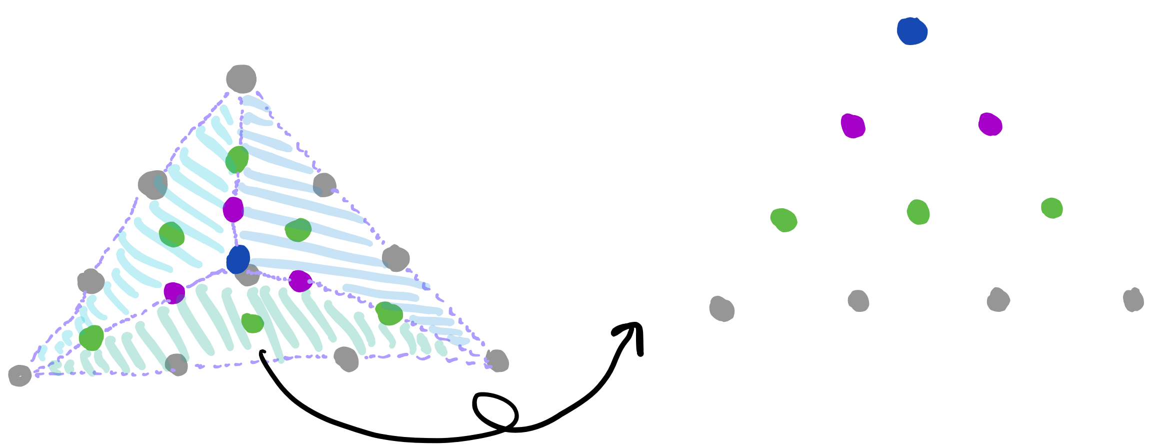
Subdivision

Why subdivision?

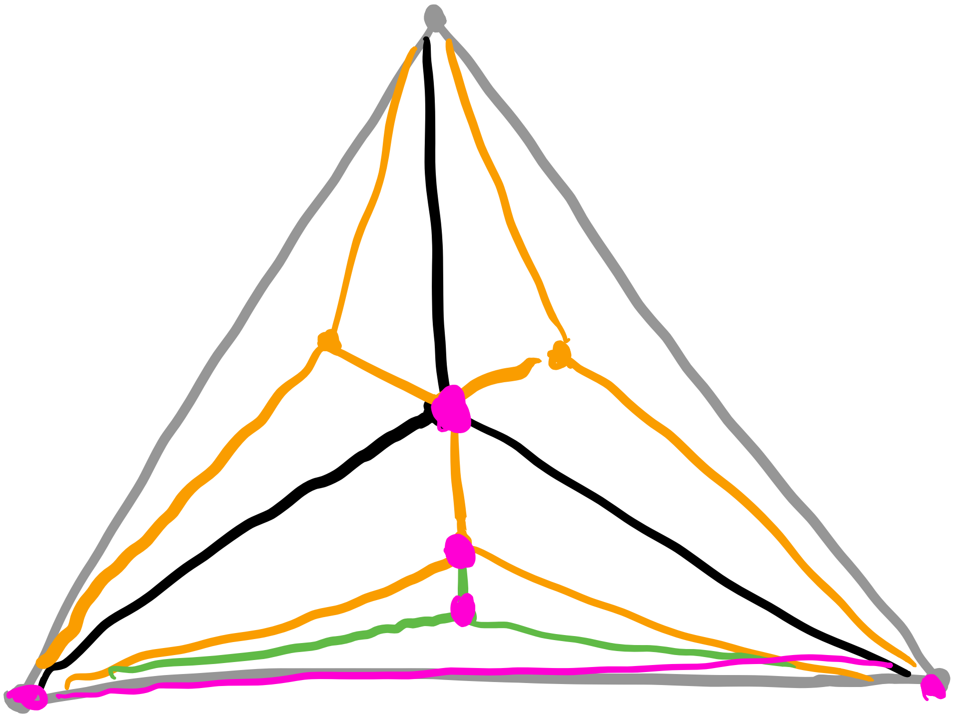
To approximate the curve or shape for numerical computing (Need higher resolution)

Traditional subdivision method.

