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Spatial Relations, Minimum Bounding Rectangles, and Spatial Data Structures

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Abstract Despite the attention that spatial relations have attracted in domains such as Spatial Query Languages, Image and Multimedia Databases, Reasoning and Geographic Applications, they have not been extensively applied in spatial access methods. This paper is concerned with the retrieval of topological and direction relations using spatial data structures based on Minimum Bounding Rectangles. We describe topological and direction relations between region objects and we study the spatial information that Minimum Bounding Rectangles convey about the actual objects they enclose. Then we apply the results in R-trees and their variations, R^+-trees and R^*-trees, in order to minimise the number of disk accesses for queries involving topological and direction relations. We also investigate queries that express complex spatial conditions in the form of disjunctions and conjunctions, and we discuss possible extensions.

1. INTRODUCTION

The representation and processing of spatial relations is crucial in geographic applications because very often in geographic space, relations among spatial entities are as important as the entities themselves. Depending on the application domain, some spatial relations may be more significant than others: *topological* relations have been used to access consistency in Geographical Databases (Egenhofer and Sharma, 1993), *ordinal* relations to describe aggregation hierarchies (Kainz et al., 1993), and *direction* relations have been incorporated in Spatial Query Languages (Papadias and Sellis, 1994a).

Topological relations describe concepts of neighbourhood, incidence and overlap and stay invariant under transformations such as scaling and rotation. They are important for numerous practical applications that involve queries of the form "find all land parcels *adjacent* to the main park". Our approach on topological relations is based on the 4-intersection model (Egenhofer and Herring, 1990). Tests with human subjects have shown evidence that this model, and its extension the 9-intersection model (Egenhofer, 1991) have potential for defining cognitively meaningful spatial predicates, a fact that renders them good candidates for commercial systems (Mark and Egenhofer, 1994). In fact, the 4- and 9-intersection models have been implemented in Geographical Information Systems (GIS); Hadzilacos and Tryfona (1992) used them to express geographical constraints, and Mark and Xia (1994) to determine spatial relations in ARC/INFO. In addition there are implementations in commercial systems such as Intergraph (MGE, 1993) and Oracle (Keighan, 1993).

Direction relations (*north*, *north_east*) deal with order in space. Unlike topological relations, there are not widely accepted definitions of direction relations. Most people will agree that Germany is *north* of Italy, but what about the relation between France and Italy? There are parts of France that are north of all parts of Italy, but is it enough for stating that France is *north* of Italy?[1] These concepts are directly applicable to geographic applications where the formalization of spatial relations is crucial for user interfaces and query optimisation strategies. Furthermore, the importance of direction relations has been pointed out by several researchers in areas including Spatial Data Structures (Peuquet, 1986), Geographic and Multimedia Databases (Papadias et al., 1994a, Sistla et al. 1994), Spatial Reasoning (Glasgow and Papadias, 1992), Cognitive Science (Jackendoff, 1983) and Linguistics (Herskovits, 1986).

This paper shows how direction and topological relations between region objects can be processed in Spatial Databases. Conventional Database Management Systems are designed to store one-dimensional data (e.g., integers, records) and, as a result, the underlying data structures are not powerful enough to efficiently handle boxes, polygons etc. On the other hand, the need to process multi-dimensional data in applications, such as Cartography, Computer-Aided Design, and Computer Vision has led to the

[1] A survey and an experimental study regarding the use of direction relations in cognitive spatial reasoning at geographic scales can be found in (Mark, 1992).

development of alternative spatial access methods. It is a common strategy in spatial access methods to store object approximations and use these approximations to index the data space in order to efficiently retrieve the potential objects that satisfy the result of a query. The paper is concerned with the retrieval of objects that satisfy certain direction and topological relations using spatial methods based on Minimum Bounding Rectangle (MBR) approximations. In particular we concentrate on R-trees and their variations.

Section 2 describes topological relations and Section 3 defines direction relations between extended objects using definitions for points. Section 4 is concerned with MBRs and spatial data structures based on them. Section 5 describes the retrieval of topological relations and Section 6 the retrieval of directions relations using MBR-based data structures. Section 7 presents the tests in actual implementations and compares the results. Section 8 discusses queries that involve both topological and direction information and Section 9 concludes with comments about future work.

2. TOPOLOGICAL RELATIONS

The representation of topological relations is necessary for several practical applications that involve *neighbourhood*, *inclusion*, *overlap* and related queries. In particular, in this section we describe topological relations between region objects without holes as defined by the 4-intersection[2] model (Egenhofer and Herring, 1990). According to this model, each object p is represented in 2D space as a point set which has an interior (denoted by p^o) and a boundary (denoted by ∂p). Let x_i be a point (for the rest of the paper points are denoted by a letter followed by a subscript). If we make the distinction between the boundary and the interior, we can define the topological relation between two objects by examining object intersections:

$$\neg\exists x_i\,(x_i \in \partial p \wedge x_i \in \partial q) \qquad \exists x_i\,(x_i \in \partial p \wedge x_i \in \partial q)$$
$$\neg\exists x_j\,(x_j \in p^o \wedge x_j \in \partial q) \qquad \exists x_j\,(x_j \in p^o \wedge x_j \in \partial q)$$
$$\neg\exists x_k\,(x_k \in \partial p \wedge x_k \in q^o) \qquad \exists x_k\,(x_k \in \partial p \wedge x_k \in q^o)$$
$$\neg\exists x_l\,(x_l \in p^o \wedge x_l \in q^o) \qquad \exists x_l\,(x_l \in p^o \wedge x_l \in q^o)$$

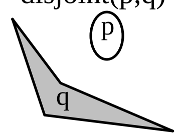
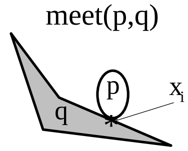
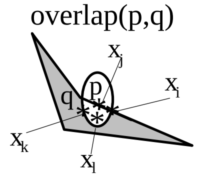
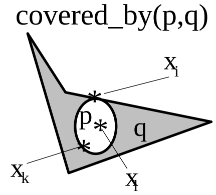
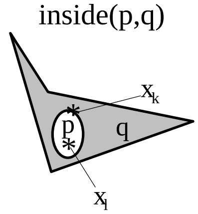
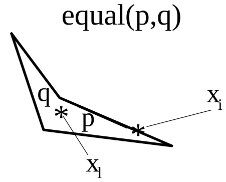
The formulae in each line are pairwise disjoint; one is true, while the other is false for any pair of objects. By taking conjunctions of four formulae, one from each line, we can create 16 pairwise disjoint topological relations between objects. However not all of these relations are valid due to the constraints imposed by the properties of the object boundaries and interiors. For instance, for all pairs of objects the following constraints must always hold:

$$[\exists x_l\,(x_l \in p^o \wedge x_l \in q^o)] \Rightarrow$$
$$[\exists x_i\,(x_i \in \partial p \wedge x_i \in \partial q) \vee \exists x_j\,(x_j \in p^o \wedge x_j \in \partial q) \vee \exists x_k\,(x_k \in \partial p \wedge x_k \in q^o)]$$
$$[\neg\exists x_l\,(x_l \in p^o \wedge x_l \in q^o) \wedge \neg\exists x_i\,(x_i \in \partial p \wedge x_i \in \partial q)] \Rightarrow$$

[2] Since we deal with region objects without holes, the 9 and 4 intersection models are equivalent in the sense that they permit the same valid binary topological relations.

$$[\neg\exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \neg\exists x_k\ (x_k \in \partial p \wedge x_k \in q^o)]$$

Out of the 16 possible relations that can be defined using the previous conjunctions only the following eight illustrated in Table 1 are valid. These relations are proved to be pairwise disjoint, and they provide a complete coverage (Randell et al., 1992). For each of the primitive relations Table 1 includes some intersection points. The term *primary object* denotes the object to be located and the term *reference object* denotes the object in relation to which the primary object is located. For the following examples the primary objects are transparent and the reference objects are grey.

disjoint(p,q)

$\neg\exists x_i\ (x_i \in \partial p \wedge x_i \in \partial q) \wedge \neg\exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \neg\exists x_k\ (x_k \in \partial p \wedge x_k \in q^o) \wedge \neg\exists x_l\ (x_l \in p^o \wedge x_l \in q^o)$
meet(p,q)

$\exists x_i\ (x_i \in \partial p \wedge x_i \in \partial q) \wedge \neg\exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \neg\exists x_k\ (x_k \in \partial p \wedge x_k \in q^o) \wedge \neg\exists x_l\ (x_l \in p^o \wedge x_l \in q^o)$
overlap(p,q)

$\exists x_i\ (x_i \in \partial p \wedge x_i \in \partial q) \wedge \exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \exists x_k\ (x_k \in \partial p \wedge x_k \in q^o) \wedge \exists x_l\ (x_l \in p^o \wedge x_l \in q^o)$
covered_by(p,q)

$\exists x_i\ (x_i \in \partial p \wedge x_i \in \partial q) \wedge \neg\exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \exists x_k\ (x_k \in \partial p \wedge x_k \in q^o) \wedge \exists x_l\ (x_l \in p^o \wedge x_l \in q^o)$
inside(p,q)

$\neg\exists x_i\ (x_i \in \partial p \wedge x_i \in \partial q) \wedge \neg\exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \exists x_k\ (x_k \in \partial p \wedge x_k \in q^o) \wedge \exists x_l\ (x_l \in p^o \wedge x_l \in q^o)$
equal(p,q)

$\exists x_i\ (x_i \in \partial p \wedge x_i \in \partial q) \wedge \neg\exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \neg\exists x_k\ (x_k \in \partial p \wedge x_k \in q^o) \wedge \exists x_l\ (x_l \in p^o \wedge x_l \in q^o)$

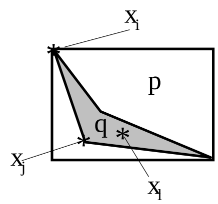
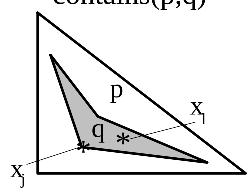
covers(p,q)  $\exists x_i\ (x_i \in \partial p \wedge x_i \in \partial q) \wedge \neg\exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \exists x_k\ (x_k \in \partial p \wedge x_k \in q^o) \wedge \exists x_l\ (x_l \in p^o \wedge x_l \in q^o)$
contains(p,q)  $\neg\exists x_i\ (x_i \in \partial p \wedge x_i \in \partial q) \wedge \exists x_j\ (x_j \in p^o \wedge x_j \in \partial q) \wedge \neg\exists x_k\ (x_k \in \partial p \wedge x_k \in q^o) \wedge \exists x_l\ (x_l \in p^o \wedge x_l \in q^o)$

Table 1 Topological relations

The disjunction of all topological relations that involve some non-empty intersection is called *not_disjoint*, i.e., not_disjoint ≡ meet ∨ overlap ∨ covered_by ∨ inside ∨ equal ∨ covers ∨ contains. The subset of topological relations that deal with the concepts of inclusion and containment have been called *ordinal* or *order* relations (Kainz et al. 1993). Based on the above definitions we can define the following ordinal relations:

– in(p,q) ≡ inside(p,q)∨ covered_by(p,q)
– consists_of(p,q) ≡ contains(p,q)∨ covers(p,q).

Topological (and ordinal) relations are very useful in geographic applications. The query “find all industrial areas of France adjacent to Italy”, for example, would require a search of industrial areas *in* France that *meet* Italy. In the next section we discuss direction relations, another class of relations that are important for practical applications.

3. DIRECTION RELATIONS

In order to define direction relations between extended objects we first define relations between points and apply these definitions using universally and existentially quantified formulae. We assume an absolute frame of reference and a pair of orthocanonic axes x and y. Our method is *projection based*, that is, direction relations are defined using projection lines vertical to the coordinate axes. Another approach is based on the *cone-shaped* concept of direction, i.e., direction relations are defined using angular regions between objects. Extensive studies of the two approaches can be found in (Frank, 1994, Hernandez, 1994).

