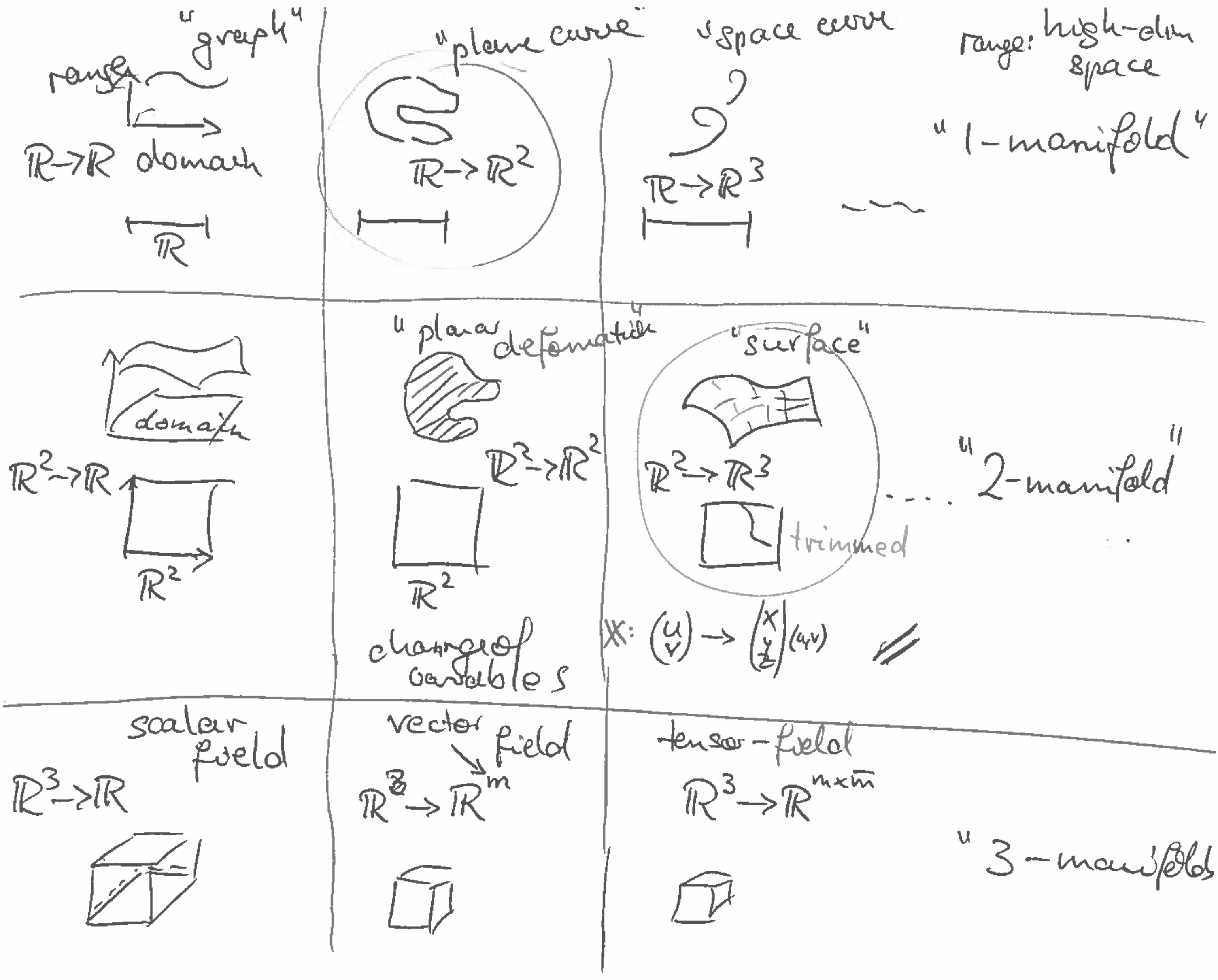


"graph"
range
R->R domain
R
"plane curve"
R->R2
"space curve"
R->R3
range: high-dim space
"1-manifold"
domain
R2->R
R2
"planar deformation"
R2->R2
R2
change of variables
"surface"
R2->R3
trimmed
X: (u v) -> (x y z)(u,v)
"2-manifold"
scalar field
R3->R
vector field
R3->Rm
tensor-field
R3->Rmxm
"3-manifolds"

"surface"

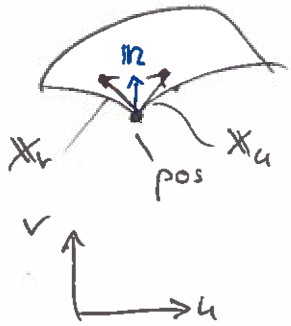
Differential Geometry

$$\mathbf{x} : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$
$$\begin{pmatrix}u\\v\end{pmatrix} \longmapsto \begin{pmatrix}x\\y\\z\end{pmatrix}(u,v)$$

$$\mathbf{x}_u := \frac{\partial \mathbf{x}}{\partial u} \in \mathbb{R}^3$$

$$D\mathbf{x} := [\mathbf{x}_u, \mathbf{x}_v] \in \mathbb{R}^{3\times 2}$$

"Jacobian" spans the tangent plane (attached at point)



$$\mathbf{n} = \frac{\mathbf{x}_u \overset{\text{cross product}}{\times} \mathbf{x}_v}{\|\mathbf{x}_u \times \mathbf{x}_v\|} \quad \text{normal}$$

"shape" ~ deviation from flat

claim:
$$D\mathbf{n}\cdot\mathbf{n} = 0 \qquad \mathbf{n}\cdot\mathbf{n} = 1 \qquad \underbrace{D(\mathbf{n}\cdot\mathbf{n})^2}_{\text{product rule}} = D1 = 0$$
$$2(D\mathbf{n}\cdot\mathbf{n}) = 0$$

Therefore
$$-D\mathbf{n} = D\mathbf{x}\,[W] \qquad W \in \mathbb{R}^{2\times 2} = \text{Weingarten map}$$

assume $\mathbf{x}_u \times \mathbf{x}_v \neq 0$

$$-(D\mathbf{x})^T D\mathbf{n} = \underbrace{(D\mathbf{x})^T(D\mathbf{x})}\; W$$

$$\begin{pmatrix} -\mathbf{x}_u\cdot\mathbf{n}_u & -\mathbf{x}_v\cdot\mathbf{n}_u \\ -\mathbf{x}_v\cdot\mathbf{n}_v & -\mathbf{x}_v\cdot\mathbf{n}_v \end{pmatrix} \qquad \begin{pmatrix} \mathbf{x}_u\cdot\mathbf{x}_u & \mathbf{x}_u\cdot\mathbf{x}_v \\ \mathbf{x}_u\cdot\mathbf{x}_v & \mathbf{x}_v\cdot\mathbf{x}_v \end{pmatrix}$$

u v

$$\begin{pmatrix}x_u\\y_u\\z_u\end{pmatrix}\cdot\begin{pmatrix}x_v\\y_v\\z_v\end{pmatrix} = x_u x_v + y_u y_v + z_u z_v$$

I = first fundamental form ~ "metric"

$$\begin{pmatrix} e & f \\ f & g \end{pmatrix}$$

II = second fud. form

$$\begin{pmatrix} E & F \\ F & G \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{n}\mathbf{x}_{uu} & \mathbf{n}\mathbf{x}_{uv} \\ \mathbf{n}\mathbf{x}_{vu} & \mathbf{n}\mathbf{x}_{vv} \end{pmatrix} \quad \text{shape}$$