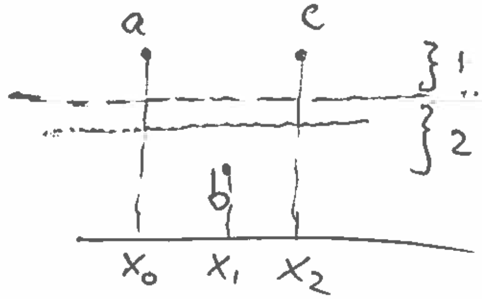
Linear least squares



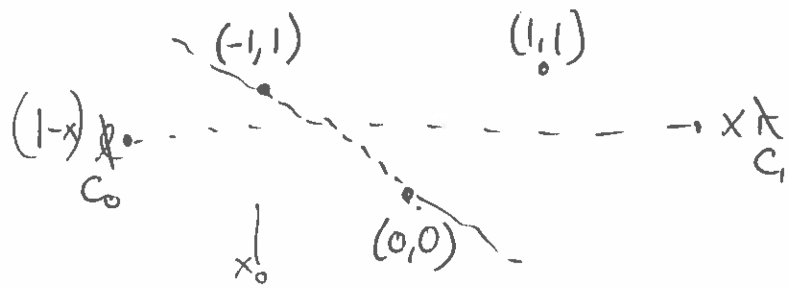
$$\min_{\ell} \left(a-\ell(x_0)\right)^2+\left(b-\ell(x_1)\right)^2+\left(c-\ell(x_2)\right)^2$$

$$\| \text{---} \|_2^2$$

$$\cdots\cdots \quad \| \qquad \|_\infty$$

Exercises (book) p42
1.1 (temperature)
1.10 $\frac{1}{1-x}-\frac{1}{1+x}$

Programming
1.5 Hopital's Rule
1.13 interest
1.18 $x_{k+1} = \cdots$



Equation of line: $(1-x_i)\,c_0 + x_i\,c_1 = y_i \qquad i=0,1,2$

$$\boxed{\begin{aligned}(1-(-1))\,c_0 + (-1)\,c_1 &= 1\\ (1-0)\,c_0 + 0\,c_1 &= 0\end{aligned}}$$

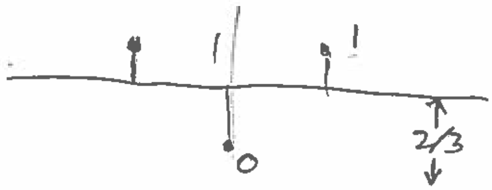
$$(1-1)\,c_0 + 1\,c_1 = 1$$

$$\underset{A}{\begin{pmatrix}2 & -1\\ 1 & 0\\ 0 & 1\end{pmatrix}}\underset{c}{\begin{pmatrix}c_0\\ c_1\end{pmatrix}} = \underset{b}{\begin{pmatrix}1\\ 0\\ 1\end{pmatrix}}$$

$$\min \underbrace{\|Ac-b\|_2^2}\, = \min\, (Ac-b)^{\overset{\text{transpose}}{T}}(Ac-b)$$

$$\boxed{0 = (Ac-b)^T A = A^T A c - b^T A}$$

$$A^T(Ac-b) + (Ac-b)^T A$$



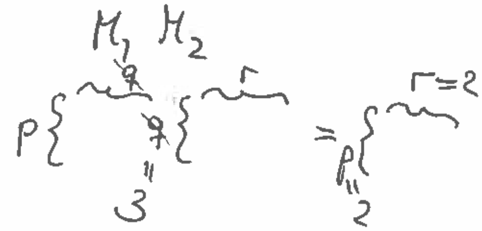
$$0 = A^T A c - A^T b$$

$$\begin{pmatrix} \quad \end{pmatrix} c = \begin{pmatrix} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} c_0 \\ c_1 \end{pmatrix}}_{c} = \underbrace{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}_{b}$$

$$A^T = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$c = \left(\tfrac{2}{3}, \tfrac{2}{3}\right)$$



$$\text{Error} = \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \ldots$$

Mo Jan 22 5pm	Lin Alg Review
Fr 26th	no class

mathematical	numerical
singular $\simeq$	ill-conditioned

Examples from page 7 of Lecture 2 ....

Remark:

$(b^T A)^T = A^T b$

so if b is a "column vector"
$A^T b$ is a "column vector"
$b^T A$ is a "row vector"

However:
we are differentiating with respect to one c_i at a time!

$(b^T A)(i) = (A^T b)(i)$

So when I collect
$(A^T A)(i,:) \setminus (b^T A)(i)$ it does not matter