Let p_i be a point of object p (p denotes the primary object), q_j be a point of object q (q denotes the reference object), and *X* and *Y* be functions that return the *x* and y coordinate of a point. If we draw projection lines from a reference point, the plane is divided into nine partitions (Figure 1)[3].

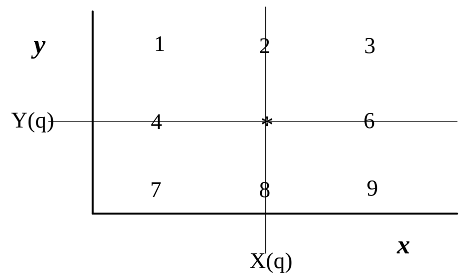


Fig. 1 Plane partitions using one point per object

Thus there are nine possible ways to put a primary point in the plane and therefore nine pairwise disjoint relations that provide a complete coverage. These relations can be defined as:

north_west(p_i,q_j) ≡ $X(p_i)<X(q_j) \wedge Y(p_i)>Y(q_j)$

restricted_north(p_i,q_j) ≡ $X(p_i)=X(q_j) \wedge Y(p_i)>Y(q_j)$

north_east(p_i,q_j) ≡ $X(p_i)>X(q_j) \wedge Y(p_i)>Y(q_j)$

restricted_west(p_i,q_j) ≡ $X(p_i)<X(q_j) \wedge Y(p_i)=Y(q_j)$

same_position(p_i,q_j) ≡ $X(p_i)=X(q_j) \wedge Y(p_i)=Y(q_j)$

restricted_east(p_i,q_j) ≡ $X(p_i)>X(q_j) \wedge Y(p_i)=Y(q_j)$

south_west(p_i,q_j) ≡ $X(p_i)<X(q_j) \wedge Y(p_i)<Y(q_j)$

restricted_south(p_i,q_j) ≡ $X(p_i)=X(q_j) \wedge Y(p_i)<Y(q_j)$

south_east(p_i,q_j) ≡ $X(p_i)>X(q_j) \wedge Y(p_i)<Y(q_j)$

The above relations correspond to the *highest accuracy* that we can achieve when reasoning in terms of points using the concept of projections, in the sense that they cannot be represented as disjunctions of other relations. Exactly one of the previous relations holds true between any pair of points. The relations are transitive and *same_position* is also symmetric. The rest form four pairs of converse relations (e.g., north_east(p_i,q_j) ⇔ south_west(q_j, p_i)). Additional direction relations can be defined using disjunctions of the nine relations:

north(p_i,q_j) ≡ north_west(p_i,q_j) ∨ restricted_north(p_i,q_j) ∨ north_east(p_i,q_j)

east(p_i,q_j) ≡ north_east(p_i,q_j) ∨ restricted_east(p_i,q_j) ∨ south_east(p_i,q_j)

south(p_i,q_j) ≡ south_west(p_i,q_j) ∨ restricted_south(p_i,q_j) ∨ south_east(p_i,q_j)

west(p_i,q_j) ≡ north_west(p_i,q_j) ∨ restricted_west(p_i,q_j) ∨ south_west(p_i,q_j)

same_level(p_i,q_j) ≡ restricted_west(p_i,q_j) ∨ same_position(p_i,q_j) ∨ restricted_east(p_i,q_j)

same_width(p_i,q_j) ≡ restricted_north(p_i,q_j) ∨ same_position(p_i,q_j) ∨ restricted_south(p_i,q_j).

[3] In general, if k is the number of points used to represent the reference object, then the plane is divided into $(2k+1)^2$ partitions; k^2 of the partitions are points, $2k(k+1)$ are open line segments and $(k+1)^2$ are open regions. The numbers refer to the case that none of the k points has a common co-ordinate with some other point.

Using the above definitions between points we define spatial relations between objects. The relation *strong_north* between objects p and q denotes that all points of p are *north* of all points of q: strong_north $\equiv \forall p_i \forall q_j$ north(p_i,q_j). Figure 2a illustrates a configuration that corresponds to the relation strong_north. As in the case of *strong_north*, all the relations between objects are defined using universally and existentially quantified formulae representing relations between points. This is different from (Papadias and Sellis, 1993; Papadias et al., 1994b) where direction relations are defined using representative points. The relation *weak_north* can be defined as: weak_north(p,q) $\equiv \exists p_i \forall q_j$ north$(p_i,q_j) \wedge \forall p_i \exists q_j$ north$(p_i,q_j) \wedge \exists p_i \exists q_j$ south(p_i,q_j). As Figure 2b illustrates, object p is *weak_north* of object q if:

– there exist some points of p which are north of all points of q (since $\exists p_i \forall q_j$ north(p_i,q_j)) and
– for each point p_i there exist points q_j such that north(p_i,q_j) (since $\forall p_i \exists q_j$ north(p_i,q_j)) and
– there exist some points of p which are south of some points of q (since $\exists p_i \exists q_j$ south(p_i,q_j)).

Fig. 2 Strong_north and weak_north relation

The relation *strong_bounded_north* is defined as: strong_bounded_north(p,q) $\equiv \forall p_i \forall q_j$ north$(p_i, q_j) \wedge \forall p_i \exists q_j$ north_east$(p_i, q_j) \wedge \forall p_i \exists q_j$ north_west(p_i, q_l). According to Figure 3a, all points of object p must be in the region bounded by the horizontal line that passes from q's northmost point and by the vertical lines that also bound q. In a similar manner we can define *weak_bounded_north* as, weak_bounded_north(p,q) $\equiv \exists p_i \forall q_j$ north$(p_i, q_j) \wedge \exists p_i \exists q_j$ south$(p_i, q_j) \wedge \forall p_i \exists q_j$ north_east$(p_i, q_j) \wedge \forall p_i \exists q_j$ north_west(p_i, q_j). As in the case of *strong_bounded_north*, object p is bounded by the vertical lines that bound q, but p is also *weak_north* q (Figure 3.b).

Fig. 3 Strong_bounded_north and weak_bounded_north relations

Strong_north_east is a variation of the *strong_north* relation which can be defined as strong_north_east(p,q) $\equiv \forall p_i \forall q_j$ north_east(p_i, q_j), i.e., all points of p must be *north_east* of all points of q. Similarly we can define *weak_north_east* relation as: weak_north_east(p,q) $\equiv \exists p_i \forall q_j$ north_east$(p_i,q_j) \wedge$

$\exists p_i \exists q_j south(p_i,q_j) \wedge \forall p_i \exists q_j north\_east(p_i,q_j)$. Figure 4 illustrates *strong_north_east* and *weak_north_east* relations.

Fig. 4 Strong_north_east and weak_north_east relations

The relation *just_north* is defined as: $just\_north(p,q) \equiv \forall p_i \; \forall q_j \; (north(p_i,q_j) \vee same\_level(p_i,q_j)) \wedge \exists p_i \; \exists q_j \; same\_level(p_i,q_j) \wedge \exists p_i \; \exists q_j \; north(p_i,q_j)$. As Figure 5a illustrates, according to this relation there must be some point of p at the *same_level* with the northmost point of q, and all the other points of p must be *north* of all points of q. We define the relation *north* between objects as: $north(p,q) \equiv \exists p_i \; \forall q_j \; north(p_i,q_j) \wedge \forall p_i \; \exists q_j \; north(p_i,q_j)$, that is, there exist points of p that are *north* of all points of q, and for each point of p there exist point(s) of q south of it (Figure 5b). It can be seen that $north \equiv strong\_north \vee weak\_north \vee just\_north$.

Fig. 5 Just_north and north relations

All direction relations between objects that we have defined refer to the direction north. The relations are not necessarily pairwise disjoint. In particular:

strong_north_east(p,q) ⇒ strong_north(p,q), strong_bounded_north(p,q) ⇒ strong_north(p,q),
weak_north_east(p,q) ⇒ weak_north(p,q), weak_bounded_north(p,q) ⇒ weak_north(p,q),
strong_north(p,q) ⇒ north(p,q), weak_north(p,q) ⇒ north(p,q), just_north(p,q) ⇒ north(p,q)

The definition of pairwise disjoint relations is not a difficult task, but it is beyond the scope of this paper that aims to show how direction relations of different resolution between extended objects can be defined and retrieved in spatial data structures. Furthermore since the above relations are concerned with direction north only, they do not provide a complete coverage. Other directions can be formed in a similar manner. Such relations are important because they allow the expression of queries in various levels of qualitative resolution. As a special case consider the example with the direction relations of the introduction and the map in Figure 6. According to the above definitions, France is *weak_north*, while Germany is *strong_north* of Italy. Denmark is *strong_bounded_north* and Romania is *weak_north_east* of Italy. All these variations of *north* may be very important for application domains where one concept of “north” is not sufficient to express all configurations. Numerous additional relations between region, line and point data can be defined and used in queries depending on the application needs. If we follow the same terminology for the other directions, we can say that UK is *strong_north_west* and Spain is *strong_bounded_west* of Italy

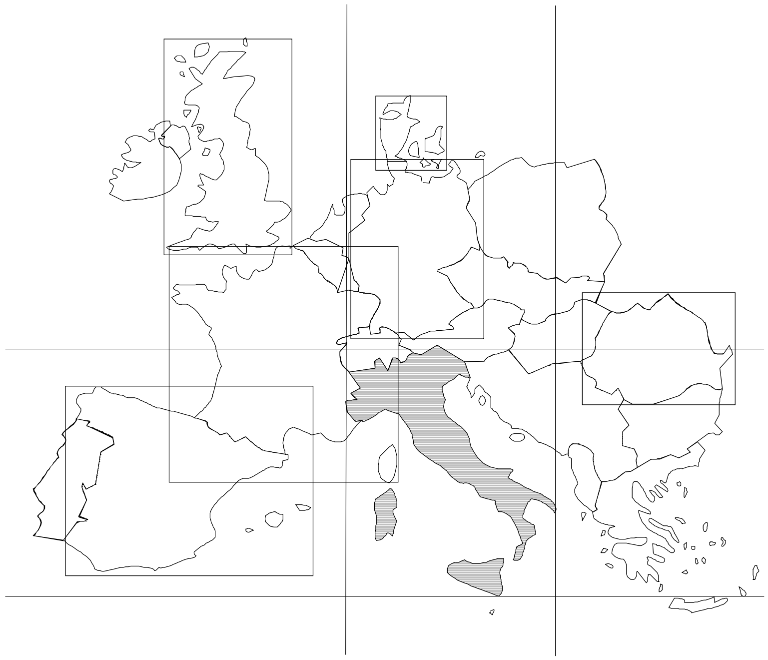


Fig. 6 A geographic example involving direction relations

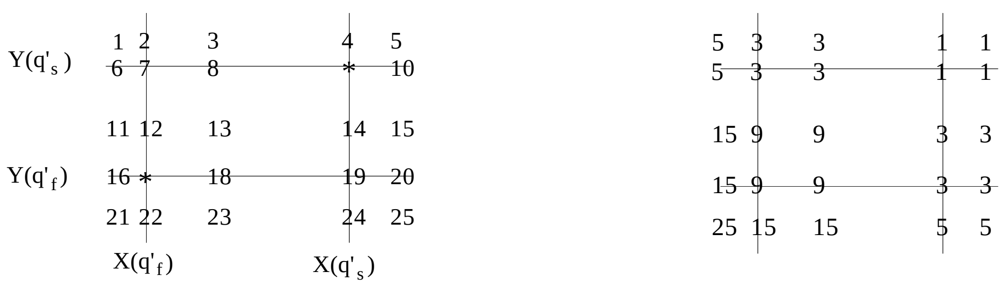
Most of the work on direction relations has concentrated on point objects (Freksa, 1992; Frank, 1992). According to this approach, each object is abstracted as one point, which may be the center of mass, the center of symmetry etc. Our work extends previous approaches to direction relations by dealing with extended objects instead of point objects. Peuquet and Ci-Xiang (1987) also designed an algorithm for the determination of direction relations between arbitrary polygons represented by minimum bounding rectangles. Unlike our approach, their method is based on the *cone-shaped* concept of direction and involves only four direction relations (*north, east*, *west* and *south*).