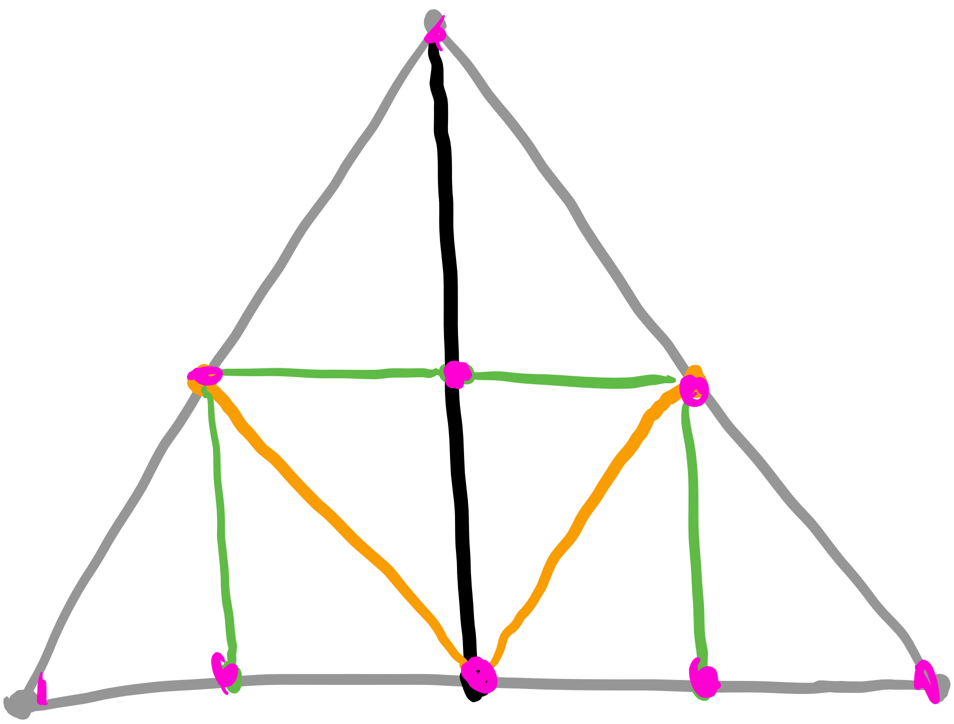
2-D de Casteljau.



Drawback of de Casteljau.

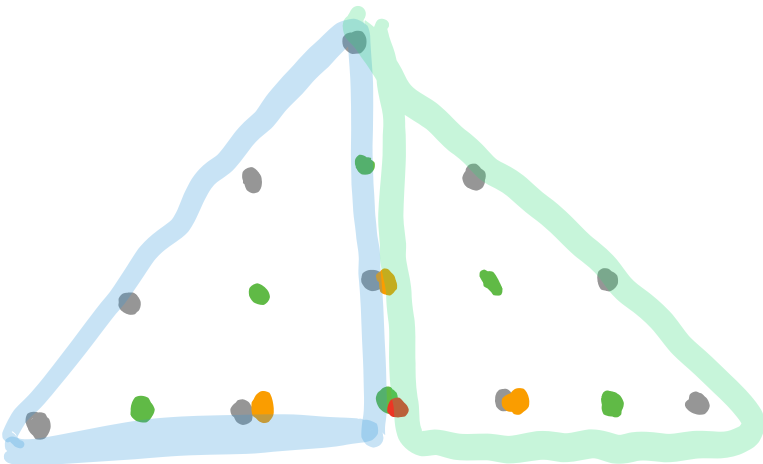


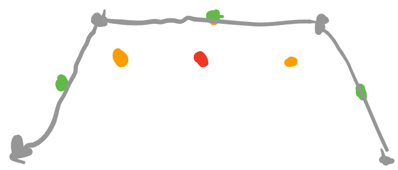
More Uniform



How to subdivide uniformly?

