Test B Geometric Modeling

Name:

IMPORTANT: You may have to make simple, reasonable assumptions to arrive at an answer. State any such assumption.
An answer 'yes' or 'no' is worth 0 points if it does not explain the reasoning.
Write answers cleanly on the space provided. Use the back of the previous page if more space is needed.

1 Bounding Constructs

[2+2 points] We define three curve segments, $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ by their Bernstein-Bézier coefficients

$$\mathbf{c}_1 : \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}3\\2\end{smallmatrix}\right], \left[\begin{smallmatrix}5\\2\end{smallmatrix}\right], \qquad \mathbf{c}_2 : \left[\begin{smallmatrix}-4\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}-2\\2\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}2\\2\end{smallmatrix}\right] \qquad \mathbf{c}_3 : \left[\begin{smallmatrix}0\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}2\\2\end{smallmatrix}\right], \left[\begin{smallmatrix}4\\2\end{smallmatrix}\right].$$

- Use *bounding constructs* to prove or disprove that the two curve segments $\mathbf{c}_1$ and $\mathbf{c}_2$ intersect.

- (*) Use *bounding constructs* to prove or disprove that the two curve segments $\mathbf{c}_3$ and $\mathbf{c}_2$ intersect.

2 curves, general

(3 points – somewhat atypical as a test question) The V-shaped function is convex. Show that one cannot, in general, expect that there exists a function that is convex, interpolates several points from a convex function and is C^1, regardless of the representation of the curve.

3 Differential Geometry

(6 points) For the curve with BB-coefficients

$$\left[\begin{smallmatrix}-1\\-1\\-1\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\-1\\-1\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\1\\-1\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\1\\1\end{smallmatrix}\right], \tag{1}$$

determine the tangent, the osculating plane, the main normal, the binormal, the curvature, and the torsion at $\left[\begin{smallmatrix}-1\\-1\\-1\end{smallmatrix}\right]$.