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Topological operations transparent for users

Stephan Winter

Department of Geoinformation, Technical University Vienna,
Gusshausstr. 27-29, A-1040 Vienna,
phone +43 (1) 5880112712, fax +43 (1) 5880112799
winter@geoinfo.tuwien.ac.at

Abstract. The topological operators defined in the spatial schemata of ISO and OGC for vector geometries can also be applied to raster geometries. This principle ability is investigated here for its semantic implications. From a user's perspective, topological operators should be applicable to features in any representation and any resolution with a predictable behavior. The conditions and requirements for transparency are presented and discussed. At the end it becomes clear that topological interoperability is given across representational borders.

1. Introduction

1.1 Motivation

An emerging issue for spatial analysis is the use of distributed, heterogeneous data sets. This issue regards for example web mapping, with applications like disaster management (which typically needs interoperability on demand), or interacting systems of different agencies or companies, with applications in administration or planning. Typical situations are:

- After a hurricane the damage shall be visualized, e.g., to coordinate rescue. Useful data are among others satellite images, topographic data sets, car navigation data sets. These data sets are typically maintained by different institutions in different systems and in different representations.

- The environmental impact of a planned factory shall be tested. Data from heterogeneous sources will be merged: cadastral maps, zoning plans, soil maps, vegetation classification maps, hydrological maps, and microclimate information, among others. Again, the data sets will be heterogeneous.

Access to remote data sets will reduce redundant local storage of data sets, and even buying data sets rarely or partly used will become unprofitable. Furthermore, with open system interfaces, as specified by the OpenGIS® Consortium (OGC), data conversion becomes obsolete between proprietary formats. The long-term goal is to hide representational issues from the user of a geographic information system. The user is interested in spatial problems, not in representational ones. Especially, when different representations behave differently such differences may be extremely confusing in combining heterogeneous data sources. Thus, one requirement for interoperability is transparency from data representation.

1.2 The problem

The focus of this paper is on topological operations (operators determining the topological relation between two spatial features). Topological operations for vector representation (Egenhofer and Herring 1990; OGC 1998a) behave significantly different from a naïve application of the same operations for raster representation. Probably for that reason, the OGC specification for raster representation (OGC 1998b) contains no topological operations. The problem is shown by three use cases:

- Consider a vector data set containing water areas, and another vector data set containing industry areas with environmental impact. Assume the following query: Is any lake *touching* an industrial zone? The common instrument to answer such a query is the nine-intersection model (Egenhofer 1989; Egenhofer and Herring 1990; Egenhofer and Franzosa 1991), which is also part of international standards (OGC 1998a; ISO 1999). The nine-intersection model goes back to the intersection sets of the interiors, the boundaries and the exteriors of two objects in concern. Within this model, the topological relation *touch* between two regions is defined by intersecting boundaries, with no overlap between the interiors (Fig. 1).
- Consider now two data sets, containing the same information in raster representation (Fig. 2). The above query cannot be answered with Egenhofer's model without a concept of a (one-dimensional) boundary, which does not exist explicitly in raster representations. In *digital topology* (Kong and Rosenfeld 1989) two-dimensional boundaries are introduced that contradict Egenhofer's assumptions. Alternatively, one could apply Egenhofer's nine-intersection method to the four existing intersection sets between the two interiors and exteriors. However, this simplifying approach allows only selecting non-overlapping water and industry regions, where non-overlap means *disjunct* or *touch* in terms of Egenhofer operators. For simple regions, the number of possible relations is reduced from eight to five (Winter 1998b). – The second prominent

model, region connection calculus (Cohn et al. 1997), supports this generalized set of relations within RCC-5; RCC is based on connection axioms only.

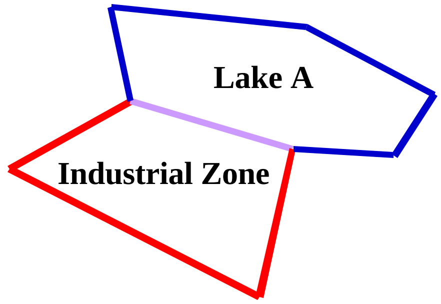

Fig. 1: Two touching regions in vector representation: they share a boundary.

