Cartography and Geographical Information Systems 21 (3): 195-212, 1994.

Modeling Spatial Relations Between Lines and Regions: Combining Formal Mathematical Models and Human Subjects Testing

David M. Mark and Max J. Egenhofer

ABSTRACT. This paper describes the results of a series of human-subjects experiments to test how people think about spatial relations between lines and regions. The experiments are centered on a formal model of topological spatial relations, called the 9-intersection. For unbranched lines and simply-connected regions, this model identifies 19 different spatial relations. Subjects were presented with two or three geometrically-distinct drawings of each spatial relation (40 drawings in all), with the line and region said to be a road and a park, respectively. In the first experiment, the task was to group the drawings so that the same phrase or sentence to describe every situation in each group. A few subjects differentiated all 19 relations, but most identified 9 to 13 groups. Although there was a great deal of variation across subjects in the groups that were identified, the results confirm that the relations grouped by the 9-intersection model are the ones most often grouped by the subjects. No consistent language-related differences were identified among 12 English-speaking subjects, 12 Chinese-speaking subjects, and 4 other subjects tested in their own native languages. A second experiment presented the subjects with a short sentence describing a spatial relation between a road and a park, and the same 40 diagrams. Each subject was asked to rate the strength of their agreement or disagreement that the sentence described each relation. For each of the two different predicates tested—"the road crosses the park" and "the road goes into the park"—there was a great deal of consensus across the subjects. The results of these experiments suggest that the 9-intersection model forms a sound basis for characterizing line-region relations, and that many spatial relations can be well-represented by particular subsets of the primitives differentiated by the 9-intersection.

1. Introduction

Over the last several decades, research on fundamental theories of spatial relations has been driven by at least three independent motivations. Mathematicians and some mathematical geographers have searched for situations that can be distinguished in a formal sense (Peuquet 1988; Herring 1991). Largely independently, cognitive scientists have described informally how spatial relations are expressed and manipulated in natural language and thought (Talmy 1983; Herskovits 1986; Retz-Schmidt 1986). And, during the same period, designers of software for geographic information systems (GISs) have developed solutions that would allow them to implement those spatial relations and concepts that are needed for the operation of actual working GISs. The last of these approaches often has produced *ad hoc* results that are difficult or impossible to generalize from or to extend. While the approach based on mathematics will generate sound definitions as the basis for query algebras, it is not clear how closely such "artificial" models represent human thinking. It seems obvious that research leading to fundamental theories of spatial relations must take human spatial cognition into account, but up to now, studies of locative expressions in cognitive science have usually dealt either with very general principles or with narrowly defined situations often involving non-geographic spaces.

What is missing in almost all of this research is the human factor. Even the cognitive scientists have typically studied published grammars, or used their own intuitions about language as a basis for formalization, and rarely have tested the concepts they develop with human subjects. Several research questions arise: What aspects of spatial relations do people pay attention to during spatial reasoning and decision-making? Of the unlimited number of possible differences that could be distinguished mathematically, what distinctions do people actually make, and what detailed situations do they group when they reason about spatial relations or describe them in natural language? How is this differentiation of spatial concepts influenced by the task that the person is trying to perform, by the native language of the person, by their culture, or by individual differences? To be general, a model of the distinctions that people make in the context of geographic problem solving, or in simply talking about geographic space and spatial relations, must include all of the required distinctions needed for spatial reasoning, for any people and for any problem domains.

A basic thesis of this paper is that human-subjects experiments can guide mathematicians and software engineers as to which distinctions are worth making, and which are not. It describes such experimental work, which we believe demonstrates that the *interplay* between formal mathematics and human subjects testing is of eminent value in the search for fundamental theories of spatial relations.

2. Spatial Relations

Calls for general theories of spatial relations have been issued prominently in the GIS literature for a decade or so (Boyle *et al.* 1983; Abler 1987; Frank 1987; Peuquet 1988; NCGIA 1989). Indeed, Boyle *et al.* listed the lack of such a general theory as a major problem for the development of GIS. Thus, it is not surprising that one of the five high-priority topics for research by the proposed U. S. National Center for Geographic Information and Analysis (NCGIA) was defined to be a search for “a general theory of spatial relationships” (Abler 1987, 304). Abler went on to elaborate that the goal is “a coherent, mathematical theory of spatial relationships” (Abler 1987, 306). On the same page, he also stated:

> “Fundamental spatial concepts have not been formalized mathematically and elegantly. Cardinal directions are relative concepts, as are ideas basic to geography such as near, far, touching, adjacent, left of, right of, inside, outside, above, below, upon, and beneath.”

