I $\in \mathbb{R}^{2\times 2}$

"metric" $[x_i \cdot x_j]_{i,j \in \{u,v\}}$

$\begin{pmatrix} e & f \\ f & g \end{pmatrix} \longleftrightarrow \begin{pmatrix} E & F \\ F & G \end{pmatrix}$

II

$[n \cdot x_{ij}]_{i,j \in \{u,v\}}$ "shape"

$y'' = x^2 \Rightarrow y = \frac{x^4}{4 \cdot 3} + \alpha x + \beta$

Gauss Curvature $:= \dfrac{eg - f^2}{EG - F^2} = \dfrac{\det II}{\det I} =: K$

"invariant"

mean curvature $:= \dfrac{eG - fF + gE}{EG - F^2} =: H$

does not change if we change x to $x(\varphi)$

$\varphi : \mathbb{R}^2 \to \mathbb{R}^2$

Gauss – Bonnet

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principal direction — eigenvector — $v_1\, v_2$; $v_1 \perp v_2$

— u — curvatures — eigenvalues — κ_1, κ_2 of W

$\kappa_1 \cdot \kappa_2 = K$

$\kappa_1 + \kappa_2 = H$