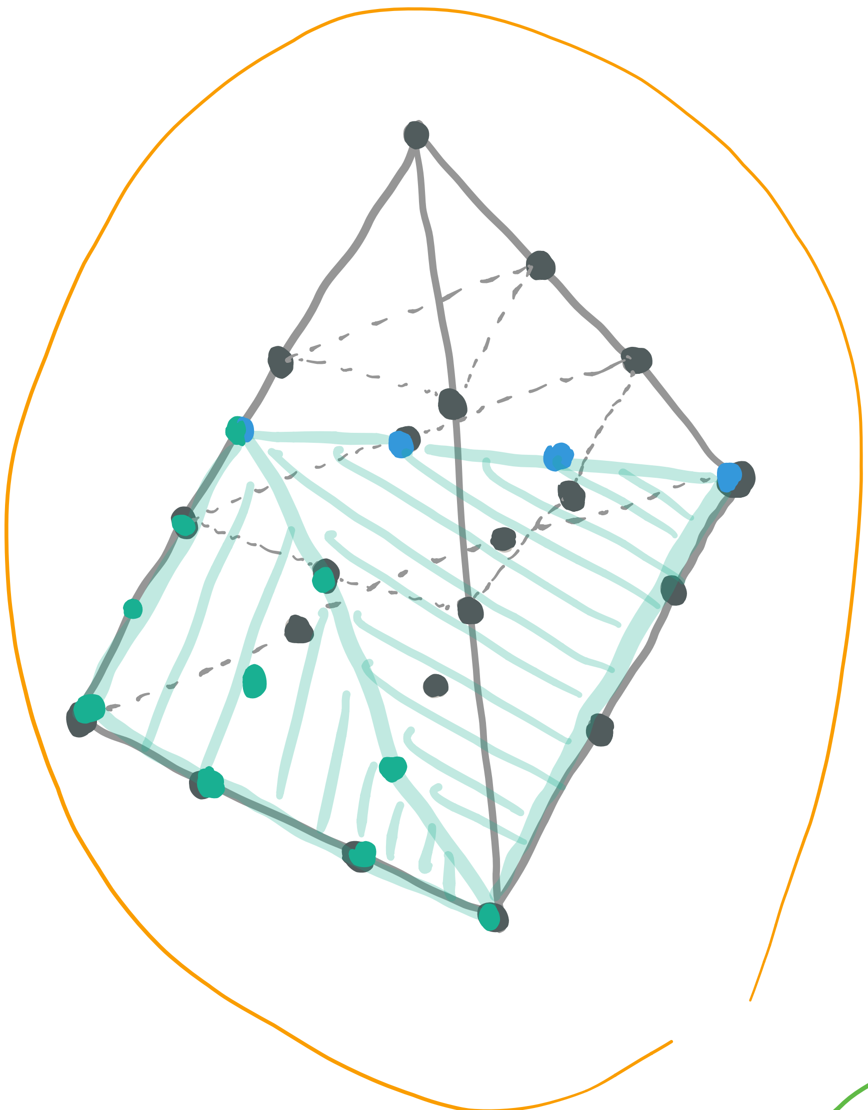
Let's say we have Cubic Bézier Triangle...

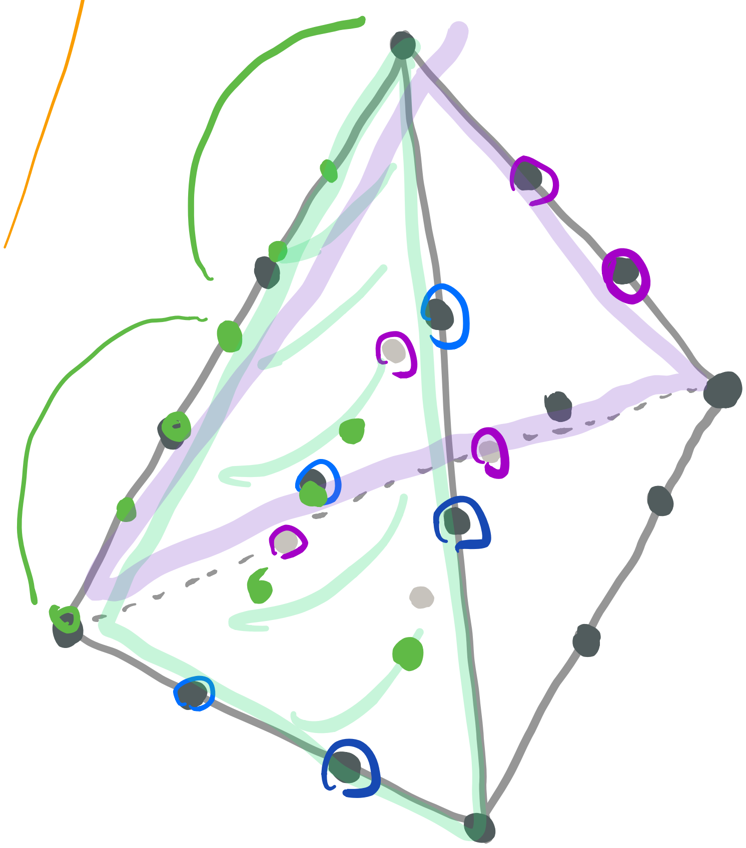




Can we do this in 3-D ?

YES !





