$Ax = b \qquad x \in \mathbb{R}^n$

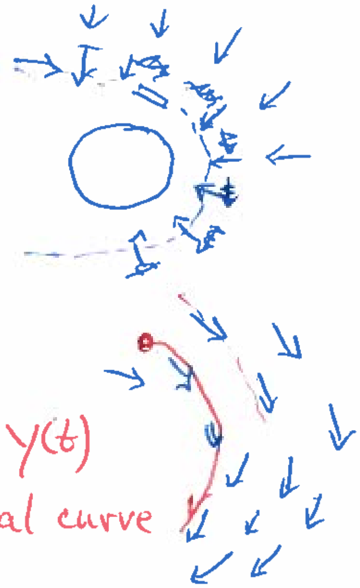
$\begin{matrix} y \\ t \end{matrix} \rightarrow f(t) \qquad c \in \mathbb{R}^n$

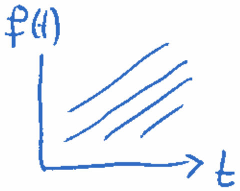
$y' = \underline{f(y,t)} \qquad t \in \mathbb{R}$ <u>ODE</u>

field (force)

$y(0) \leftarrow$

$f(y,t) = y'$





$y' = t$

$(t, f(t))$ graph of function

$t \rightarrow [x(t), y(t)]$ plane curve

(x, y, t)

<u>Ex:</u> $x(t) = \cos(t)$
$y(t) = \sin(t)$

$y''(t) = \text{force}(t) \longrightarrow \begin{cases} \tilde{y}'(t) = \text{force}(t) \\ y'(t) = \tilde{y}(t) \end{cases}$

$Y'(t) = \begin{pmatrix} \text{force}(t) \\ \tilde{y}(t) \end{pmatrix}$

in general

find coefficients of

$$y(t) = \sum_{i=0}^{n} \underline{\underline{y_i}}\, b_i(t)$$

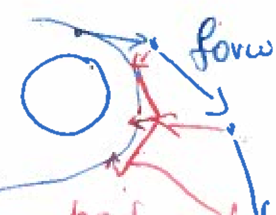
Euler 1st order

DERIVATION

$\cancel{f(y(t+h), t+h)\ f(y,t) + h}\ y(t+h) \approx y(t) + h \underbrace{y'(t)}_{f(y(t),t)} + \cancel{h^2}$

forward Euler $\boxed{y^+ = y + h f(y,t)}$

backward Euler $y^+ + y + h f(y^+, t^+)$ solve



$t^+ := t^{i+1};\ y^+ = y^{i+1}$
$t := t^i;\ y = y^i$

$\boxed{\text{R-k4}}$ Runge-kutta

<u>Initial Value Problem</u>