$d=1$

$\boxed{i:\ i+d+1}$

B-spline definition by recursion

Let

$$t_{(i:j)} := t_i, t_{i+1}, \ldots, t_j \tag{1}$$

$\frac{u-t_i}{t_{i+3}-t_i}$ $\quad \frac{u-t_{i+1}}{t_{i+4}-t_{i+1}}$

be a nondecreasing sequence of scalars, i.e. $t_{k+1} \geq t_k$. The scalars are called *knots*. Then the B-spline of degree d is defined recursively as follows.

$$B(u|t_{(i:i+1)}) := \begin{cases} 1 & \text{if } t_i \leq u < t_{i+1} \\ 0 & \text{otherwise,} \end{cases}$$

t_i $\quad t_{i+1}$

$$B(u|t_{(i:i+d+1)}) := \ell(u|_{i,i+d})B(u|t_{(i:i+d)}) + (1 - \ell(u|_{i+1,i+d+1}))B(u|t_{(i+1:i+d+1)})$$

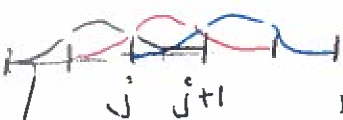
$i : i+d$

$i+1 : i+d+1$

$$\text{where } \ell(u|_{i,j}) := \begin{cases} \frac{u-t_i}{t_j-t_i} & \text{if } t_i \neq t_j \\ 0 & \text{otherwise.} \end{cases}$$

X_5– Verify that for $t_{j+d+1} > t_j$, $B(u|t_{(j:j+d+1)}) > 0$ on the interval (t_j, t_{j+d+1}).

$$c_{i:i+d} := (1 - \ell(u|_{i,i+d}))\ c_{i-1:i+d} + \ell(u|_{i,i+d})\ c_{i:i+d+1} \tag{3}$$
$$=^* \frac{t_{i+d} - u}{t_{i+d} - t_i} c_{i-1:i+d} + \frac{u - t_i}{t_{i+d} - t_i} c_{i:i+d+1}.$$



Here $=^*$ indicates the case that $t_{i+d} \neq t_i$. While, formally, all $c_{i:j}$ are functions of u, in the following context u will be the parameter of evaluation, and therefore a fixed number so that the expressions in (3) will be constants.

To evaluate the spline $s(u|t_{(i:i+d+n+1)})$ at a value

$$u \in [t_j, t_{j+1}) \subset [t_{i+d}, t_{i+n+1}),$$

we compute $c_{j:j+1}$ by repeatedly applying Equation (3) (cf.). It is convenient

$B(u|t_{j-2} t_{j-1} t_j t_{j+1})$
$= B(u|t_{(j-2:j+1)})$