pre-conditioning = scaling

$\boxed{\text{do page 58 ff}}$

Sparse matrix

$\longrightarrow$ wiki   space curves

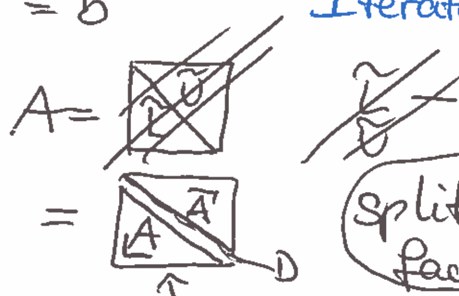
$$Ax = b$$

answer x_0 $b - Ax_0 = \underbrace{\text{residual}}_{r_0} \neq 0$

$Ax = r_0$ $\boxed{\text{iteration}}$

answer x_1 $x_0 + x_1 + \dots$

$x_0 = 0$ $Ax = b$ Iterative Solvers of $Ax=b$



split not factor

row index > col index

$$(\lfloor A + D)\,x = b - \underbrace{\bar{A}\,x_0}_{O(n^2)}$$

back solve for $x \xrightarrow[\text{yields}]{} x_1$

$$(\lfloor A + D)\,x_{i+1} = b - \bar{A}\,x_i$$

if A is diagonally dominant then $x_{i+1}, x_{i+2}, \dots \longrightarrow$ solution

$$\{x_i\}_{i=0..\infty} \xrightarrow{\text{converges}} x^* = [A^{-1}b]$$

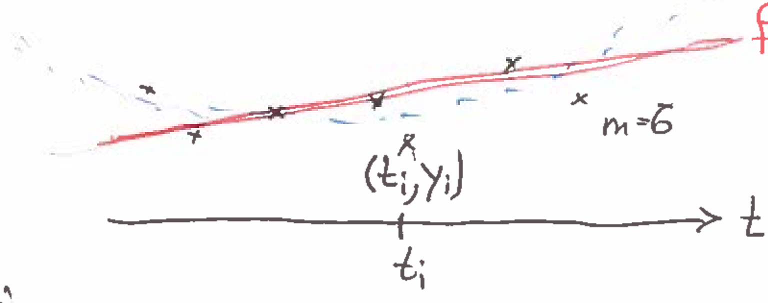
Gauss-Seidel

Gauss - Seidel $\quad \triangleright \quad x_{i+1} = b - \nabla x_i$

Jacobi iteration $\quad \diagdown \quad x_{i+1} = b - \nabla x_i$

Successive Over Relaxation

diagonally dominant



$$f(t,x) = \sum_{j=0}^{n} x_j \phi_j(t)$$

for example

$$f = x_0 t^0 + x_1 t^1 + x_2 t^2 + \cdot x t^n$$
$$= \sum_{j=0}^{n} x_j t^j$$

$n=1$: Linear polynomial

$n=2$ / quadratic poly- - -

$$\underset{x=(x_0, x_1, \ldots x_n)}{\min \ \cancel{\text{find}}} \sum_{i=0}^{m} \left(y_i - f(t_i, x)\right)^2$$

$$\cancel{\#\#} = \min_{x} \|y - f(x)\|_2^2$$

Linear least squares:

NOT

f depends linearly on x

$$f = (x_0 | \cdots | x_n) \begin{pmatrix} \phi_0(t) \\ \vdots \\ \phi_n(t) \end{pmatrix}$$

$$= x_0 \phi_0(t) + \ldots + x_n \phi_n(t)$$

non-linear least squares

$$\sum x_0 e^{x_1 \frac{2\pi}{n} t \sqrt{-1}}$$

$$\min_{x} \|\underset{\underset{b}{\parallel}}{y} - \underset{\underset{A}{\parallel}}{\left(\phi_j(t_i)\right)} x\|^2 \Rightarrow A^T(b - Ax) = 0$$

Vandermonde

$$A^T A x = A^T b$$

Normal Equ's