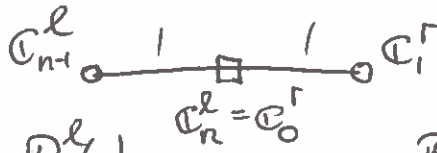
Geometric Continuity can generalize parametric continuity



still "smooth" = tangent is same

$$P^l(u)\Big|_{u=1} = 3\,P^r(u)\Big|_{u=0}$$

$$\left(P^l(u)\right)'(1) = \left(P^r(3u)\right)'(0) = 3\,(P^r)'(0)$$

$\rho : \mathbb{R} \to \mathbb{R}$, $u \to \rho(u)$ ← change of variables

$$\left(P^l(u)\right)'\Big|_{u=1} = \left(P^r(\rho(u))\right)'\Big|_{u=0} = \rho'(u)\,(P^r)'(\rho(u))\Big|_{u=0}$$

change of variables

$$\left(P^l(u)\right)'' = \left(P^r \circ \rho(u)\right)'' = \left(\rho'(u)\,(P^r)'_{\rho(u)}\right)'$$

P^l P^r

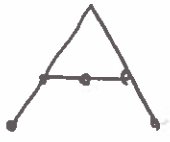
composed with

$$= \left(\rho'(u)\,(P^r)''_{\rho(u)}\,\rho'(u)\right) + \rho''(u)\,(P^r)'_{\rho(u)}$$

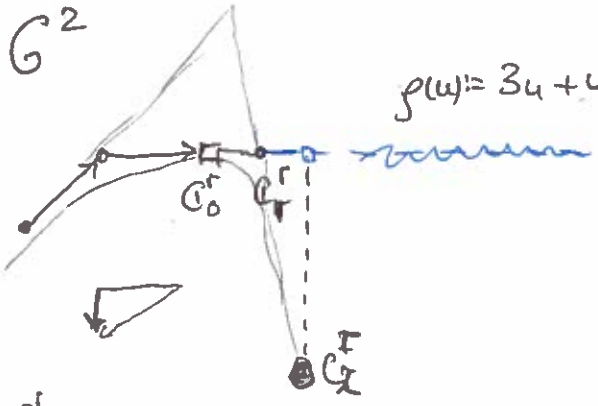
G^1  $\rho(u) := 3u$ $\quad (P^l)' = 3\,(P^r)'$

C^1 

G^2 $\quad \rho(u) := 3u + u^2$ $\quad (P^l)'' = \left(3\,(P^r)''\,3\right) + 2\,(P^r)'$

C^2 

$(3+2u)\Big|_{u=0}$



Shape

Faà di Bruno's Law

Shape & speed