4. MINIMUM BOUNDING RECTANGLES

Usually, spatial access methods instead of the actual objects, store approximations that need only a few points for their representation. Such approximations are used to efficiently retrieve candidates that could satisfy a query. Depending on the application domain there are several options in choosing object approximations. Brinkhoff et al. (1993) compare the use of *rotated minimum bounding rectangles*, *convex hulls* and *minimum bounding n-corner convexes* in spatial access methods. In this paper we examine methods based on the traditional approximation of Minimum Bounding Rectangles. MBRs have been used extensively to approximate objects in Spatial Data Structures and Spatial Reasoning because they need only two points for their representation; in particular, each object *q* is represented as an ordered pair (q'_f, q'_s) of points that correspond to the lower left and the upper right point of the MBR *q'* that covers q (q'_f and q'_s are called the *edge* points - q'_f stands for the first and q'_s for the second point of the MBR). While MBRs demonstrate some disadvantages when approximating non convex or diagonal objects, they are the most commonly used approximations in spatial applications. In the next subsection we investigate the possible spatial relations between MBRs.

4.1 Spatial Relations between MBRs

When we use two points for the representation of the reference (region) object, the plane is divided into 25 partitions. Four partitions are points, 12 partitions are line segments and 9 partitions are open regions. The character * in Figure 7a denotes the edge points of the reference MBR and the numbers correspond to partitions. Therefore there are 25 ways to locate a primary point in the plane. If the primary object is also represented by a pair of points ordered on the x and y axis (i.e., it is the MBR of a region object) the number of possible relations between the two MBRs is 169. Figure 7b illustrates how the placement of the first point of the primary object in a partition, constrains the number of possible partitions for the second point. If for instance p'_f is in partition 8 of Figure 7a, there are only three valid partitions for p'_s, namely 3, 4 and 5. The sum of all numbers in Figure 7b is 169, that is, there are 169 unique placements for the edge points and therefore 169 possible relations between MBRs (this number is the square of possible relations between time intervals introduced in Allen 1983).



a **b**

Fig. 7 Plane partitions using two points per object

Figure 8 illustrates the possible configurations between MBRs. Each configuration corresponds to a unique placement of points p'_f and p'_s with respect to q' R_{3-9}, for instance, denotes that p'_f is in partition 13 and p'_s in partition 3.

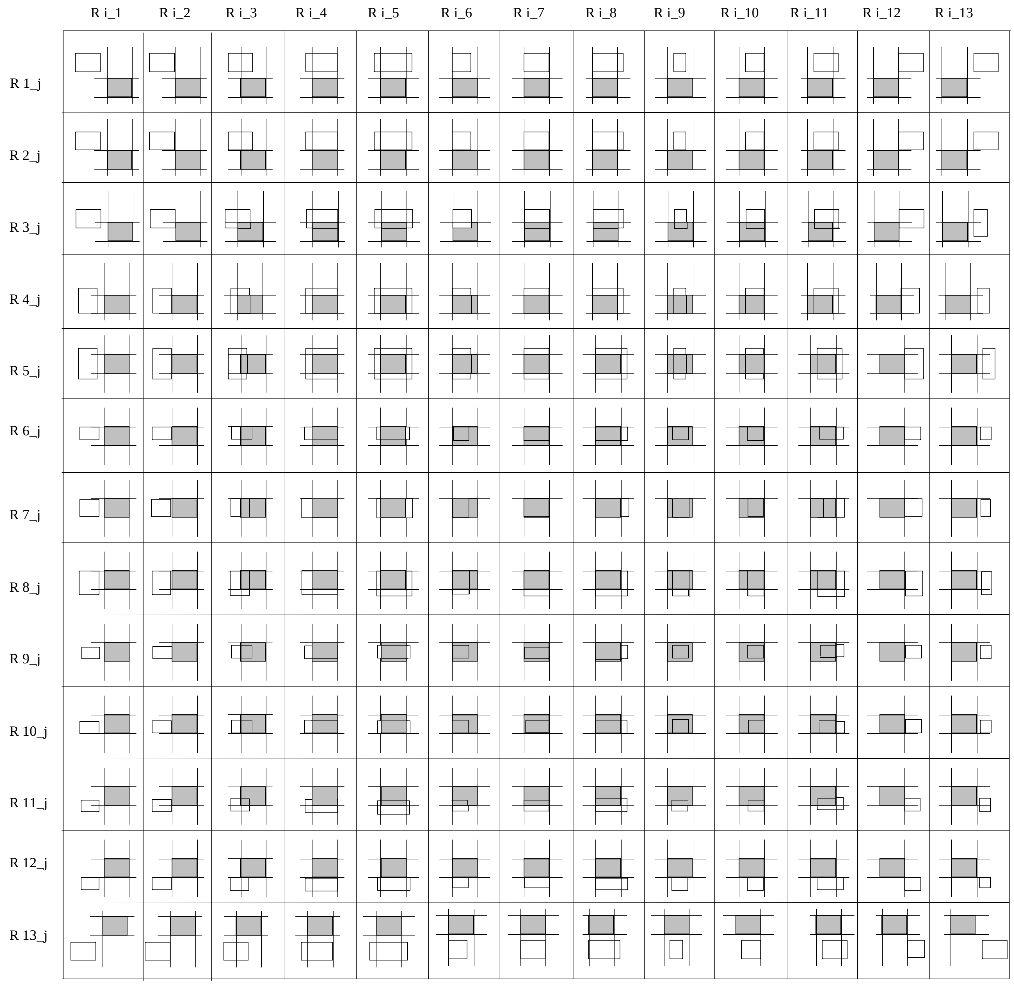


Fig. 8 Possible relations between MBRs

Figure 9 illustrates the corresponding topological relation for the 169 configurations of Figure 8. For instance, all the configurations of the first row (R_{1_j} where j can take any value from 1 to 13) correspond to the *disjoint* relation, and the total number of configurations in which the MBRs are *disjoint* is 48. On the other hand, only relation R_{5_5} corresponds to the topological relation *contains*.

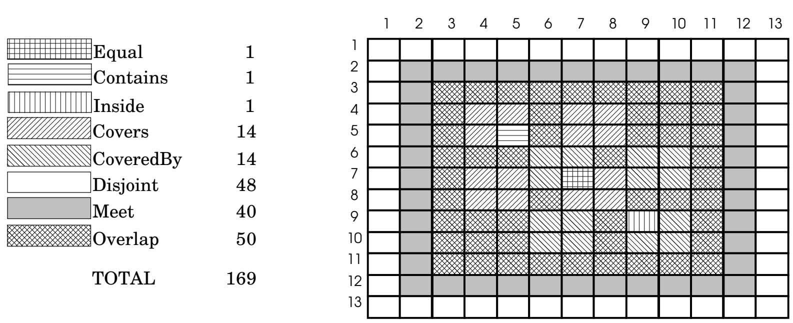


Fig. 9 Topological relations between MBRs

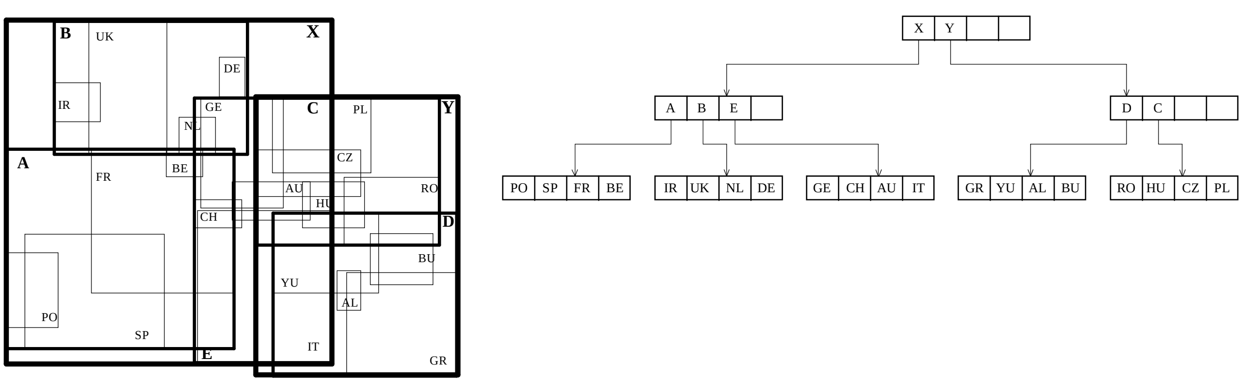
The topological relation between MBRs yields useful information about the topological relation between actual objects. If it is known that the MBRs are *disjoint*, it can be concluded that the objects they enclose are also *disjoint*. Furthermore, depending on the relative positions of the edge points, several conclusions can be drawn about the direction relationships between the actual objects, as for instance:

- $north(p'_f, q'_f) \Rightarrow \forall p_i\ \exists q_j\ north(p_i, q_j)$,
- $north(p'_f, q'_s) \Rightarrow \forall p_i\ \forall q_j\ north(p_i, q_j)$,
- $north(p'_s, q'_f) \Rightarrow \exists p_i\ \exists q_j\ north(p_i, q_j)$ and
- $north(p'_s, q'_s) \Rightarrow \exists p_i\ \forall q_j\ north(p_i, q_j)$.

For example, $north(p'_f, q'_s)$ implies that all points of object p are *north* of all points of object q. This relation between p and q was call called *strong_north* in Section 3. In Figure 8, *strong_north* is mapped onto relations $R_{1\text{-}j}$ where j can take any value from 1 to 13 because these relations correspond to the only configurations in which $north(p'_f, q'_s)$. Therefore, the result of the query "find all objects *strong_north* of object q" consists of all objects whose MBRs satisfy the relation $R_{1\text{-}j}$. In sections 5 and 6 we discuss in detail the topological and direction information that MBRs convey about the actual objects they enclose and how this information can be used for query optimisation techniques in MBR-based data structures.

4. 2 Spatial Data Structures based on MBRs

The most promising group of spatial data structures based on MBRs includes R-trees (Guttman, 1984) and their variations. The R-tree data structure is a height-balanced tree, which consists of intermediate and leaf nodes (R-trees are direct extensions of B-trees in k-dimensions). The MBRs of the actual data objects are assumed to be stored in the leaf nodes of the tree. Intermediate nodes are built by grouping rectangles at the lower level. An intermediate node is associated with some rectangle which encloses all rectangles that correspond to lower level nodes. Each pair of nodes may satisfy any of the 169 configurations of Figure 8. Figure 10 shows how the map of Figure 6 is represented by MBRs and stored in an R-tree. We assume a branching factor of 4, i.e., each intermediate node contains at most four entries. At the lower level MBRs of countries (denoted by two letters) are grouped into five intermediate nodes A, B, C, D and E. At the next level, the five nodes are grouped into two larger nodes X and Y. Figure 10b illustrates the corresponding R-tree.



a b

Fig. 10 Some MBRs grouped into intermediate rectangles and the corresponding R-tree

The fact that R-trees permit overlap among node entries sometimes leads to unsuccessful hits on the tree structure. The R+-tree (Sellis et al., 1987) and the R*-tree (Beckmann et al., 1990) methods have been proposed to address the problem of performance degradation caused by the overlapping regions or excessive dead-space respectively. To avoid this problem, R+-tree achieves zero overlap among intermediate node entries by allowing partitions to split nodes. The trade-off is that more space is required because of the duplicate entries and thus the height of the tree may be greater than the original R-tree. On the other hand, the R*-tree permits overlap among nodes, but tries to minimise it by organising rectangles into nodes using a more complex algorithm than the one of the original R-tree.

Figure 11 illustrates how each method would organise a set of rectangles into intermediate nodes. The original R-tree would create two intermediate nodes A and B. Node A contains MBRs 1,2,3 and 6 (the grey MBRs) and node B contains 7, 4 and 5 (Figure 11a). The R+-tree algorithm splits rectangles that fall on some partition line (rectangles 2 and 7) achieving zero overlap between A and B. This results in duplicate entries that may increase the number of intermediate nodes (if we assume a branching factor of 4 we have an extra node C) (Figure 11b). The R*-tree algorithm groups rectangles in a different way in order to achieve smaller overlap and coverage area of the intermediate nodes compared to the original R-tree algorithm (Figure 11c).

