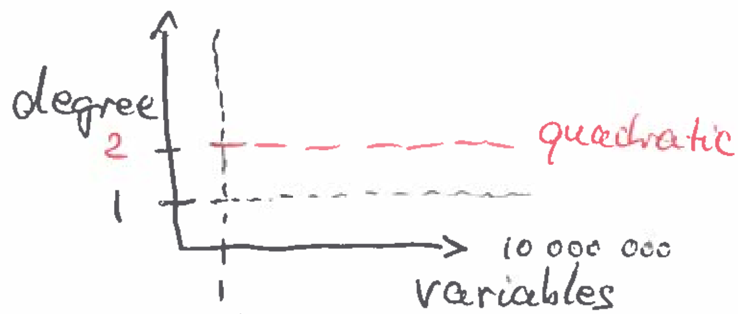
Chapter 6

Optimization

First the big $\boxed{\text{pic}}$

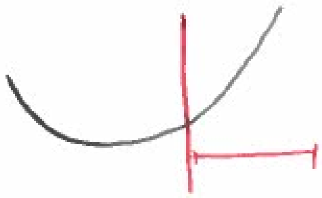


(A) easy:

$\min f(x)$
when $f \in C^2$

$C^k \cong$ space of k times continuously differentiable functions

(B) hard:
constraints



$A\begin{pmatrix}x\\y\\z\end{pmatrix} = b$

(1) linear constraint with $=$ $\left[\text{constraint } \boxed{A}\begin{pmatrix}x\\ \\ \end{pmatrix} = \square\right]$

$(A_1\ A_2)\begin{pmatrix}x_1\\x_2\end{pmatrix} = \square$

$A_1 x_1 = \square - A_2 x_2$

$x_1(x_2)$

$$\boxed{\min_{x_1 x_2} f(x_1, x_2) \quad Ax = b}$$

$\Updownarrow$

$$\boxed{\min_{x_2} f(x_1(x_2), x_2)}$$

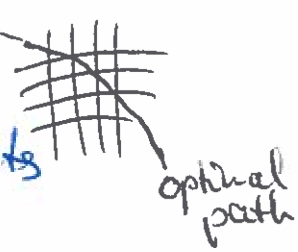
"simulated annealing"

(2) non-linear constraint ○ $x^2+y^2=1$

$\longrightarrow$ linearize

(3) integer programming

(4) inequality constraints



$$A = \begin{pmatrix} 2 & 8 \\ 2 & 2 \end{pmatrix} = 2 \underbrace{\begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}}_{B}$$

$$Av = \lambda v = \lambda I v$$

$$\Leftrightarrow \underbrace{(A - \lambda I)}\, v = 0 \quad \uparrow_{\neq 0}$$

$$0 = \det(A - \lambda I) = \det \begin{pmatrix} 2-\lambda & 8 \\ 2 & 2-\lambda \end{pmatrix} =$$

$$= (\lambda - 2)^2 - 16$$

$$\lambda = \begin{cases} -2 \\ 6 \end{cases}$$

$$(A - (-2)I) = \begin{pmatrix} 4 & 8 \\ 2 & 4 \end{pmatrix} \qquad 4v_1 + 8v_2 = 0$$

$$\lambda = -2,\ v = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\lambda = 6,\ v = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

power method:

$$x^1 = A x^0 \qquad x^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x^1 \to \frac{x^1}{\|x^1\|_2}$$

$$\downarrow$$

$$x^\infty \to$$

inverse power: ("$x^1 = A^{-1} x^0$")

$$A x^1 = x^0$$

$$x^1 = \frac{x^1}{\|x\|}$$

$$\frac{x^T A x}{x^T x} \qquad Av = \lambda v \qquad -Av = -\lambda v$$

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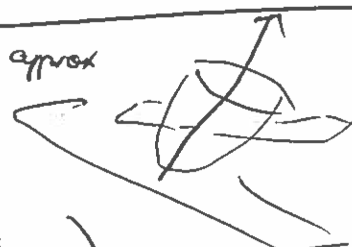
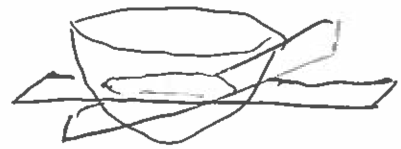
quadratic approx

$$\min f(x)$$

$$\left(\text{s.t. } g(x) = 0 \right)$$

linearize





$Ax - b = 0$

$$f: \mathbb{R}^m \to \mathbb{R}^n \quad \underbrace{\nabla f}_{J} = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}, \ldots, \frac{\partial f}{\partial x_m} \right) = F$$

$$F(x) = 0 \Rightarrow \qquad F(x^i) = -\nabla F(x^i) \cdot h$$

$$\underbrace{\nabla F}_{H} := \begin{pmatrix} \frac{\partial^2 f}{\partial x \partial x_1} & \frac{\partial^2 f}{\partial x \partial x_1} & \cdots \\ & \left(\frac{\partial^2 f}{\partial x_i \partial x_j} \right) & \end{pmatrix}_{\substack{i = 1..m \\ j = 1..m}}$$