The successful proposal for the NCGIA featured this topic prominently in its proposed research program, and stated that “the search for ‘fundamental spatial concepts’ must be conducted in the cognitive sciences in parallel with searches in mathematics” (NCGIA 1989). Cognitive science can be characterized as follows:

> “Cognitive science is a new field that brings together what is known about the mind from many academic disciplines: psychology, linguistics, anthropology, philosophy, and computer science. It seeks answers to such questions as: What is reason? How do we make sense of our experiences? What is a conceptual system and how is it organized? Do all people use the same conceptual system? If so, what is that system? If not, exactly what is there that is common to the way all human beings think? The questions aren’t new, but some recent answers are.” (Lakoff 1987, xi)

Research at the NCGIA during 1989 and 1990 made considerable progress on the formal side (Egenhofer 1989a, 1989b; Egenhofer and Herring 1990; Egenhofer and Franzosa 1991), but that work was not linked to cognitive principles. NCGIA researchers also conducted research on spatial cognition, but concentrated primarily on wayfinding (Freundschuh 1989, 1991; Gould 1989; Mark 1989; Freundschuh et al. 1990; Gopal and Smith 1990; Mark and Gould 1992), and did not directly address the sorts of fundamental spatial relations needed for GIS from a cognitive perspective.

Recently, there has been a burst of publications that attempt to extend the formal work noted above by formalizing fairly fine distinctions among spatial relations (Pigot 1990; Egenhofer and Herring 1991; Svensson and Zhexue 1991; Clementini et al. 1992; Hadzilacos and Tryfona 1992; Hazelton et al. 1992). Although the distinctions made in these papers may be valid, and perhaps exhaustive within specific domains, mathematical methods alone cannot establish whether these are the most appropriate distinctions to make for human spatial reasoning and problem-solving. These and other formal developments must be evaluated and refined through human-subjects experiments, and we have begun a series of such experiments to attempt to do such evaluations, the first of which are reported in this paper.

2.1 Previous Published Definitions of Spatial Relations Between Lines and Regions

Most categorizations of spatial relations distinguish between topological relations, such as inclusion or overlap, and metrical relations, such as distance and directions (Pullar and Egenhofer 1988; Worboys and Deen 1991). In this paper, we concentrate on topological relations, although some of the relations we examine may have other geometric constraints. Peuquet (1988), Frank (1991), and Freksa (1992) provide contributions to some important aspects of metrical spatial relations.

Spatial query languages contain many spatial predicates (Frank 1982; Roussopoulos et al. 1988; Herring *et al.* 1988; Egenhofer 1991; Raper and Bundock 1991); however most of these predicates lack formal definitions, at least in the publications describing them. While a great deal of attention has been paid in the GIS literature to spatial relations between two regions (Freeman 1975; Claire and Guptill 1982; Peuquet and Zhan 1987; Egenhofer and Franzosa 1991; Hernández 1992), and to spatial relations between points and regions (the classic “point-in-polygon” problem, for example), there has been relatively little published work on relations between lines and regions.

In a relatively early paper on spatial abstract data types, Cox *et al.* (1980) mentioned just three pairs of Boolean relations, stating that their arguments could be points, lines, or regions. They called these relations “equality,” “sharing,” “exclusivity,” and their negations, but did not give definitions, other than to note that equality and sharing are symmetric whereas exclusivity is not. They give point-in-polygon (region) as a special case of “sharing,” which implies that “sharing” is true if the objects have one or more points in common. “Equal” would appear to be a subset of “sharing,” but this cannot be confirmed from the information included in the published paper.

In the late 1980s, several other papers appeared that defined spatial relations. Güting (1988) listed three Boolean spatial relations between a line and a region: “inside,” “outside,” and “intersect.” Roussopoulos *et al.* (1988) listed three pairs of Boolean spatial relation operators between a line and a region: “intersect”/“not-intersect;” “within”/“not-within;” and “cross”/“not-cross.” Once again, however, no details are given in the paper for the exact definition of these predicates. Menon and Smith (1989) included metric spatial relations between points and lines (distance, direction), but no Boolean predicates. Bennis *et al.* (1991) added the idea of asymmetric spatial relations between line objects and region objects, that is, some spatial relations apply between a line and a region, but not between a region and a line. For example, a region can be left-of or right-of a directed line that is coincident with part of its boundary, but without an external reference frame, a line cannot be left-of (or right-of) any region. The other Boolean spatial relations Bennis *et al.* presented were “overlap” and “inclusion,” and to these they added the metrical relations “distance” and “direction.”

