Test A Geometric Modeling

Name:

IMPORTANT: You may have to make simple, reasonable assumptions to arrive at an answer. State any such assumption.
~~An answer 'yes' or 'no' is worth 0 points if it does not explain the reasoning.~~
Write answers cleanly on the space provided. Use the back of the previous page if more space is needed.

1 Bézier form

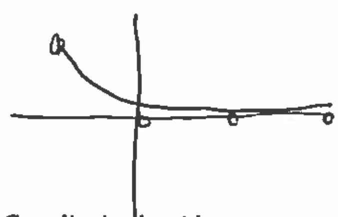
A planar polynomial curve piece p in Bernstein-Bézier form on the interval $[0..1]$ has coefficients (control points)

$$\begin{bmatrix}-2\\4\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}4\\0\end{bmatrix}.$$

- (1 points) What is the degree of this polynomial piece?

  3

- (1 points) *Sketch* the curve piece (the endpoints and the tangents need to be exact).



- (2 points) Using *DeCasteljau's algorithm*, compute the position $p(t)$ at $t = 1/2$ (show the 'triangle' diagram of coefficients!).

$$\begin{pmatrix}-1\\2\end{pmatrix} \quad \begin{pmatrix}1\\0\end{pmatrix} \quad \begin{pmatrix}3\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \quad \begin{pmatrix}2\\0\end{pmatrix}$$

$$\begin{pmatrix}1\\\frac{1}{2}\end{pmatrix}$$

$t_{unn} = \begin{pmatrix}2\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix}$ $n = \begin{pmatrix}+1\\2\end{pmatrix}/\sqrt{5}$

$\begin{pmatrix}1\\\frac{1}{2}\end{pmatrix}$

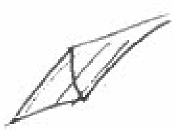
- (1 points) Compute the (upward unit) *normal* of p at $t = 1/2$.

- (1 points) If the coordinates of the control points are scaled by a factor of 2 in the x direction, what is the position $p(t)$ at $t = 1/2$?

  $\begin{pmatrix}1\\1/2\end{pmatrix} \xrightarrow{x \times 2} \begin{pmatrix}2\\\frac{1}{2}\end{pmatrix}$

- (1 points) How does *affine invariance* help when transforming curves?

  transform control coefficients
  yields transform of curve



- (3 points) What is the convex hull property? Explain for a curve segment of degree 3. How does the convex hull property help when testing whether two curves intersect? If CH not intersect then curve segments not intersect

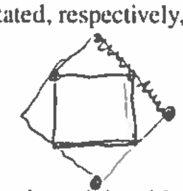
4 [1] [0] for degree
draw [1]
linear $x = 1$, $y = (1/2)^3 4 = 1/2$
1
2 0
3 1 -1
4 2 0 -2
0 0 0 4
0 0 2
0 1
1/2
pts [2] half if not de Casteljau
tangent: (0,1)-(2,0) = (-2,1), normal (1,2)/sqrt5
stretch by 2
just the control point x-coordinate needed to be stretched
above

2 Area

- (1 point) Carefully sketch the polynomial piece with Bézier coefficients

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \tag{1}$$

and its three copies rotated, respectively, by $\pi/2$, π and $3\pi/2$ about $(0,0)$.



- (1 points) Do the curve pieces join with matching derivatives? Why?

Yes, because end ~~points~~ coeffs are averages of middle coeffs

eg $\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \left(\begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix}\right) / 2$

- (2 points) Use the *convex hull property* to estimate the area enclosed by the four curve segments: Give a close upper bound and a close lower bound on the area.

CH (area of diamond) $= (\sqrt{32})^2$ edge length $\left\| \begin{pmatrix} 0 \\ -4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} \right\|_2 = \sqrt{32}$

Inner $= \underline{16}$ (4×4 length, area of square)

- (4 points) Compute the area enclosed by the four curve segments from the Bézier coefficients. (Hint: $\int_0^1 y(u)x'(u)du$ is the area contribution of a curve segment $x(u), y(u)$.)

16 (square) $+ 4 \times$ (area $) = 16 + 4 (4 \quad) = 16 + 16 \, \frac{0+2+0}{3} = 16 \frac{5}{3}$