

$$A := \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{pmatrix} = A - \lambda I$$

$$\lambda = 2$$

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$\det A = 9 - 6\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 6\lambda + 8$$

$$\lambda_{1,2} = \frac{6 \pm \sqrt{36 - 4 \cdot 8}}{2}$$

Find  $\lambda$ :  $\begin{matrix} B \\ \underbrace{A - \lambda I} \end{matrix}$  is not of full rank

$$= 3 \pm 1 = \begin{cases} 4 \\ 2 \end{cases}$$

Then there exists  $x \neq 0$ :  $(A - \lambda I)x = 0$

~~characteristic polynomial~~

$A^k \overset{\substack{\text{apply } k \text{ times} \\ \uparrow \\ \text{i-th eigenvector}}}{v_i} = A^{k-1} \lambda_i v_i = \lambda_i^k v_i$

$$x = \sum_{i=1}^n \alpha_j v_j = \alpha V$$

$$A^k x = A^{k-1} A x = A^{k-1} \sum \alpha_j \underbrace{\lambda_j v_j}_{A v_j} = A^{k-1} \left( \alpha_1 \lambda_1 v_1 + \sum_{j=2}^n \alpha_j \lambda_j v_j \right)$$

$$= \lambda_1^k \left( \alpha_1 v_1 + \sum_{j=2}^n \alpha_j \left( \frac{\lambda_j}{\lambda_1} \right)^k v_j \right)$$

$$\lim_{k \rightarrow \infty} A^k x = \lambda_1^k \alpha_1 v_1 \quad \text{if } |\lambda_1| > |\lambda_j| \quad j > 1 \quad (A - \lambda_1 I)$$