Self-Test cap{4,6}930: Geometric Modeling

Name:

If you need to make simple, reasonable assumptions to arrive at an answer, state any such assumption.
An answer 'yes' or 'no' is worth 0 points if it does not explain the reasoning.
Take this test twice.

- In red give answers without google and co.
- In black give answers after using any source (specify).

1 Coordinates

Consider the barycentric combination

$$\mathbf{b} := \sum_{j=0}^{2} \alpha_j \mathbf{b}_j, \quad (\alpha_j) := (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (\mathbf{b}_j) := ([\begin{smallmatrix}1\\1\end{smallmatrix}], [\begin{smallmatrix}2\\2\end{smallmatrix}], [\begin{smallmatrix}3\\1\end{smallmatrix}]). \tag{1}$$

- Draw the barycentric combination as the sum of a point and two vectors.
- Draw the convex hull of the three points

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2 Affine Maps

- Construct a matrix that maps $(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix})$ to $(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix})$.
- Prove that an affine map leaves barycentric combinations invariant.
- Determine an affine map that maps the points of Eq. (1) to the unit triangle $[\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}]$
- What (roughly) is the shear map for generating the slanted ”r” in Fig 2.4 of the book?

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3 Functions

Consider the functions

$$b_i : \mathbb{R} \to \mathbb{R}, \quad b_i(t) := \binom{3}{i}(1-t)^{3-i}t^i, \quad i = 0, \ldots, 3, \tag{2}$$

$$\text{where } \binom{n}{i} := \frac{n!}{(n-i)!i!} \text{ and } n! := n \cdot \ldots \cdot 2 \cdot 1.$$

- Prove that the functions b_i are polynomials.
- Prove that the polynomials b_i are linearly independent.

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4 Bonus

A ratio-preserving map leaves the ratio of three collinear points constant.
Prove: every ratio-preserving map is affine.