Nevertheless, from a user's perspective, the intention of the topological query is the same as for regions in vector representations. Instead of semantic translation of the query result, depending on the actual representation of the data sets, a transparent behavior of topological operations for both representations is required. In deed, it is possible to re-interpret raster representations as regular cell complexes (Kovalevsky 1989; Winter 1995; Winter and Frank 2000), where Egenhofer's nine-intersection model is applicable without restriction. If both raster representations are compatible (having the same orientation and resolution), the operator can be implemented as a convolution.

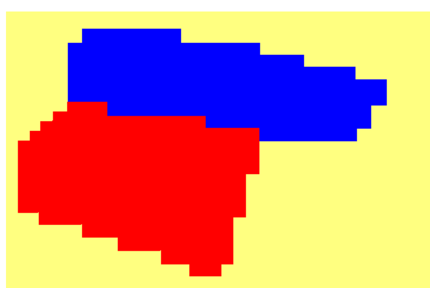

Fig. 2: Two touching regions in raster representation.

- Consider finally two spatial data sets, one of simple features, and one a grid coverage. Direct evaluation of the topological relation seems to be impractical for two reasons: (1) due to the implicit nature of boundaries in one of the representations, the step to make them explicit needs additional time and memory, and (2) due to the discrete and regular nature of one of the representations, the boundary would be highly aggregated, requiring a lot of numerical intersection tests. Therefore, to determine topological relations between a simple feature and a gridded feature it could be useful to transform either the simple feature into a (compatible) grid, or the gridded feature into a simple feature. As long as both is possible, it would be desirable to get consistent results independent of the chosen way of conversion (Fig. 3). This goal can be achieved only if in both representations the same operation is defined, which is here the nine-intersection.

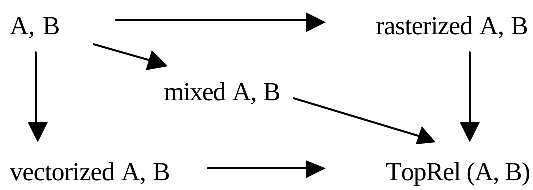

Fig. 3: Topological relations are transparent if the diagram commutes, i.e., if for given two regions A and B each kind of representation shows the same behavior in operations. (Note that no statement is made about the 'true' relation; the results along different paths even have not be equal.)

With a given mathematical approach that allows applying the same topological operations on vector and raster representations, the special contribution of this paper is an investigation semantic implications of the diagram in Fig. 3. If the operations work for vector features and for compatible raster features, do they work also on non-compatible raster features, or on mixed cases (a vector and a raster feature)? What are the conditions for a commuting diagram? What does commuting mean here? It is certainly the behavior we are interested in our argumentation; it is not the result which have to be equal. The driving force is the question: can representational aspect be kept transparent for the user? First, the hybrid representation as basic concept is recalled. Then a general concept of a feature in coverage – a generalization of raster – is introduced. With this concept semantic implications of representing a feature are investigated. It will be argued that discreteness of representations is common for both, vector and raster, so in principle the diagram in Fig. 3 commutes: the behavior is the same. However, due to abstraction and simplification in (independent) observations the diagram does not commute in practical applications: results along different paths (or along the same path but with different data sources) are most probably different. This is not a representational issue, so the interfaces for topological relations between features of any representation are a valuable extension to existing interoperability specifications (OGC 1998b), with a common, sound and intuitive semantics.

1.3 Approach

The OpenGIS Simple Feature Specification (OGC 1999b), covering a subset of features in vector representations, presents a set of interfaces for the Egenhofer operations in IDL, the interface definition language for CORBA (OMG 1999), such as:

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Boolean touches (in Geometry other);
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This interface of a vector geometry takes another vector geometry as input parameter and checks whether both geometries touch, returning true or false. The proprietary data format is hidden in the simple feature interface, i.e., systems offering this interface should produce a result consistent to the specified model. – Other versions of this implementation specification exist for SQL, and for COM.

This set of interfaces for the Egenhofer operations is missing in the raster specification (OGC 1999a), for some reasons: topology in raster is substantially different from point sets (Kong and Rosenfeld 1989). The hypothesis of this paper is that, by applying a mathematically sound approach of unifying raster and vector representation behavior, the interfaces for simple features can be used as interfaces for features in raster representations also, and that doing so an intuitive transparent behavior of topological operations is provided.

