$$p(t) := \sum_{i=0}^{n} c_i \underbrace{b_i^n(t)}_{\in \mathbb{R}} \cong \sum_{\substack{i+j=n \\ i,j\geq 0}} c_{ij}\, b_{ij}(t)$$

$$b_{ij}(t) = \left(\frac{n!}{i!\,j!}\right)(1-t)^i\, t^j$$

$$Dp(t) = \sum_{i=0}^{n-1} d_i\, b_i^{n-1}(t) \cong \sum_{i+j=n-1} d_{ij}\, b_{ij}(t)$$

proof: 5.16/17

or directly *

$$\boxed{d_{ij} = n\left(c_{i+1,j} - c_{i,j}\right)} = n\left(c_{i+1} - c_i\right)$$

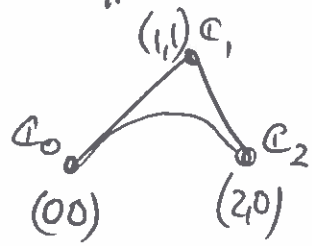
$i,j+1$ i,j

$n-1-i$

$n-1-(i+1) = n-i-2$

$$p(0) = c_{n0} \qquad p(1) = c_{0n}$$

$$(Dp)(0) = d_{\cancel{n-1,0}}\ 0 = n\left(c_{1\cancel{0}} - c_{0\cancel{0}}\right)$$



$$(Dp)(1) = n\left(c_{n\cancel{0}} - c_{n-1}\right)$$

Differentiation = differencing of coefficients

$$(Dp)\binom{0}{0} = 2\left[\binom{1}{1} - \binom{0}{0}\right] = \binom{2}{2}$$

$$(D^2 p_2)(0) = 1\left\{2[c_2 - c_1] - 2[c_1 - c_0]\right\} = 2\left[c_2 - 2c_1 + c_0\right]$$

$$(D^2 p_2)(1) = 2\left[c_2 - 2c_1 + c_0\right]$$

Exercise: write $D\, b_i^n(t) = \dfrac{n!}{i!\,(n-i)!} D\left[(1-t)^{n-i}\, t^i\right]$ as

$$= n\left(b_{i-1}^{n-1} - b_i^{n-1}\right)$$