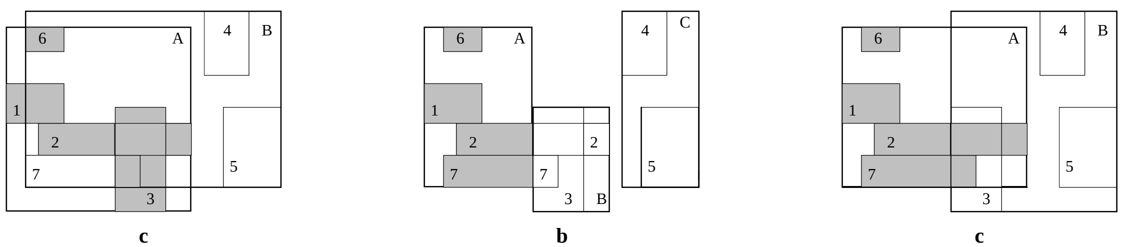


Fig. 11 Some rectangles and the corresponding grouping in the R-tree variants

So far, R trees and their variations have been used for queries involving only the relations *disjoint* and *not_disjoint*. If these are the only relations of interest, then when the MBRs of two objects are *disjoint* we can conclude that the objects that they represent are also *disjoint*. If the MBRs however share common points, no conclusion can be drawn about the spatial relation between the objects (they can be *disjoint* or *not_disjoint*). For this reason, spatial queries involve the following two step strategy:

1. *Filter step*: The tree is used to rapidly eliminate objects that could not possibly satisfy the query. The result of this step is a set of candidates which includes all the results and possibly some false hits.
2. *Refinement step*: Each candidate is examined (by using computational geometry techniques). False hits are detected and eliminated.

Brinkhoff et al. (1993b) extended the above strategy to include a second filter step with finer approximations than MBR (e.g. *convex hulls*) in order to exclude some false hits from the set of candidates. The present work uses the original two-step strategy to explore a larger, and more detailed set of direction and topological relations. Such an investigation is important because spatial queries frequently require the kind of qualitative resolution distinguished by the topological and direction relations of sections 2 and 3.

5. IMPLEMENTATION OF TOPOLOGICAL RELATIONS

Since the MBRs are only approximations of the actual objects, the topological relation between MBRs does not necessarily coincide with the topological relation between the objects. In order to answer the query "find all objects p *equal* to object q" we need to retrieve all MBRs that are *equal* with q', because only they may enclose objects that satisfy the query. According to Figure 9 these MBRs satisfy the relation R_{7-7} with respect to q'. On the other hand, the retrieved MBRs may enclose objects that satisfy the relations *equal*, *overlap*, *covered_by*, *covers* or *meet* with respect to q (see Figure 12). A refinement step is needed when dealing with topological relations, if the MBRs of the retrieved objects are not *disjoint*.

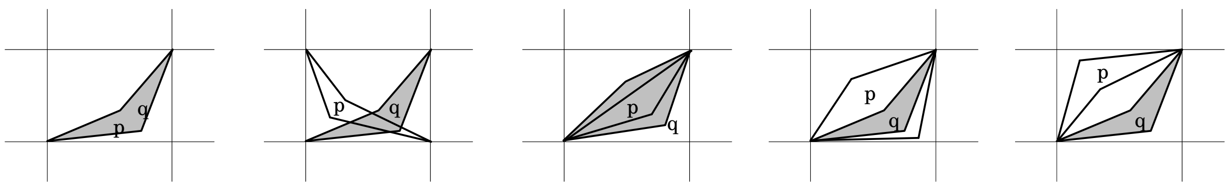


Fig. 12 Possible relations for objects when the MBRs are *equal*

The relations *contains* and *inside* also involve the retrieval of MBRs that satisfy unique configurations. In particular, the objects that *contain* q can only be in MBRs that *contain* q' (MBRs that satisfy the relation R_{5-5} with respect to q'), while the objects *inside* q, can only be in MBRs that are *inside* q' (MBRs that satisfy the relation R_{9-9} with respect to q'). As in the case of *equal*, a refinement step is needed because:

– contains(p',q') $\Rightarrow$ disjoint(p,q)$\vee$meet(p,q)$\vee$overlap(p,q)$\vee$contains(p,q)$\vee$covers(p,q) and

– inside(p',q') $\Rightarrow$ disjoint(p,q)$\vee$meet(p,q)$\vee$overlap(p,q)$\vee$inside(p,q) $\vee$covered_by(p,q)

The rest of the relations involve the retrieval of more than one MBR configurations. In case of *covers*, all MBRs that satisfy the relations R_{i-j} where i and j in {4,5,7,8} with respect to q' should be retrieved (MBRs that satisfy the relations *covers*, *contains*, *equal* with respect to q'). As Figure 13 illustrates, these MBRs may enclose objects that *cover* the reference object. Similarly the retrieval of *covered_by* involves all MBRs that satisfy the relations R_{i-j} where i,j in {6,7,9,10} with respect to q' (MBRs that satisfy the relations *covered_by*, *inside*, *equal*). The remaining relations involve the retrieval of a large number of MBRs. In the case of *disjoint*, for instance, all the MBRs may enclose objects that are *disjoint* with the reference object.

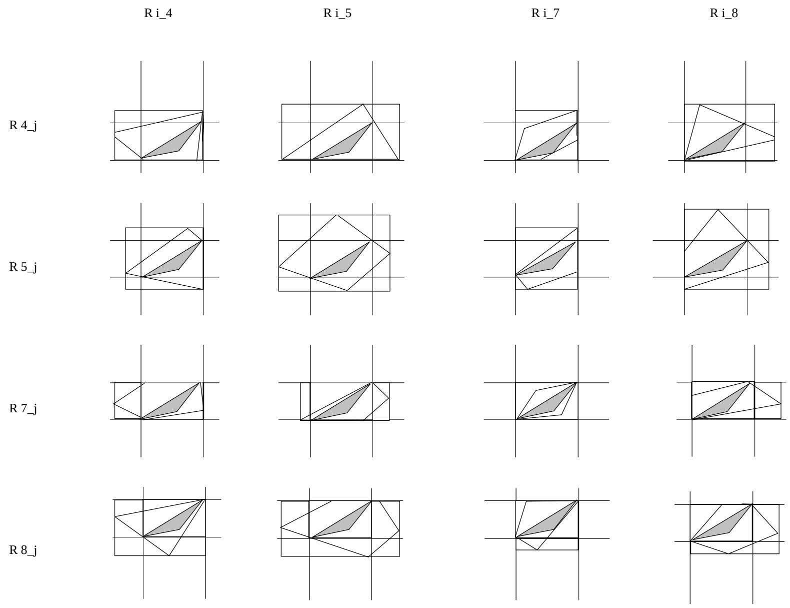
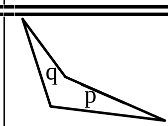
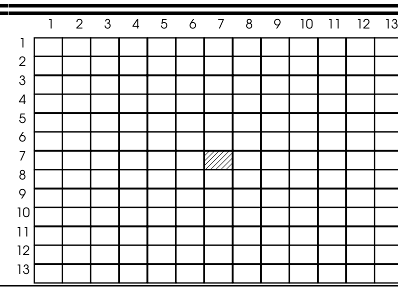
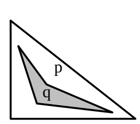
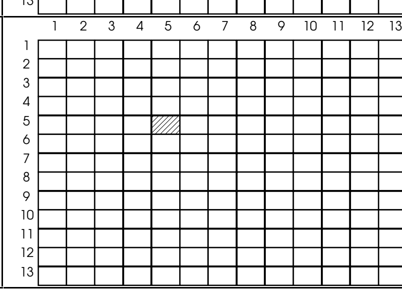


Fig. 13 Configurations of MBRs that yield the relation *covers* between actual objects

We followed the same procedure for the relations *meet* and *overlap*. Table 2 shows the conclusions that can be drawn if we use the concept of projections to study topological relations. The first column of Table 2 contains the topological relation for the actual objects and the second column a configuration that satisfies the relation. The third column describes the corresponding relation(s) between MBRs and the fourth column illustrates the subset of the 169 configurations that satisfy this relation (the grey configurations are to be retrieved for the query "retrieve all objects that satisfy the relation of the first column"). The table refers to contiguous or non-contiguous region objects. If we assume contiguity we can restrict the output MBRs for some relations (Papadias et al., 1994c).

Topological relation between actual objects p and q	**Illustration**	**Leaf nodes p' to be retrieved**	**Illustration of the corresponding projections**
equal(p,q)		equal(p' ,q')	
contains(p,q)		contains(p' ,q')	

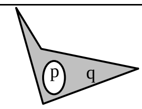
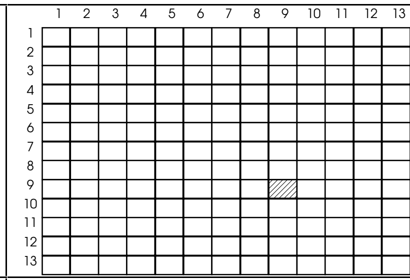
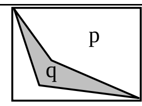
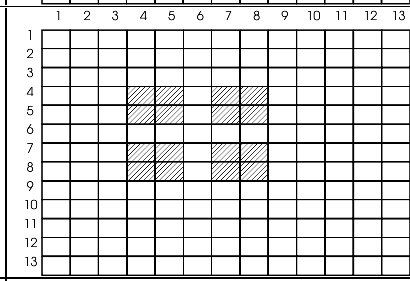
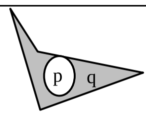
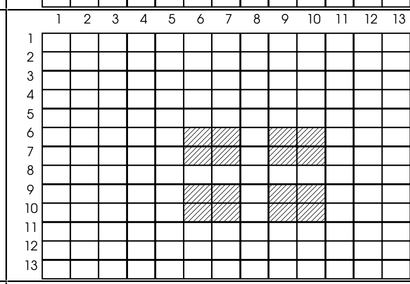
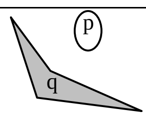
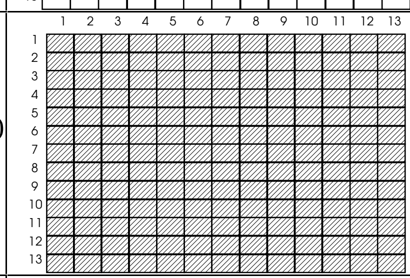
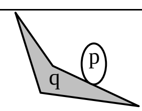
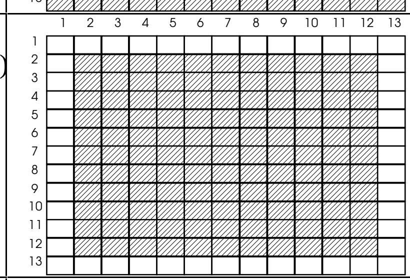
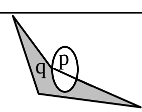
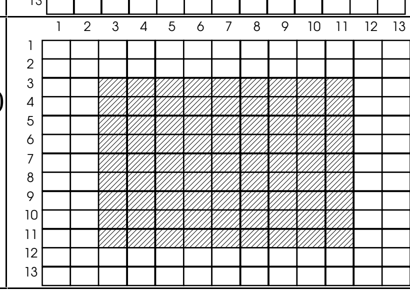
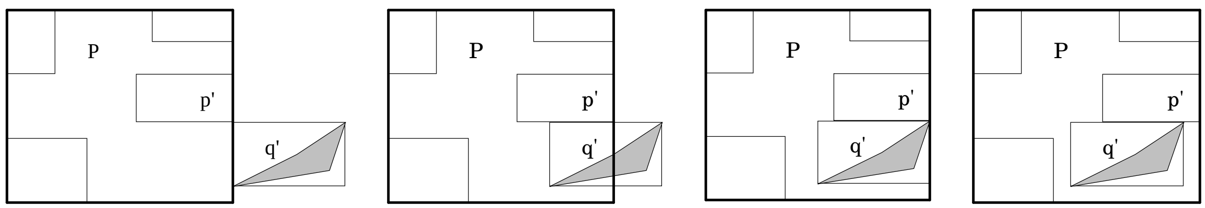
inside(p,q)		inside(p' ,q')	
covers(p,q)		covers(p' ,q'∨)contains(p' ,q'∨) equal(p' ,q')	
covered_by(p,q)		covered_by(p' ,q'∨)inside(p' ,q' ∨ equal(p' ,q')	
disjoint(p,q)		disjoint(p' ,q'∨)meet(p' ,q'∨) overlap(p' ,q'∨)covered_by(p' ,q' ∨ inside(p' ,q'∨)equal(p' ,q'∨) covers(p' ,q'∨)contains(p' ,q')	
meet(p,q)		meet(p' ,q'∨) overlap(p' ,q'∨) covered_by(p' ,q'∨)inside(p' ,q'∨) equal(p' ,q'∨)covers(p' ,q'∨) contains(p' ,q')	
overlap(p,q)		overlap(p' ,q'∨)covered_by(p' ,q' ∨ inside(p' ,q'∨)equal(p' ,q'∨) covers(p' ,q'∨)contains(p' ,q')	