The limitation to *features* in raster representations concerns the general duality of features (entities) and fields (Couclelis 1992): raster representations representing fields require (semantically and technically) at least a conceptualization to features, e.g., by thresholds or by intervals, before applying topological operations. The consequences for general raster representations are investigated here in detail.

To be consistent with the notions of the specifications of OGC, we speak of *simple features* for (simple) vector geometries, and of *grid coverage* for raster representation, also synonymously for a specific raster image. OGC uses *coverage* in a generic sense, among them images, grids, irregular point sets, and triangulation networks.

2. Previous Work

A mathematical approach to unified topological behavior of vector and raster was developed in other work (Kovalevsky 1989; Winter 1995; Winter 1998a; Winter and Frank 2000). This approach shall be investigated from a user's perspective, and is presented here in short.

A grid coverage representing sets of features is considered to partition space into square pixels, and pixels are attached to features by a vector of nominal attribute values (Stevens 1946). In a coverage, a feature *a* is selected by considering all pixels with an attribute value *a* as part of the feature, and all other pixels as exterior of *a*. In this interpretation of a grid coverage, pixels are considered as (two-dimensional) surfaces.

Consider the grid coverage of nominal scale as a regular cell complex: each pixel is enclosed by four nodes and four edges of unit-length. In this approach, pixels still belong to the interior or to the exterior of a feature, but in addition edges and nodes can belong to its boundary. An intersection of two compatible regular cell complexes yields nine intersection sets; the behavior is exactly the same as in irregular cell complexes like simple features. For that reason we speak of a unifying approach. An interesting aspect of this approach is that it is not necessary to store edges and nodes:

they can be determined on-the-fly during the intersection (Winter and Frank 2000). The cell complex is only another *interpretation* of the grid coverage.

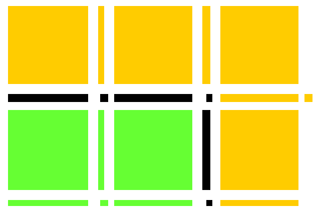

Fig. 4: Part of a raster representation, completed to a cell complex. In black: boundary.

3. Formal definition of coverage features

In this section the nominal scale of a grid coverage function, the surface view on a grid coverage, and compatibility of two coverages are formally defined. These definitions underlie the mathematical concept of the unifying approach but were not made explicit before. They determine the semantics of a raster feature.

The structure of a two-dimensional grid is defined by a function $\mathbf{p} = \mathbf{o} + a\mathbf{u} + b\mathbf{v}$, where $\mathbf{p}$, $\mathbf{o}$, $\mathbf{u}$, $\mathbf{v}$ are vectors in a reference system, and a and b are integers. The vector $\mathbf{o}$ is the origin of the grid, and $\mathbf{u}$ and $\mathbf{v}$ span the basis of the grid. We delimit ourselves to finite grids, and furthermore to rectangular grids with the ranges:

$$\text{range}\,(a) = [\,0 \ldots n\text{-}1\,]$$
$$\text{range}\,(b) = [\,0 \ldots m\text{-}1\,]$$

with positive integers n and m. Then the vector (n, m) is the size of the (rectangular) grid.

A coverage is a feature represented by a two-dimensional continuous function which is generally unknown. A grid coverage is an approximation of a coverage; it is represented by a function with points in a grid structure as domain, and with vectors of coverage values as range (instead of a vector-valued function one can imagine a vector of elementary functions). The vectors of values must have a constant dimension, and homogeneous types all over the grid. In contrast to coverage functions, grid coverage functions are observable (countable finite). However, not all coverage values need to be observed in the real world; there may be also elementary coverage functions deriving new values from observed values (as, e.g., derivates, or intersections).

The discrete function of a grid coverage can be supplemented by an interpolation function. Domain of the interpolation function is the continuous space, and the coverage values are its boundary values.

Definition: An elementary grid coverage function is of *nominal* scale if the interpolation function for the coverage values is the nearest neighborhood function (nominal coverage).

The nominal coverage has one interesting property: the single grid coverage value holds for a surface that is called pixel in images. This surface $\mathbf{s}$ covers the area:

$$\mathbf{s} = [\,\mathbf{o} + (a\text{-}1/2)\mathbf{u} + (b\text{-}1/2)\mathbf{v} \ldots \mathbf{o} + (a\text{+}1/2)\mathbf{u} + (b\text{+}1/2)\mathbf{v}\,]$$

A special case of a nominal coverage is a binary coverage.

