Solve for (40, 30 points)

(a) Quadratic objective function and linear constraints:

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) = (4x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2$$

subject to

$$\begin{aligned} x_1 + 3x_2 &= 0, \\ x_3 + x_4 - 2x_5 &= 0, \\ x_2 - x_5 &= 0. \end{aligned}$$

(b) Quadratic objective function and nonlinear constraints:

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) = 4x_1^2 + 2x_2^2 + 2x_3^2 - 33x_1 + 16x_2 - 24x_3$$

subject to

$$\begin{aligned} 3x_1 - 2x_2^2 &= 7, \\ 4x_1 - x_3^2 &= 11. \end{aligned}$$

Bonus problem (25 points) see simplex method

6.18 Use a library routine for linear programming to solve the following problem:

$$\max_{\boldsymbol{x}} f(\boldsymbol{x}) = 2x_1 + 4x_2 + x_3 + x_4$$

subject to the constraints

$$\begin{aligned} x_1 + 3x_2 + x_4 &\le 4 \\ 2x_1 + x_2 &\le 3 \\ x_2 + 4x_3 + x_4 &\le 3 \end{aligned}$$

and

$$x_i \ge 0, \quad i = 1, 2, 3, 4.$$

Solve for (30 points) [tips for (b) in class]

7.4 An experiment has produced the following data:

t	0.0	0.5	1.0	6.0	7.0	9.0
y	0.0	1.6	2.0	2.0	1.5	0.0

We wish to interpolate the data with a smooth curve in the hope of obtaining reasonable values of y for values of t between the points at which measurements were taken.

(a) Using any method you like, determine the polynomial of degree five that interpolates the given data, and make a smooth plot of it over the range $0 \le t \le 9$.

(b) Similarly, determine a cubic spline that interpolates the given data, and make a smooth plot of it over the same range.

(c) Which interpolant seems to give more reasonable values between the given data points? Can you explain why each curve behaves the way it does?

(d) Might piecewise linear interpolation be a better choice for these particular data? Why?