Table 2 MBRs to be retrieved for topological relations

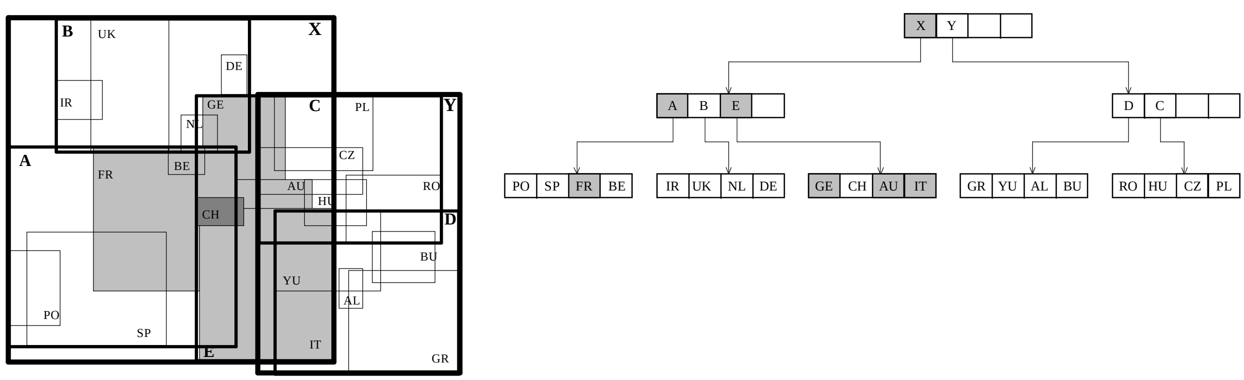
The above mapping from topological relations between actual objects to relations between MBRs refers to leaf nodes. In order to retrieve topological relations using R-trees one needs to define more general relations that will be used for propagation in the intermediate nodes of the tree structure. For instance, the intermediate nodes P that could enclose MBRs p' that *meet* the MBR q' of the reference object q, should satisfy the more general constraint $meet(P,q') \vee overlap(P,q') \vee covers(P,q') \vee contains(P,q')$. Figure 14 illustrates examples of such configurations.



a **b** **c** **d**

Fig. 14 Intermediate nodes that may contain MBRs that *meet* the MBR of the reference object

In our geographic example, if we want to find the countries that *meet* Switzerland, we can directly exclude all countries in intermediate node Y because they are *disjoint* with the MBR CH of Switzerland. Similarly sub-node B is excluded while nodes A and E are selected for propagation. The output MBRs of the search are IT, AU, GE and FR because they *overlap* with CH and therefore the actual objects may *meet* according to Table 2. The grey MBRs in Figure 15a correspond to potential candidates for the query, while the grey nodes of Figure 15b are the ones that satisfy the search conditions.



a **b**

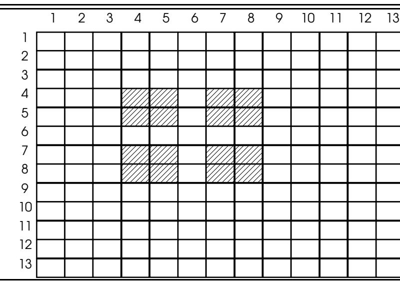
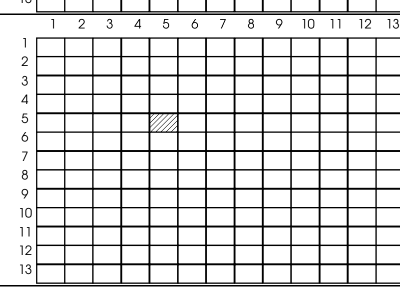
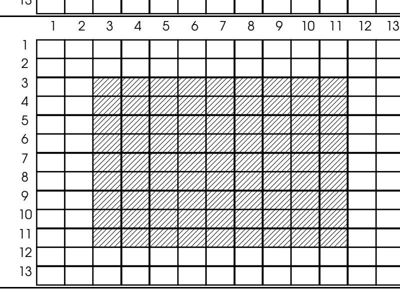
Fig. 15 An example search for topological queries

Following this strategy, the search space is pruned by excluding the intermediate nodes P that do not satisfy the previous constraint. Table 3 presents the relations that should be satisfied between an intermediate node P and the MBR q' of the reference object, so that the node will be selected for propagation.

Relation between MBRs p' and q'	**Relation between intermediate node P (that may enclose MBRs p') and reference MBR q'**
equal(p',q')	equal(P,q') ∨ covers(P,q') ∨ contains(P,q')
contains(p',q')	contains(P,q')
inside(p',q')	overlap(P,q') ∨ covered_by(P,q') ∨ inside(P,q') ∨ equal(P,q') ∨ covers(P,q') ∨ contains(P,q')
covers(p',q')	covers(P,q') ∨ contains(P,q')
covered_by(p',q')	overlap(P,q') ∨ covered_by(P,q') ∨ equal(P,q') ∨ covers(P,q') ∨ contains(P,q')
disjoint(p',q')	disjoint(P,q') ∨ meet(P,q') ∨ overlap(P,q') ∨ covers(P,q') ∨ contains(P,q')
meet(p',q')	meet(P,q') ∨ overlap(P,q') ∨ covers(P,q') ∨ contains(P,q')
overlap(p',q')	overlap(P,q') ∨ covers(P,q') ∨ contains(P,q')

Table 3 Topological relations for the intermediate nodes

The same relation between intermediate nodes and the reference MBR exists for all the levels of the tree structure. For instance, the intermediate nodes that could enclose other intermediate nodes P that satisfy the general constraint *meet(P,q')* ∨ *overlap(P,q')* ∨ *covers(P,q')* ∨ *contains (P,q')* should also satisfy the same constraint. This conclusion can be easily extracted from the above table and is applicable to all the topological relations of Table 3. Notice that the first column of Table 3 contains the relations between MBRs and not between actual objects. By combining Tables 2 and 3 we calculate the intermediate nodes to be retrieved given the relation between the actual objects (see Table 4).

Topological relation between actual objects p and q	**Intermediate nodes P to be searched**	**Illustration of the corresponding projections**
equal(p,q)	equal(P,q') ∨ covers(P,q') ∨ contains(P,q')	
contains(p,q)	contains(P,q')	
inside(p,q)	overlap(P,q') ∨ covered_by(P,q') ∨ inside(P,q') ∨ equal(P,q') ∨ covers(P,q') ∨ contains(P,q')	

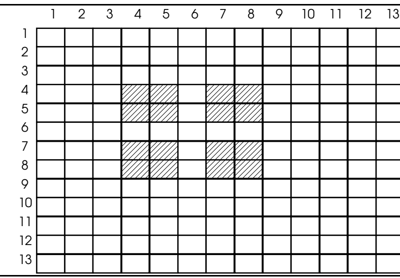
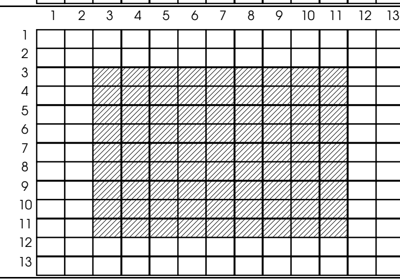
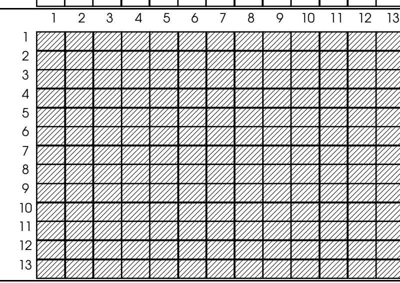
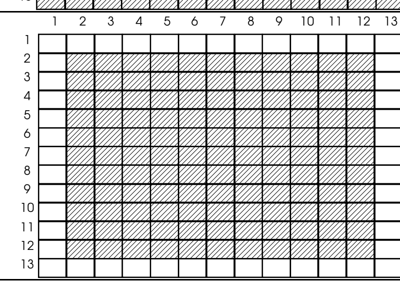
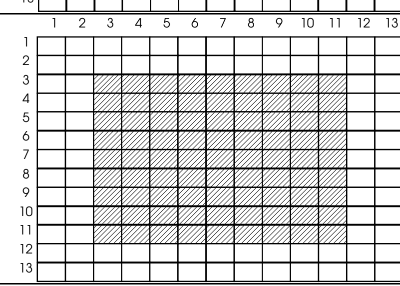
covers(p,q)	equal(P,q') ∨ covers(P,q') ∨ contains(P,q')	
covered_by(p,q)	overlap(P,q') ∨ covered_by(P,q') ∨ inside(P,q') ∨ equal(P,q') ∨ covers(P,q') ∨ contains(P,q')	
disjoint(p,q)	disjoint(P,q') ∨ meet(P,q') ∨ overlap(P,q') ∨ covered_by(P,q') ∨ inside(P,q') ∨ equal(P,q') ∨ covers(P,q') ∨ contains(P,q')	
meet(p,q)	meet(P,q') ∨ overlap(P,q') ∨ covered_by(P,q') ∨ inside(P,q') ∨ equal(P,q') ∨ covers(P,q') ∨ contains(P,q')	
overlap(p,q)	overlap(P,q') ∨ covered_by(P,q') ∨ inside(P,q') ∨ equal(P,q') ∨ covers(P,q') ∨ contains(P,q')	

Table 4 General relations for the intermediate nodes

The ordinal relations defined in Section 2 can also be processed by the above method. Consider the query "find all land parcels *in* a given area". The land parcels of the result should be *inside* or should be *covered_by* the area, because the interpretation of *in* is *inside* ∨ *covered_by*. The set of the MBRs to be retrieved in this case is the union of the MBRs to be retrieved by each of the relations that belong to the disjunction. Furthermore, the MBRs of the result are the same ones that would be retrieved if the relation of interest were *covered_by*. This is because the MBRs to be retrieved for *inside* constitute a subset of the

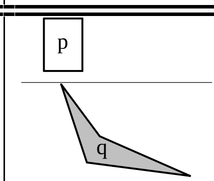
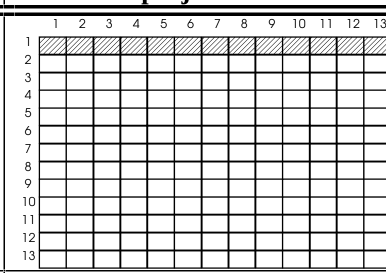
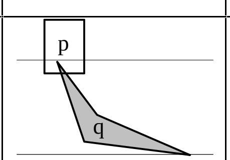
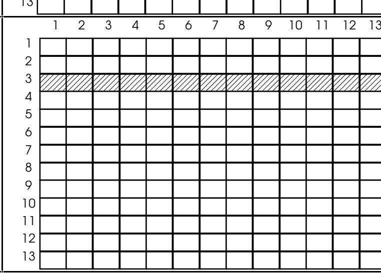
MBRs for *covered_by* (see Table 2). Similarly the MBRs to be retrieved for *consists_of* are the ones that are retrieved for *cover*. Therefore, the retrieval times for the ordinal relations *in* and *consists_of* are the same as *covered_by* and *covers* respectively.

