Sample Test 3 cot4501 Numerical Analysis

ID:

Name:

No computers or calculators. 1 page of notes.
If you need to make a simple, reasonable assumption to arrive at an answer then state any such assumption.
Write answers cleanly on the space provided. Use the back of the previous page if more space is needed.

1   Constrained Optimization

Find the *minimum* of

$$\min_{x_1,x_2} \quad (x_1 + x_2)x_1$$
$$\text{s.t.} \quad x_1^2 - x_2 = 0$$

- (15 points) Use the substitution approach.

- (20 points) Use the Lagrangian approach.

Interpolation

1. (7 points)

   Is it ever possible for two distinct polynomials to interpolate the same n data points? Explain.

2. (16 points) Determine the interpolating polynomial for the points $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ in (i) monomial (power) form, (ii) Lagrange form, (iii) Newton form, (iv) Bernstein-Bézier form. How does the result differ?

3. (12 points) Choose among the piecewise cubic Hermite interpolant, the interpolating polynomial in Lagrange form and in Newton form. Explain your choice.

   (a) If the interpolant is to be maximally smooth and the derivative easy to evaluate.

   (b) If the interpolant is to be most easily computed.

   (c) If the interpolant is to be easily computed and match derivatives.

2 Quadrature and Numerical Differentiation

- (7 points) True or false? Explain.
  Since it is based on polynomial interpolation of one degree higher, the trapezoid rule is always more accurate than the midpoint rule.

- (12 points) Compute $\int_0^1 (t - 1/2)^3$ with the midpoint rule, Simpson's rule and the Bernstein-Bézier form.

- (6 points) When computing the Newton form of an interpolant one computes divided differences. Illustrate the relation between the derivative (say at $t = 0$) and the divided difference (for equally spaced sample points about $t = 0$) for the polynomial $p(t) := t^2$?