

# soft (deformable) bodies

- Generalization: **soft (deformable) bodies**
  - **Deformed by force**: car body, punched or shot at.
  - **Prone to stress**: piece of cloth, flag, paper sheet.
  - **Not solid**: snow, mud, lava, liquid.



<http://www.games73.com/media/games73.com/files/2012/05/Crysis-3-soft-physics-demo-o-thumb-610x239.jpg>

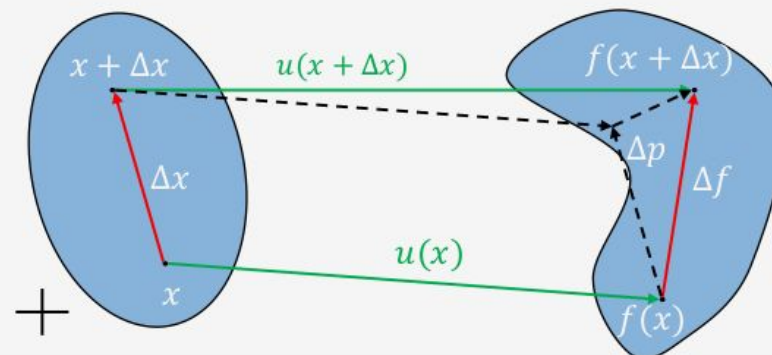


<http://i.huffpost.com/gen/1480563/images/o-DISNEY-facebook.jpg>



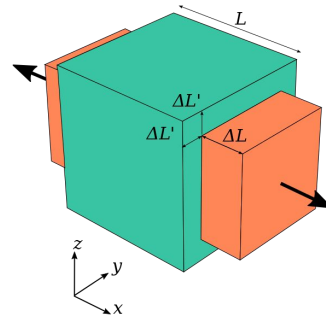
Grinspun et al. "Discrete Shells"

Relative displacement field:  $f(\vec{x}) = \vec{x} + u(\vec{x})$ .



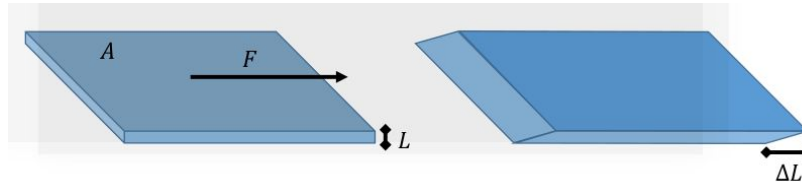
# soft (deformable) bodies

Poisson's ratio



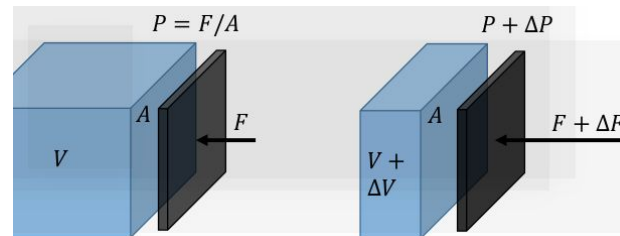
linear stress / linear strain      force/area

Young's modulus



volume stress / volume strain

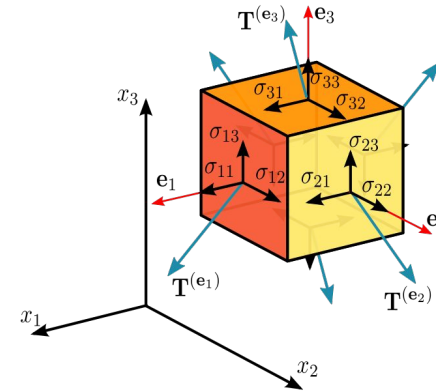
Bulk modulus



# soft (deformable) bodies

Strain Tensor:  $df'df + df + df'$

Stress Tensor: force/area  
normal stress, shear stress



Hook's law:  $\sigma = C\bar{\varepsilon}$   $C: 9 \times 9$

force = -k

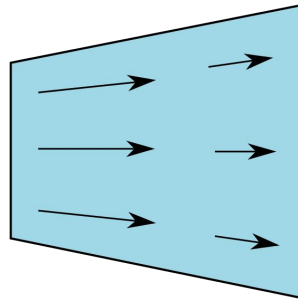
Linear Elasticity:  $\int \bar{\varepsilon} C \bar{\varepsilon}$

Dynamic Elastic:  $F = m a$

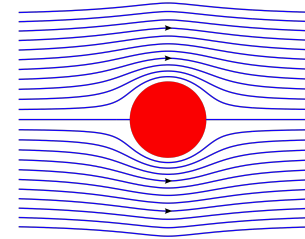
density  $u'' = \text{div}(\text{stress}) + \text{external body force}$

# Flow

➤ steady:  $u' = 0$



➤ incompressible:  $\text{div}(u) = 0$



➤ irrotational  $\text{curl}(u) = 0$



## Navier-Stokes Equations

- Representing the **conservation of mass** and momentum for an **incompressible** fluid ( $\nabla \cdot u = 0$ ):

$$\begin{array}{c}
 \text{Inertia (per volume)} \\
 \overbrace{\rho(u_t + u \cdot \nabla u)} \\
 \begin{array}{cc}
 \text{Unsteady} & \text{Convective} \\
 \text{acceleration} & \text{acceleration}
 \end{array}
 \end{array}
 =
 \begin{array}{c}
 \text{Divergence of stress} \\
 \overbrace{\nabla \cdot (\nu \nabla u) - \nabla p + f} \\
 \begin{array}{ccc}
 \text{Viscosity} & \text{Pressure gradient} & \text{External} \\
 & & \text{body forces}
 \end{array}
 \end{array}$$

- $p$ : pressure field
- $\nu$ : kinematic viscosity.
- $f$ : body force per density (usually just gravity  $\rho g$ ).