Clementini et al. (1994) studied the use of MBR approximations in query processing involving topological relations. Furthermore, they defined a minimal subset of the nine intersections that can optimally determine the relation between the actual objects taking in account the frequency of the relations. Their findings can be used during the retrieval step in order to minimise the cost for the computation of intersections between potential candidates.

6. IMPLEMENTATION OF DIRECTION RELATIONS

As in the case of topological directions, for directions we need to define a mapping function that maps relations between actual objects onto relations between MBRs. In section 4, we mentioned, for example, that the relation *strong_north* is mapped onto the 13 configurations of the first row of Figure 8. Unlike *strong_north*, which is mapped onto a set of MBR configurations, other direction relations are mapped onto unique relations. *Weak_bounded_north*, for instance, is mapped onto relation R_{3-9}, that is, objects *weak_bounded_north* of a reference object q can only be in MBRs that satisfy the relation R_{3-9} with respect to q' .

Table 5 illustrates the mappings from direction relations between actual objects to MBR relations. The first column contains the eight direction relations of Section 3 and the second column illustrates a configuration of objects that satisfies the relation. The third column describes the corresponding relations between MBRs using relations between the edge points. The last column illustrates the MBRs of the third column graphically.

Relation between actual objects p and q	**Illustration**	**Leaf nodes p' to be retrieved**	**Illustration of the corresponding projections**
strong_north(p,q)		north(p'_f,q'_s)	
weak_north(p,q)		north(p'_s,q'_s) ∧ north(p'_f,q'_f) ∧ south(p'_f,q'_s)	

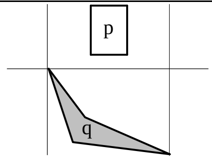
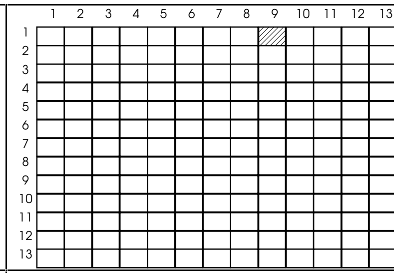
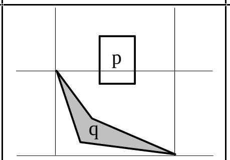
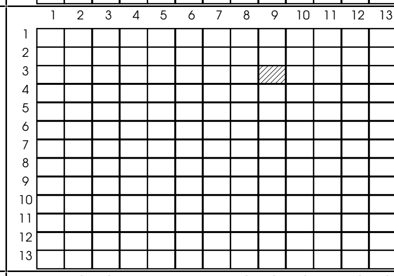
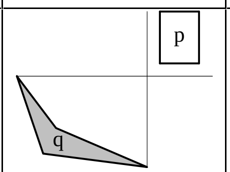
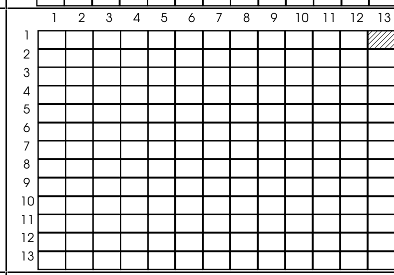
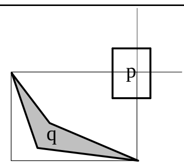
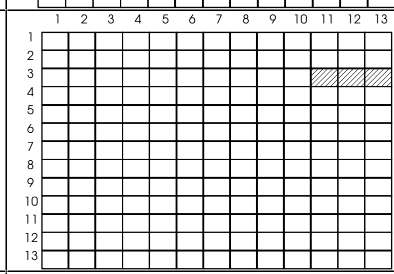
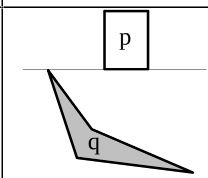
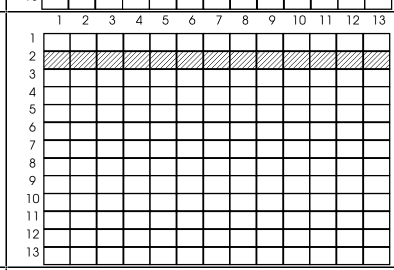
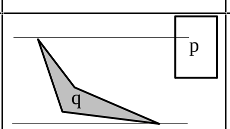
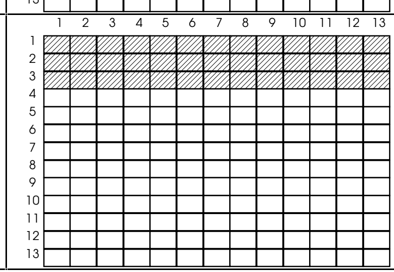
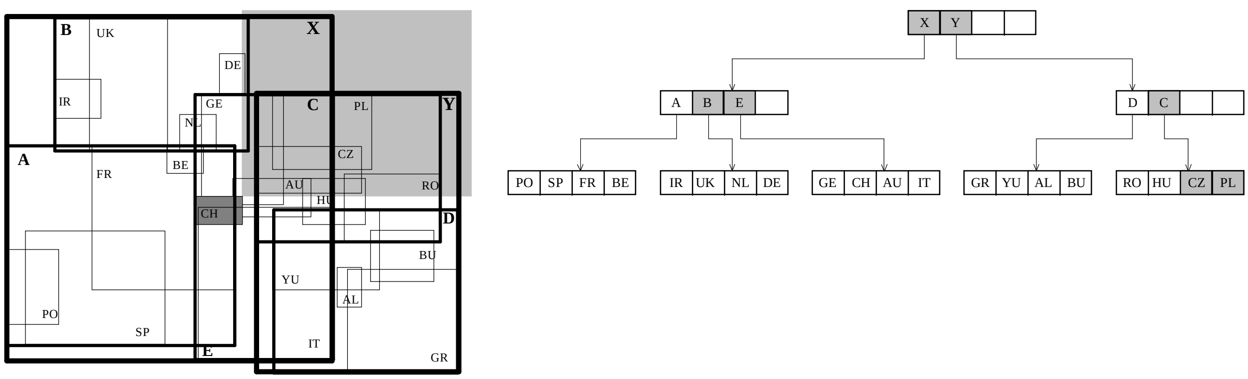
strong_bounded_north (p,q)		north(p'_f,q'_s) ∧ north_east(p'_f,q'_f) ∧ north_west(p'_s,q'_s)	
weak_bounded_north (p,q)		south(p'_f,q'_s) ∧ north_east(p'_f,q'_f) ∧ north_west(p'_s, q'_s)	
strong_north_east(p,q)		north_east(p'_f,q'_s)	
weak_north_east(p,q)		north_east(p'_s,q'_s)∧ north_east(p'_f,q'_f) ∧ south(p'_f,q'_s)	
just_north(p,q)		same_level(p'_f,q'_s)	
north(p,q)		north(p'_f,q'_f) ∧ north(p'_s,q'_s)	

Table 5 MBRs to be retrieved for direction relations

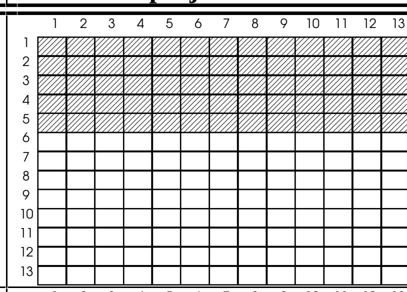
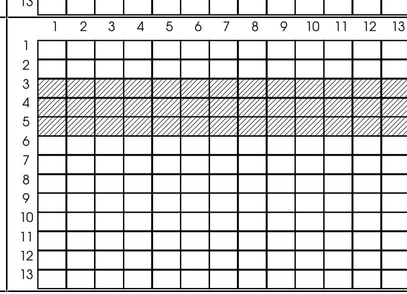
Similar to topological relations, to retrieve direction relations using R-trees one needs to define more general relations that will be used for propagation in the intermediate nodes of the tree structure. For instance, to find all countries p *strong_north_east* of Switzerland we need to retrieve all the MBRs p' within the leaf nodes that satisfy the constraint north_east(p'_f,CH_s), that is, the MBRs in the grey area of Figure 16a. However, the intermediate nodes P that could contain such MBRs should satisfy the constraint north_east(P_s, CH_s). In the R-tree of Figure 16b, intermediate nodes X and Y may both contain candidate MBRs and are selected for propagation. In node Y, only intermediate node C satisfies the constraint north_east(C_s,CH_s) and is searched, while the remaining nodes are directly excluded (C contains the only objects that satisfy the query, namely CZ and PL). Similarly, the searched sub-nodes of X are B and E.



a **b**

Fig. 16 An example search for direction queries

Table 6 describes the direction constraint for the intermediate nodes and the corresponding configurations to be searched for each direction relation between actual objects.

Relation between actual objects p and q	Intermediate nodes P to be searched	Illustration of the corresponding projections
strong_north(p,q)	north(P_s,q'_s)	
weak_north(p,q)	north(P_s,q'_s)$\wedge$ south(P_f,q'_s)	

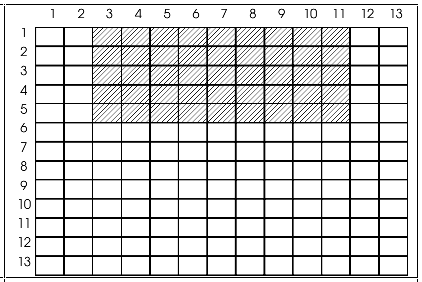
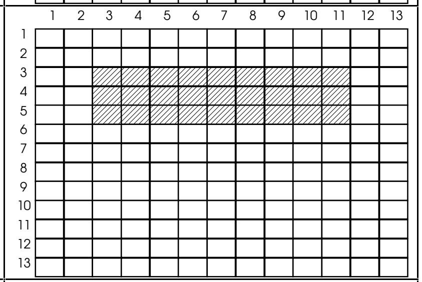
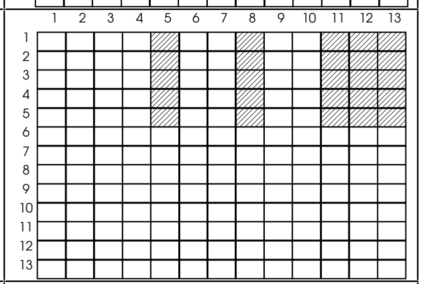
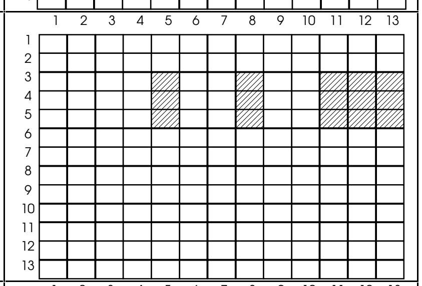
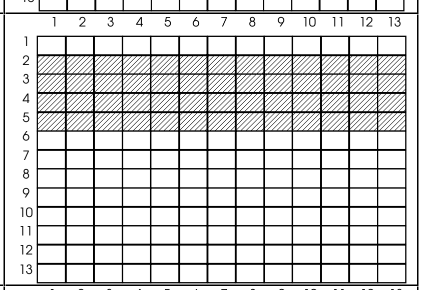
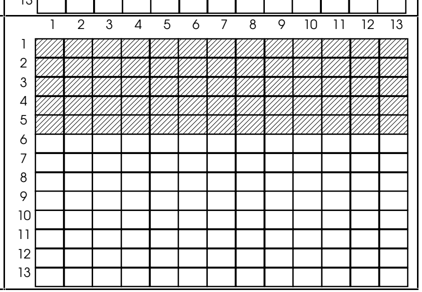
strong_bounded_north (p,q)	north(P_s,q'_s) $\wedge$ west(P_f,q'_s) $\wedge$ east(P_s,q'_f)	
weak_bounded_north (p,q)	north(P_s,q'_s) $\wedge$ south(P_f,q'_s) $\wedge$ west(P_f,q'_s) $\wedge$ east(P_s,q'_f)	
strong_north_east(p,q)	north(P_s,q'_s) $\wedge$ east(P_s,q'_s)	
weak_north_east(p,q)	north(P_s,q'_s) $\wedge$ south(P_f,q'_s) $\wedge$ east(P_s,q'_s)	
just_north(p,q)	(north(P_s, q'_s) $\wedge$ ((south(P_f, q'_s) $\vee$ same_level(P_f, q'_s))	
north(p,q)	north(P_s,q'_s)	

Table 6 Intermediate nodes to be retrieved for direction relations

Queries that involve direction relations may need a refinement step because the relations between MBRs are not always adequate to express the direction relation between the actual objects. Figure 17a illustrates a configuration of objects whose MBRs satisfy the relation R_{3_9}, but the objects do not satisfy the relation *weak_bounded_north* because formula $\forall p_i\ \exists q_j$ north_east(p_i,q_j) is not true. A refinement step is required to eliminate the false hits in this case. The refinement step is also needed queries involving *weak_north_east*. The result of this query consists of all MBRs in the configurations R_{3_11}, R_{3_12}, and R_{3_13}. The refinement step detects the false hits for the MBRs that satisfy the relation R_{3_13}, because it is not always the case that $\forall p_i\ \exists q_j$ north_east(p_i,q_j) is true (Figure 17b). The rest of the MBRs do not require a refinement step, i.e., all retrieved MBRs correspond to objects that satisfy the query.

a **b**

Fig. 17 Refinement step for direction relations

In sections 5 and 6 we have shown how topological and direction relations between objects are mapped onto relations between MBRs. In the next section we apply the previous results in R-trees and their variations and compare the retrieval times.

