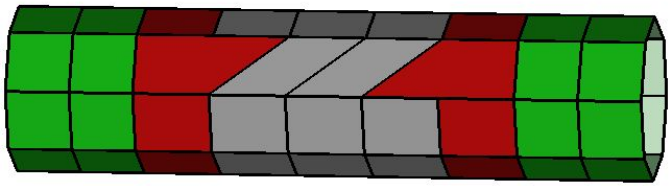
1. Elegant nested G-refinement
2. Interactive speed for higher-order
3. Optimal max-norm convergence at irregularities
4. Trimmed patches
5. General C1 trivariate with irregularities
6. T-junctions
7. non-NURBS box splines

Thank you

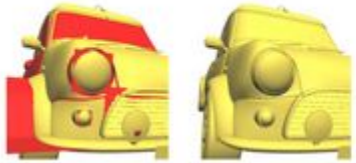
IGA -- what needs work: 6. T-junctions



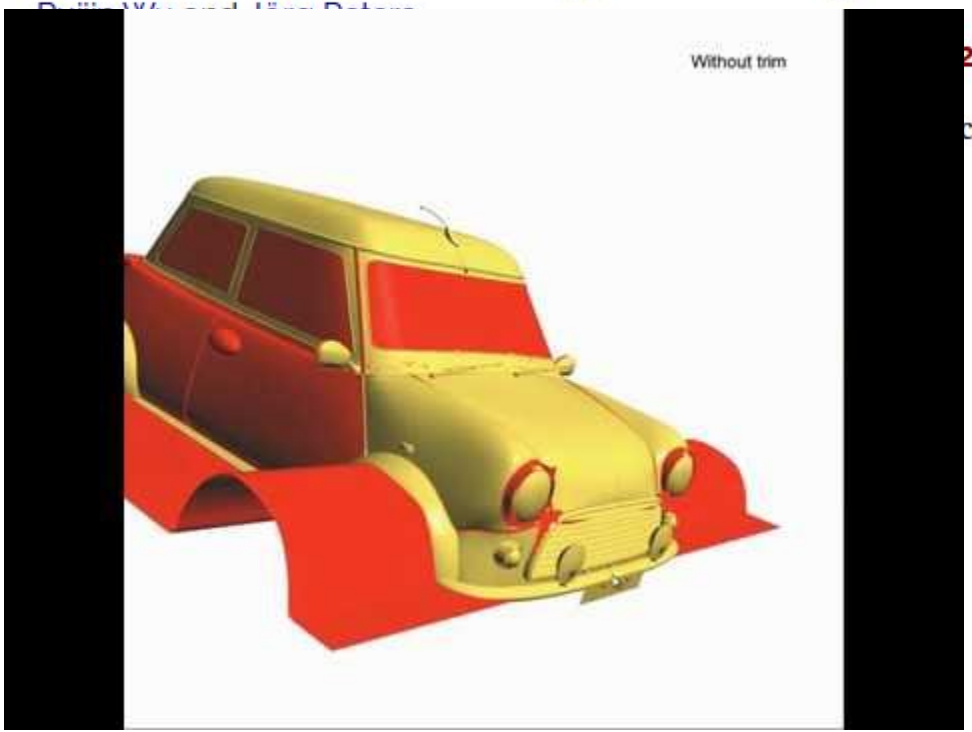
T-splines (in fact all hierarchical splines) fail to produce a smooth parameterization for this simple mesh [KPP 16] !



IGA -- what needs work: 4. Trimmed patches



" Correct resolution rendering of trimmed spline surfaces "



2014 talk

ces re-approximate