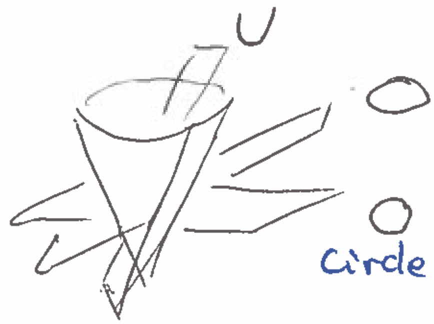
Rational curves

$\frac{\mathbb{P}(u)}{w(u)}$



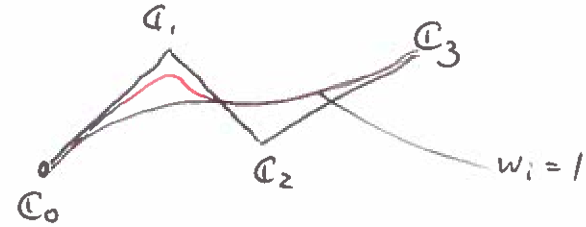
$\bigcirc$ circle $\sim \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ trig $\sim \begin{pmatrix} \frac{1-t^2}{1+t^2} \\ \frac{2t}{1+t^2} \end{pmatrix}$ rational

1-1 circle $\sim$ line

$$1 = x^2+y^2 = \left(\frac{1-t^2}{1+t^2}\right)^2 + \left(\frac{2t}{1+t^2}\right)^2 = \frac{1-2t^2+t^4+4t}{1+2t^2+4t^4}$$

$$\frac{\sum_{i=0}^{n} \mathbb{C}_i w_i b_i(u)}{\sum_{i=0}^{n} w_i b_i(u)}$$

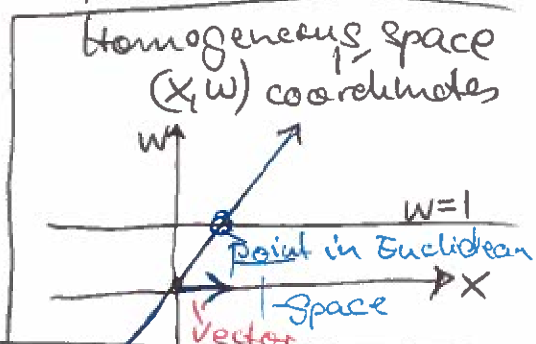
weight



what if $w_1 \to \infty$, all other $w_i = 1$?

$$= \frac{\sum \mathbb{C}_i \frac{w_i}{w_1} b_i(u)}{\sum w_i/w_1 \, b_i(u)} \xrightarrow{w_1 \to \infty} \frac{1 \cdot \mathbb{C}_1 \cancel{b_1(u)}}{1 \cdot \cancel{b_1(u)}} = \mathbb{C}_1$$

(typically assume $\forall w_i > 0$)
none < 0



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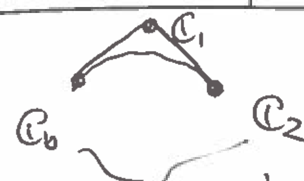
circle



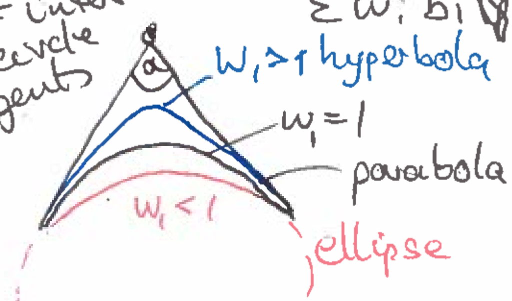
$w_0 = w_1 = w_2 = 1$
$\Rightarrow$ parabola

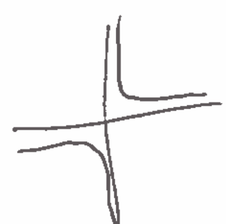
claim:
always set $w_0 = 1$ ✓ (divide)
always set $w_0 = w_2 = 1$ ✓ (change of variable)

$$\frac{\sum \mathbb{C}_i w_i b_i(\alpha t + \beta)}{\sum w_i b_i(\gamma t + \delta)}$$

$$\frac{\mathbb{C}_0 b_0(u) + \mathbb{C}_1 w_1 b_1(u) + \mathbb{C}_2 b_2(u)}{b_0(u) + w_1 b_1(u) + b_2(u)}$$

$\mathbb{C}_1$ = intersection of circle tangents





what is w_1?
$\cos \alpha$