7. TEST RESULTS

Summarising, the processing of a query of the form "find all objects p that satisfy a given topological or direction relation with respect to object q" in R-tree-based data structures involves the following steps:

1. Compute the MBRs p' that could enclose objects that satisfy the query. This procedure involves Table 2 for topological and Table 5 for direction relations.
2. Starting from the top node, exclude the intermediate nodes P which could not enclose MBRs that satisfy the relations of the second step and recursively search the remaining nodes. This procedure involves Table 4 for topological and Table 6 for direction relations.
3. Follow a refinement step: a) for all MBRs except those in configuration R_{i_j} where i or j in {1,13} when dealing with topological relations, b) MBRs in configurations R_{3_9} and R_{3_13} when dealing with the direction relations *weak_bounded_north* and *weak_north_east* respectively.

In order to experimentally quantify the performance of the above algorithm, we created tree structures by inserting 10000 MBRs randomly generated. The node capacity (branching factor) is 50 entries per node. We tested three data files:

- the first file contains *small* MBRs: the size of each rectangle is at most 0,02% of the global area
- the second file contains *medium* MBRs: the size of each rectangle is at most 0,1% of the global area

- the third file contains *large* MBRs: the size of each rectangle is at most 0,5% of the global area.

The search procedure used a search file for each data file containing 100 rectangles, also randomly generated, with similar size properties as the data rectangles. We used the previous data files for retrieval of topological and direction relations in R, R^+ and R* trees and we recorded the number of disk accesses (the standard measure of efficiency in data structures). In the implementation of R-trees we selected the quadratic-split algorithm and we set the minimum node capacity to m=40%; in the implementation of R*-trees we set m=40% while in the implementation of R^+-trees the "minimum number of rectangle splits" was selected to be the cost function. These settings seem to be the most efficient ones for each method (Beckmann et al., 1990; Sellis et al., 1987). In the rest of the section we present the results for topological and direction relations.

7.1 Topological Relations

We will start by the number of hits per search, that is, the number of retrieved MBRs for each relation. The number of hits per search is inversely proportional to the selectivity of the relation and it is related to the number of disk access. Usually the least selective relations require the greatest number of disk accesses. Table 7 illustrates the number of hits per search for topological relations.

data	number of hits per search							
size	disjoint	meet	overlap	covered_by	inside	equal	covers	contains
small	10000	3,32	2,76	1,06	0,03	1,00	1,07	0,02
medium	10000	11,70	10,37	1,30	0,21	1,00	1,26	0,17
large	10000	56,89	53,56	2,75	1,47	1,00	2,52	1,25

Table 7 Retrieved MBRs per topological relation for each data file

Disjoint is the least selective relation since it involves the larger number of output MBRs. Notice that the sum of retrieved MBRs in each row is larger than the total number of MBRs in the database because the same MBR may be retrieved for more than one topological relations between two objects; the refinement process that filters out the inappropriate MBRs is beyond the scope of this paper. Table 8 illustrates the number of disk accesses per search for topological relations in R- trees and their variations.

data	data	disk accesses per search							
structure	size	disjoint (d)	meet (m)	overlap (o)	covered_by (cb)	inside (i)	equal (e)	covers (cv)	contains (cn)
R-trees	*small*	296	4,13	4,04	4,04	4,04	3,58	3,58	3,42
	medium	297	5,35	5,26	5,26	5,26	3,92	3,92	3,75
	large	298	9,16	8,99	8,99	8,99	4,31	4,31	4,13
R^+-trees	*small*	638	3,39	3,28	3,28	3,28	2,81	2,81	2,75
	medium	1176	4,47	4,19	4,19	4,19	2,39	2,39	2,28
	large	5373	21,39	19,13	19,13	19,13	2,33	2,33	2,25
R*-trees	*small*	304	3,65	3,57	3,57	3,57	3,13	3,13	2,91
	medium	296	4,70	4,60	4,60	4,60	3,53	3,53	3,32
	large	293	8,52	8,24	8,24	8,24	3,87	3,87	3,63

Table 8 Results of tests on topological relations

With the exception of *disjoint*, the improvement of the retrieval using R-trees compared to serial retrieval is immense; especially for small size MBRs the improvement is almost two orders of magnitude[4]. The difference drops as the MBRs become larger. The increase in the size, increases the density and, therefore, the possibility that the reference object is not *disjoint* with other MBRs or intermediate nodes. The retrieval of *disjoint* is, as expected, worse than serial retrieval because this relation requires the retrieval of all the nodes of the tree structure. Clearly, a "real" system would do an exhaustive search of all the MBRs for *disjoint* instead of using the tree structure.

According to Table 8, topological relations can be grouped in three categories with respect to the cost of retrieval. The first group contains *disjoint* which is the most expensive relation and should be processed by serial search. The second group consists of *meet*, *overlap*, *inside* and *covered_by*; they all require similar retrieval times. The third group consists of the three relations (*equal*, *covers*, *contains*) which are the least expensive to process. The cost difference between the second and the third group increases as the size of the MBRs becomes larger. Some of these results can be understood if we observe the retrieved configurations for each relation. For instance in Section 5, we mentioned that the MBRs to be retrieved for *contains* are a subset of the MBRs for *covers*. Figure 18 illustrates the subset relations with respect to the output MBRs. According to Figure 18 *disjoint* is expected to be more expensive than *meet* since it involves the retrieval of all the MBRs for *meet* and some more. *Meet* is slightly more expensive than *overlap* (although they belong to the same group) and so on. Figure 18 also illustrates the three groups with respect to the cost of retrieval.

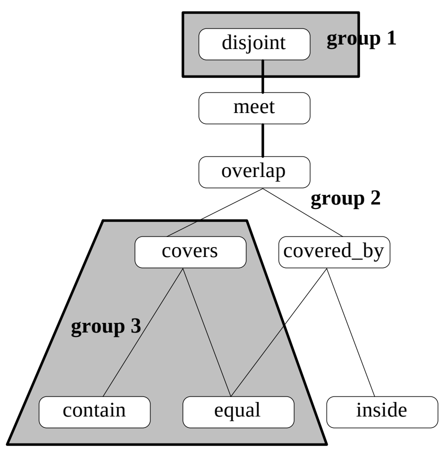


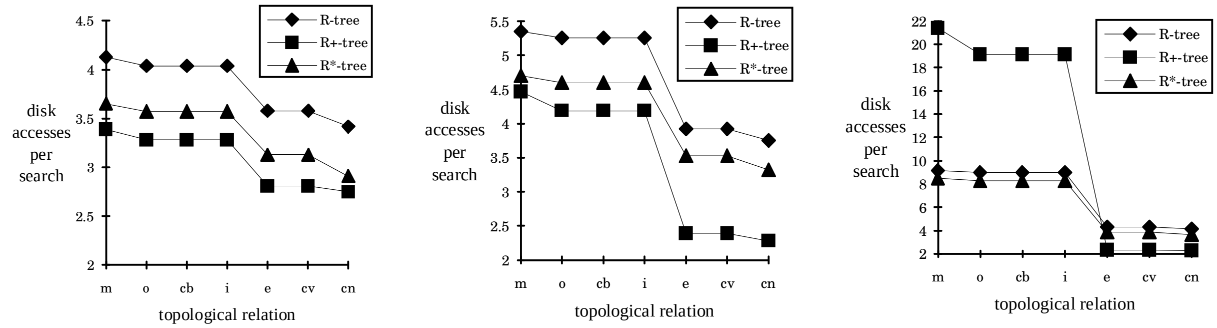
Fig. 18 Subset relations according to the output MBRs

Notice that the number of disk accesses depends more on the intermediate nodes to be searched, than the MBR configurations to be retrieved. The relation *inside* is more expensive than *covers*, although it retrieves only one output MBR configuration (relation $R_{9\text{-}9}$ with respect to q').This is due to the large number of intermediate nodes that could contain MBRs that satisfy the relation $R_{9\text{-}9}$. Relations that

[4]The number of disk accesses per search using serial retrieval is equal to 200 for all relations (10000 entries / 50 page capacity).

involve the search of the same intermediate nodes, such as *inside*, *overlap* and *covered_by*, have similar or identical disk accesses, despite the fact that the leaf MBRs are different (the same observation can be made for direction relations).

The comparison of the various R-tree-based structures follows, in general, the conclusions drawn in the literature for *disjoint* and *not_disjoint* relations, i.e., the variations R^+-trees and R*-trees outperform the original R-trees. Figure 19 (a-c) illustrates a graphical representation of the performance of each method, for small, medium and large data size respectively.



a b c

Fig. 19 Performance comparison of R-tree variants on topological relations

According to Figures 19a-b, R^+-trees perform better than R*-trees. However this advantage is lost if the duplicate entries generate one extra level in the tree structure, as happened for the large data size of our tests (figure 19c). In such case the performance of R^+-trees is inferior for the most expensive relations but remains competent for the least expensive ones. Furthermore, when the data density becomes high it is possible that all of the entries in a full node coincide on the same point of the plane. In such cases R^+ trees do not work (Greene, 1989). This happened for some data sets involving large MBRs during our tests. The irregular behaviour of R^+-trees is due to the lack of overlap between nodes, which results to the quick exclusion of the majority of the intermediate nodes when one of the least expensive relations is retrieved. On the other hand, for the expensive relations there is no significant gain and the performance is worse compared to the other structures because of the great number of nodes in the tree.