Definition: A nominal coverage is *binary* if the domain of coverage values consists of exactly two elements.

The binary coverage is a suited representation for single features: feature selection or feature extraction labels the interior and the exterior by the two values.

Definition: A *feature* in a nominal coverage is the set of pixels with a common coverage value (grid feature).

A grid feature is always two-dimensional. The definition allows features consisting of unconnected components. However, it is guaranteed that raster features – or the topological hull of them – are simple features in the sense of OGC. Given two grid features, the topological relation will be determined by intersecting the two grid coverages. Such an operation is reduced to a convolution if the two grid coverages are compatible.

Definition: Two grid coverages are called *compatible*, if they have the same common discrete spatial reference system.

The spatial reference system enforces the same resolution and the same orientation of the base vectors, and discreteness of the system enforces corresponding grid point positions. Remaining differences in origin or size can be adjusted by (virtually) extending both covered rectangles to their common bounding box. The extended part of each coverage contains only exterior of a feature. – For features in non-compatible grid coverages, one of the grid coverages can be transformed into the system, orientation and resolution of the other. The transformation is practically done into the system of the higher resolution, to keep the details. The transformation includes re-sampling of coverage values.

4. Semantic Issues

In this section we investigate the semantic consequences of the formal requirements, especially the term 'feature' in a (grid) coverage. This is the main contribution of this paper, a necessary prerequisite to discuss the use of the (mathematically sound) unifying approach in Sect. 2. The strategy of this investigation aims at deducing the semantics of topological properties of raster features from the properties of vector features. This is reasonable since the Egenhofer operators (on vector features) have been proven to be cognitive salient (Mark and Egenhofer 1994; Mark et al. 1995; Shariff et al. 1998). Therefore a transparent behavior should be based on vector behavior. Thus the semantics of topological relations in general needs not to be discussed here again.

4.1 Raster Features

One of the remaining problems is the semantics of features in grid coverage. The generic form of coverage is a function (Sect. 3). It is determined by a set of locations (points or higher-dimensional geometries), and a vector of coverage values at each location. The range of the coverage function may be of any type of scale (Stevens

1946): coverage represents properties of locations, but not properties of features (Couclelis 1992).

What is conceived as a feature in a two-dimensional function, depends on

- the specific domain and context of a user's query. The user belongs to an information community agreeing in a specific conceptualization of the world into a set of features (classes, not instances).
- the ability to observe the user's concept in the signal of the function. This item regards the fitness for use of the data set, and concerns precision.

In principle, carving out a feature from a two-dimensional signal means a mapping from (at least) an ordinal domain to a nominal range, where the nominal scale represents the set of identities. The nominal scale can contain a null value (symbol) also, to represent locations that do not belong to a feature in the requested context.

The mapping to nominal range may consist of several functions, attaching several nominal attributes to locations. A vector of nominal values at a location allows that the location refers to several features, or: features may overlap. An example is the area of a hurricane and the area of a city in two elementary nominal coverage functions: they may exist at a point of time (partly) on the same location. However, a single nominal coverage function always represents a partition of space.

Typical mapping functions are linear constraints on coverage values $v(x)$ or on derivations of coverage values.

- The simplest case regards a nominal coverage (e.g., a map), where a feature (e.g., *forest*) may be determined by selecting all pixels of a unique value ('green'). The linear constraint is

  $$\text{'forest'} = \{x \mid v(x) = \text{'green'}\}$$

  This example works on ordinal scales also.
- Another example regards an ordinal coverage, e.g., a remote sensing image, where a feature (e.g., *forest*) may be determined by selecting all pixels in an interval of values:

  $$\text{'forest'} = \{x \mid 127 < v(x) < 135\}$$

  or, using a vector of local values:

  $$\text{'forest'} = \{x \mid r(x) < 100 \wedge g(x) < 150 \wedge b(x) < 100\}$$
- A third example regards an ordinal coverage, e.g., a height image, where a feature (e.g., *plain area*) may be determined by derivatives of the coverage values, in this case by slope values:

  $$\text{'plain area'} = \{x \mid \partial v(x) < 0.1\}.$$

However, mapping functions may be arbitrarily complex, and they may use vectors of coverage values. They have in common that they exploit homogeneity of location attributes by some means. This mapping is often called feature selection, image classification, or feature extraction. The result is often not a feature instance, but merely a feature class, because the nominal value characterizes locations of homogeneous properties, not feature identities. To identify single features further processing is needed; maybe by identifying connected components, by form criteria, by aggregation or disaggregation, or similar techniques.