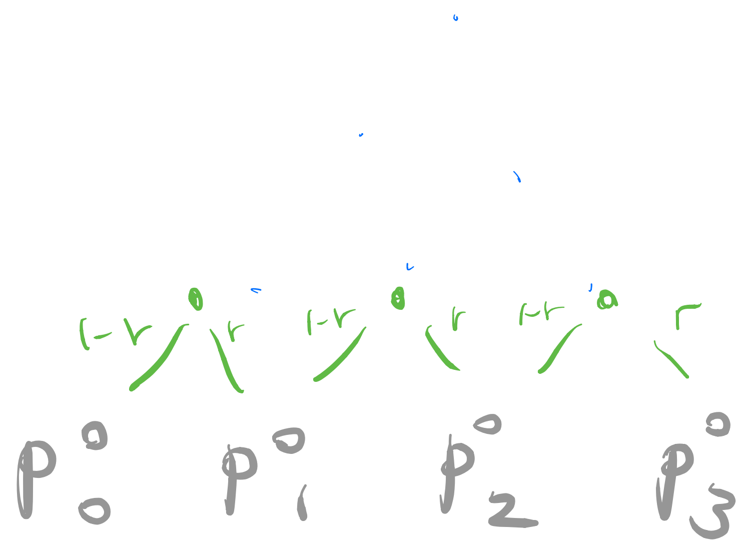
Subdivision

When we have N control points and want to subdivide it's corresponding curve, we can generate its degenerated triangle.

$$\{p_j^k(r)\}, \quad j+k \leq N.$$

$$\begin{cases} p_j^0(r) = p_j \\ p_j^{k+1}(r) = (1-r)p_j^k(r) + r p_{j+1}^k(r) \end{cases}$$

ex:

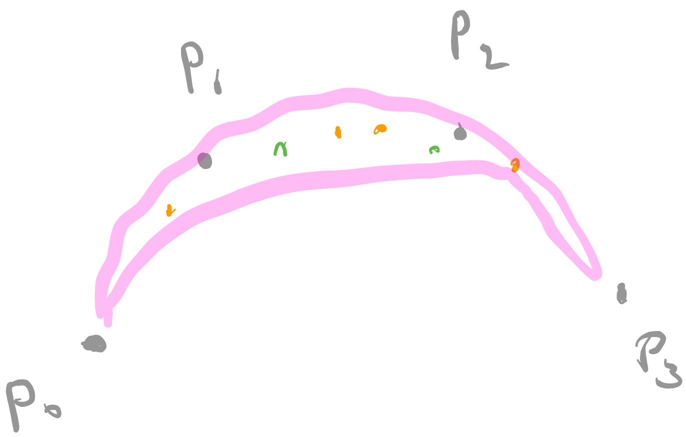


Using Degenerated Bézier Triangle to subdivide Bézier Curves.

◎ $B^N[P_j^K(r)](s,t) = B^N[P_i](\underline{s+rt})$

(proved by paper).

Which is saying



How to get Bézier Curve from Bézier Triangle?

For 2-dimentional $(m=2)$

$$\sum^{N} B_{ij}(\xi,t)P_{ij}$$

ex: $N=3$, $i+j \leq 3$

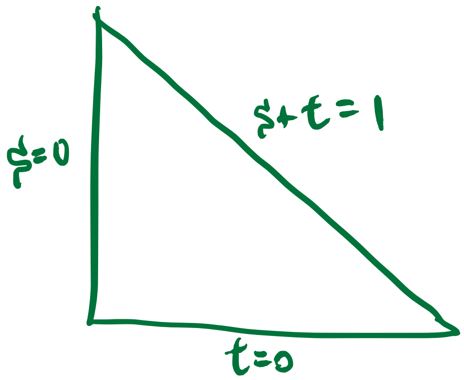
P_{30}

P_{20} P_{21}

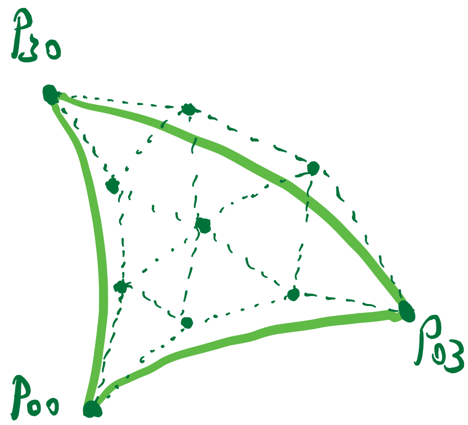
P_{10} P_{11} P_{12}

P_{00} P_{01} P_{02} P_{03}

Control Points.



Parameter Space.



Bézier Simplex.

We get edges when...

$\xi=0$ or $t=0$ or $\xi+t=1$ (MATLAB DEMO)