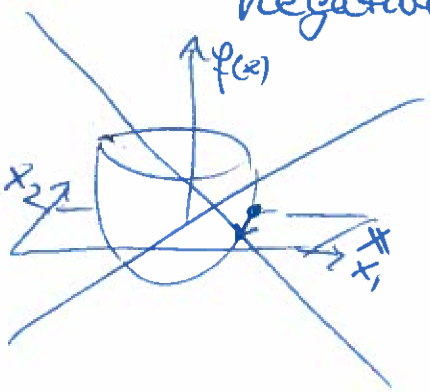
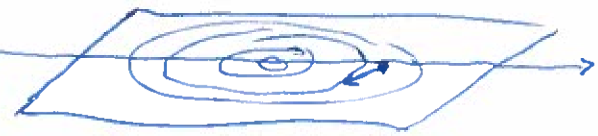
negative gradient $-\nabla f := -\begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_m} \end{pmatrix}$





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coefficient (weight) of ϕ_j

interpolant $f(t) = \sum_{j=1}^{n} x_j \phi_j(t) = x \cdot \phi$ $\qquad \phi := \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$

IN: $\boxed{\{t_i\}_{i=1..m}^{\text{sample}} \text{ choice of } \{\phi_j\}_{j=1..n} \ \{y_i\}_{i=1..m}^{\text{data}}}$

OUT: $\{x_j\}_{j=1..n}$

$i = 1..m \qquad y_i = x \cdot \phi(t_i) \qquad x := \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

$y = \underbrace{[\phi]}_{[\phi_j(t_i)]_{i=1..m,\, j=1..n}} x$

Ex: $5 + 3t + t^2$ (x_1, x_2, x_3; ϕ_2, ϕ_3)

$\phi_0 = 1$
$\phi_1 = t$
$\phi_2 = t^2$

monomial

power form of polynomial

$\phi_j(t) = (t)^j$

Vandermonde

full

$x = \phi \backslash y$

Lagrange form

$j=2, n=3$

$\phi = \begin{cases} \phi_2 = \dfrac{(\cdot - t_1)(\cdot - t_3)}{(t_2-t_1)(t_2-t_1)} \\ \phi_2(t_1) = 0 \qquad \phi_2(t_2) = 1 \\ \phi_2(t_3) = 0 \end{cases}$

[redacted]

$y = I x$

⊕ $\boxed{\text{easy factoring}}$

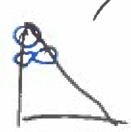
⊖ slow evaluation

Newton form ~~of~~

⊖ need to factor

⊕ nested multiplication

$= \boxed{\text{fast evaluation}}$



$\phi_j(\cdot) := \prod_{k=1}^{j-1} (t - t_k)$

$p(t) := \sum_{j=1}^{n} x_j \phi_j(t)$

y_1
$y_2 \quad \frac{y_2 - y_1}{t_2 - t_1} \quad f[t_1, t_2]$
$\vdots$
$y_n \quad \frac{y_n - y_{n-1}}{t_n - t_{n-1}}$

divided difference table