4.2 Granularity of representations

The user is in responsibility to define the mapping function for the features she is interested in. This is critical for some reasons:

- the real world is complex and of high variability,
- the real world is continuous and by no means of regular structure,
- the real world is dynamic,
- the concept of the user is vague,
- the mathematical formulation is distinct from the concept the user has in mind,
- the formulation by constraints requires clear cuts (thresholds).

Consider for example a hurricane. Such a meteorological phenomenon is as far a feature as it is given a name by people. But it is also a local maximum in a continuous wind field, and it is difficult to decide where at a time *t* the hurricane begins and ends in the field. Furthermore, different information communities could have a different concept of the hurricane. Thresholds are chosen with some arbitrariness. The hurricane is an extreme example of the vagueness of human concepts. Exact knowledge of the location of features is possible in principle for some *fiat* objects created in the mind only (Smith 1995), with the problem not being observable physically. A lack of definition of a feature continues in indeterminate boundaries of its location (Burrough and Frank 1996; Bittner 1999). A second component of uncertainty is due to error in observing the coverage values. Among different error components, at least noise is inevitable. There is a lot of research in modeling error, uncertainty, or imprecision, proposing rough sets (Pawlak 1982), fuzzy sets (Zadeh 1965), or stochastic methods (Koch 1999). None of these methods are considered in interoperability standards up to now. However, observing two features independently may disturb their topological relation under these circumstances.

Therefore, features taken or derived from coverage functions are discrete approximations of real world features. Features taken or derived from *grid* coverage functions are *regular* discrete approximations. But features observed by measurement and put in vector representation suffer from all the same factors. They are also discrete approximations of real world features. Granularity (level of detail) influences topological operations since slivers may occur, either of regular (raster) or of irregular shape (vector)). That means that effects from granularity are not specific to raster representation, but concern any representation in the same way. Transparency can be guaranteed only with respect to the behavior of the operators, not to the results for two features in any representation.

4.3 Discreteness of representations

The desired commutability of Figure 3 cannot be guaranteed generally, due to the fact that each representation of a feature is an approximation. Independent approximations disturb topological dependencies between the features. Nevertheless, here is pointed out that this is a problem of generalization, not of the (kind of) representation.

Representation of real numbers in finite computers is finite. In CAD systems it is even common using integers to define coordinate space. Thus vector and raster representation have the same coordinate space, a grid. The difference between vector and raster representation is mainly in the realization of implicit elements: lines are generated by (by default linear) interpolation functions. In vector representation, lines may connect any two points; in raster representation, lines may connect only (four-) neighbored grid points (Fig. 4).

For the nine-intersection model in vector representation, algorithms exploit (1) coincidence of points, (2) coincidence of point and line, and (3) intersection of lines. Accepting the finite nature of vector representation, the latter two require approximations of line points on the grid of the coordinate space (Güting and Schneider 1993; Hölbling et al. 1998). For the nine-intersection model in compatible raster representations, an algorithm profits from pixel-wise correspondence of area elements; points and lines can be inferred. Intersections of elements in all three dimensions are exact, having each grid cell represented explicitly. In comparison, the nine-intersection model exploits in both representations only information at grid points, and infers the rest.

It is clear then that at a conceptual level the nine-intersection model works identical on both representations: the diagram of Figure 3 commutes at the elementary grid level. With regard to discreteness, transparency of topological operators can be guaranteed.

5. Conclusions and Further Work

This paper bridges the gap between the results of formal research and applicability. In a thorough investigation we show that the mathematically sound approach presented elsewhere is a useful addition to existing interoperability standards, with an intuitive semantics.

In detail it is shown that the user's concepts of topological behavior can be preserved by transparent behavior of topological operations on vector and raster representations. The value of this demonstration even increases considering that types of coverages other than grids, such as triangles, Thiessen polygons (OGC 1998b), or quadtrees, fit to the surface view and can be interpreted topologically as cell complexes. Coverages with a spatial domain of irregular point sets can be treated in the same manner: nearest neighborhood interpolation maps the domain to Thiessen polygons.

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