7.2 Direction Relations

We used the previous data files for retrieval of direction relations in R, R^+- and R*-trees. Table 9 illustrates the number of hits per search. In general, the selectivity of direction relations that we have defined is less than the selectivity of topological relations (and as a consequence the retrieval times are worse due to enhanced retrieval). *North* is the least selective direction relation because it retrieves almost half the MBRs in the files. On the other hand, *weak_bounded_north* is the most selective relation. The remaining relations have a wide range of selectivity so that they are representative of other direction relations as well.

data	number of hits per search							
size	sn	wn	sbn	wbn	sne	wne	jn	n
small	4936,55	38,98	8,12	0,13	2669,62	20,95	10,22	4985,75
medium	4647,52	100,07	8,12	0,34	2260,36	50,64	9,97	4578,19
large	4646,17	228,10	61,87	2,78	2383,6	128,64	10,18	4884,45

Table 9 Retrieved MBRs per direction relation for each data file

Table 10 illustrates the results. According to Table 10, R^+-trees have the worst performance for most queries. Especially for the least selective relations, their performance is worse than serial retrieval. This happens because of the redundant nodes that have to be searched for large query windows. Furthermore the nature of direction queries does not take advantage of the fact that intermediate nodes in R^+- trees do not overlap. The performance degrades as the MBRs become larger because the increase in density increases the possibility that two nodes are not *disjoint* and therefore the number of duplicate nodes. On the other hand, R and R* -trees perform significantly better (with R* slightly outperforming R trees).

data	data	disk accesses per search							
structure	size	sn	wn	sbn	wbn	sne	wne	jn	n
R-trees	*small*	163,08	25,69	16,50	3,81	96,29	15,01	26,36	163,08
	medium	155,20	27,73	19,09	4,71	86,72	16,56	28,31	155,20
	large	159,33	38,28	26,87	7,48	95,65	24,47	38,84	159,33
R^+-trees	*small*	328,94	35,93	18,27	3,19	187,94	21,61	37,27	328,94
	medium	577,39	46,13	33,63	3,52	296,29	24,83	48,13	577,39
	large	2512,93	107,62	150,35	7,38	1394,39	61,30	118,41	2512,93
R*-trees	*small*	162,47	21,72	14,70	3,35	94,80	13,24	22,36	162,47
	medium	151,83	26,54	16,15	4,24	83,21	15,16	27,21	151,83
	large	159,37	33,66	25,86	6,44	94,41	21,04	34,46	159,37

Table 10 Results of tests on direction relations

In case of directions as well, we can define groups of relations with respect to the cost of retrieval. *North*, *strong_north* and *strong_north_east* are the most expensive to process but still the number of disk accesses for such queries in R and R* trees is 50% - 80% compared to serial retrieval. *Weak_north*, *weak_north_east*, *just_north* and *strong_bounded_north* belong to the second group and require about 10% of the accesses for serial retrieval. The third group consists of the relation *weak_bounded_north* which requires about 2% of accesses. Furthermore, the notion of subsets in MBR retrieval that we illustrated in Figure 18 for topological relations exists for direction relations also. Figure 20 illustrates the three groups and the subset relations with respect to retrieved MBRs.

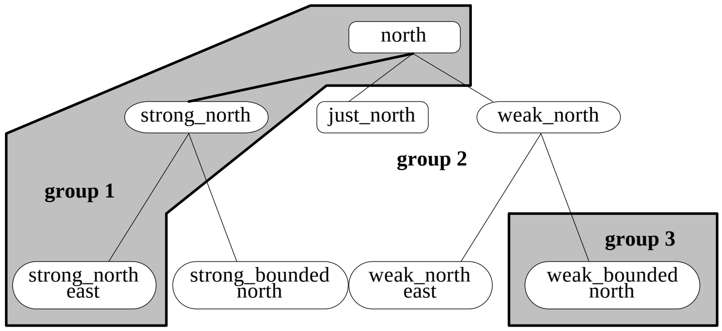


Fig. 20 Subset relations according to the output MBRs

Figure 21 illustrates the results graphically using a logarithmic scale. Even for the least selective relations, the number of disk accesses in R and R* trees is significantly better than serial retrieval. This fact renders R and R* trees suitable data structures for the retrieval of direction relations in addition to topological relations. Furthermore, unlike topological relations where the refinement step is the rule, the only direction relations that require a refinement step (a computationally expensive process) are *weak_north_east* and *weak_bounded_north*.

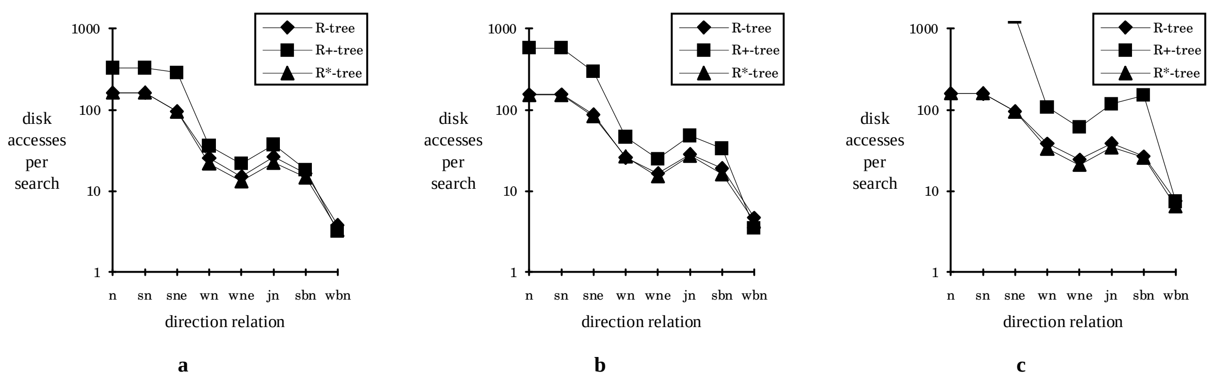


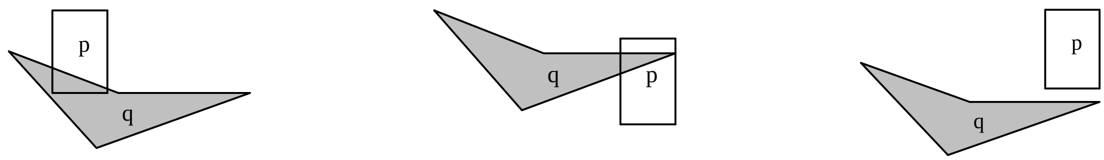
Fig. 21 Performance comparison of R-tree variants on direction relations

Summarising, we can argue that R* trees is the best data structure to process topological and direction relations since they exhibit a stable behaviour and their performance is consistently better than R-trees. Despite the fact that R^+ trees are not suitable for direction relations, they can be used for the retrieval of topological relations in applications involving low density of MBRs because the have the best performance in such cases. However, as the size of MBRs becomes larger with respect to the embedding space, the duplicate nodes that are created lead to performance degradation for the most expensive topological relations.

The previous sections refer to queries involving either topological or direction relations but not both. In the next section we discuss how queries that involve both topological and direction information can be processed in R-tree-based data structures.

8. QUERIES INVOLVING TOPOLOGICAL AND DIRECTION INFORMATION

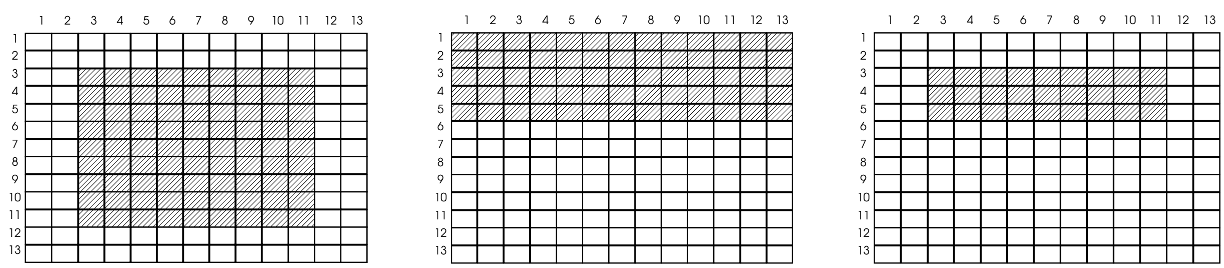
There are often practical situations that a topological or direction relation alone does not suffice to describe a situation. Consider for example the query “find all objects that are *north* and *overlap* object q”. Object p in Figure 22a should be retrieved because it satisfies both sub-conditions of the query. On the other hand, object p of Figure 22b, does not belong to the result because it is not *north* of q. Similarly object p of Figure 22 c is *north* but does not *overlap* q.



a **b** **c**

Fig. 22 Query involving direction and topological information

Queries that involve conjunctions of topological and direction relations are easy to process given the tables for the individual relations. The only difference is that the MBRs to be retrieved belong to the intersection of the MBRs to be retrieved for each relation. Figure 23a illustrates the MBRs for the relation *overlap* (see also Table 2), and Figure 23b illustrates the MBRs for the relation *north* (see also Table 5). Figure 23c illustrates the MBRs to be retrieved for *overlap* and *north*.



a **b** **c**

Fig. 23 MBRs to be retrieved for query involving overlap *and* north

Similarly, the intermediate nodes to be searched should also belong to the intersection of the intermediate nodes for each relation. Thus, queries that involve conjunctions of topological and direction relations are cheaper to process. Furthermore, for some conjunctions an empty result can be returned without processing the query. For example, the output of the query “find all objects that are *strong_north* and *overlap* object q” is empty because strong_north(p,q) $\Rightarrow \neg$ overlap(p,q). Although we treated them separately, depending on the definitions of the relations, topological information can be extracted from direction relations and vice-versa. Table 11, illustrates the topological relations that are consistent with each direction relation.

strong_north⇒disjoint
weak_north⇒disjoint∨overlap∨meet
strong_bounded_north⇒disjoint
weak_bounded_north⇒disjoint∨overlap∨meet
strong_north_east⇒disjoint
weak_north_east⇒disjoint∨overlap∨meet
just_north⇒disjoint∨meet
north ⇒disjoint∨overlap∨meet

Table 11 Topological information that can be extracted from direction relations

Queries containing conjunctions of topological and direction relations that are not consistent have an empty result. In case of disjunctions of direction and topological relations (e.g., “find all objects that are *north* or *overlap* object q”) the result consists of the union of MBRs that would be returned for each sub-condition (Figure 24). The intermediate nodes to searched should also belong to the union of the intermediate nodes for each relation.

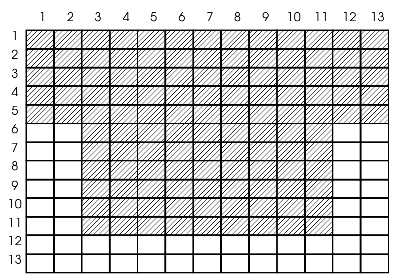


Fig. 24 MBRs to be retrieved for query involving overlap *or* north

Summarising, the processing of complex queries that involve conjunctions and disjunctions of direction and topological relations is straightforward, given the MBRs to be retrieved for each sub-condition. Furthermore, information about the sets of consistent relations can be used to perform semantic query optimisation.

9. CONCLUDING REMARKS

The paper shows how topological and direction relations can be retrieved from spatial data structures based on MBRs. First we described topological and direction relations between region objects and me mapped these relations onto relations between MBRs. Then we used these mappings to retrieve objects that satisfy topological and direction constraints in R, R^+ and R* trees. We found that R* is the MBR-based data structure with the best overall performance. Finally, we demonstrated how complex queries involving conjunctions and disjunctions of topological and direction relations can be processed.

Our approach on topological relations is based to the 4-intersection model. This model is the prevalent model for topological relations in the literature and has been used in a wide range of applications such as Spatial Reasoning (Sharma et al., 1994) and consistency checking in Geographic Databases (Egenhofer and Sharma, 1993). Furthermore, experimental studies have shown that it has the potential for defining cognitively meaningful spatial predicates, a fact that renders it attractive for user interfaces.

We also defined a set of direction relations between extended objects, because the previous sets of direction relations that have been proposed either refer to point objects (Frank, 1994, Hernandez 1994) or they do not provide adequate qualitative resolution to distinguish between situations that may be important for practical applications (Peuquet and Ci-Xiang, 1987). As an example we used the geographic map of Europe and relations such as strong_north(Germany, Italy) and weak_north(France, Italy). The set of direction relations that we defined was just an initial attempt towards the definition of direction relations between extended objects. We do not argue that it has a cognitive motivation but similar sets that match more the user needs and expectations can be defined accordingly.

In this paper we have concentrated on region objects. In order to model linear and point data we need further extensions because the topological relations that can be defined, as well as the number of possible projection relations between MBRs, depend on the type of objects. Egenhofer (1993), for instance, defined 33 relations between lines based on the 9-intersection model, while Papadias and Sellis (1994b) have shown that the number of different projections between a region reference object and a line primary object is 221. The ideas of the paper can be extended to include linear and point data, objects with holes etc.

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