2.2 The "9-Intersection" Definition of Topological Spatial Relations

Recently, Egenhofer and Herring (1991) extended a previously-published formal categorization of topological spatial relations between two spatial regions (Egenhofer and Herring 1990; Egenhofer and Franzosa 1991) to account for binary relations in two-dimensional space IR^2 between objects other than regions, such as between two lines, or between a line and a region. For line-region relations the following definitions are relevant:

- A *line* is a sequence of 1...*n* connected 1-cells—connection between two geometrically independent 0-cells (nodes)—such that they neither cross themselves nor form cycles. A line defined in this way is equivalent to a non-directed "Chain" in the U.S. Spatial Data Transfer Standard (SDTS) (Fegeas *et al.* 1992). Nodes at which exactly one 1-cell ends will be referred to as the *boundary* of the line, whereas nodes that are an end point of more than one 1-cell are interior nodes. The *interior* of a line is the union of all interior nodes and all connections between those nodes. Finally, the *exterior* is the difference between the embedding space IR^2 and the union of the interior and boundary (Figure 1). In this paper, we focus on simple lines, which have exactly two boundary nodes.

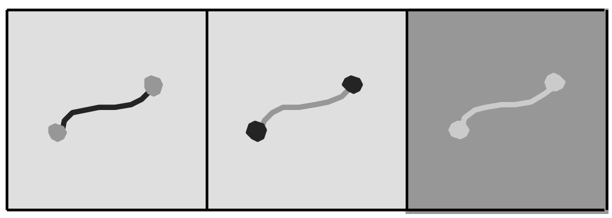

Figure 1. Interior, boundary, and exterior of a line in IR^2.

- A *region* is defined as a connected, homogeneously 2-dimensional 2-cell; this is termed a "GT-Polygon" in SDTS. Its *boundary* forms a Jordan curve separating the region's *exterior* from its *interior* (Figure 2).

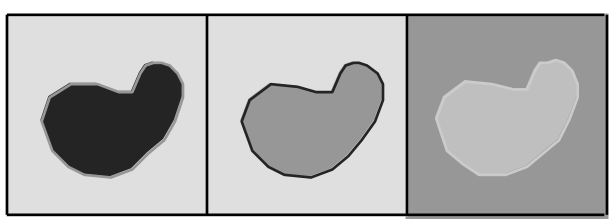

Figure 2. Interior, boundary, and exterior of a region in IR^2.

The *9-intersection* describes binary topological relations in terms of the intersections of the interiors, boundaries, and exteriors of the two spatial objects. The nine possible intersections among the six object parts (each of the line's interior, boundary, and exterior with each of the region's interior, boundary, and exterior) are preserved under topological transformations and provide a framework for the formal description of their topological relationship.

A variety of topological invariants can be applied to analyze the intersections. The most general topological invariant is the distinction of the content (emptiness or non-emptiness) of the intersections. This can be concisely represented as a 3x3 "bitmap" (Figure 3).

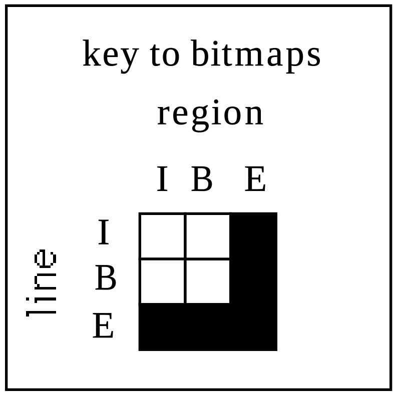

Figure 3. The 3x3 “bitmap” of a 9-intersection. White pixels represent empty intersections and black pixels stand for non-empty pixels. The left column of each bitmap has a black square for each part of the line (from the top, the **I**nterior, **B**oundary, and **E**xterior) that intersects the **I**nterior of the line; the column in the middle indicates the same for the **B**oundary of the region; and the right column represents the three intersections with the region’s **E**xterior.

With each of these nine intersections being empty or non-empty, the model has 512 (2^9) possible relations between the objects. Most of these combinations of the nine intersections are, however, impossible for connected objects in the 2-dimensional Cartesian plane. In fact, between an unbranched line and a region, just 19 distinct topological relations are possible (Figure 4). More detailed distinctions would be possible if further criteria were employed to describe the non-empty intersections, such as the dimensions of the intersections (0- or 1-dimensional) or the number of separate components per intersection (Egenhofer and Herring 1990; Clementini *et al.* 1992). Such additional distinctions are, however, ignored in this paper.

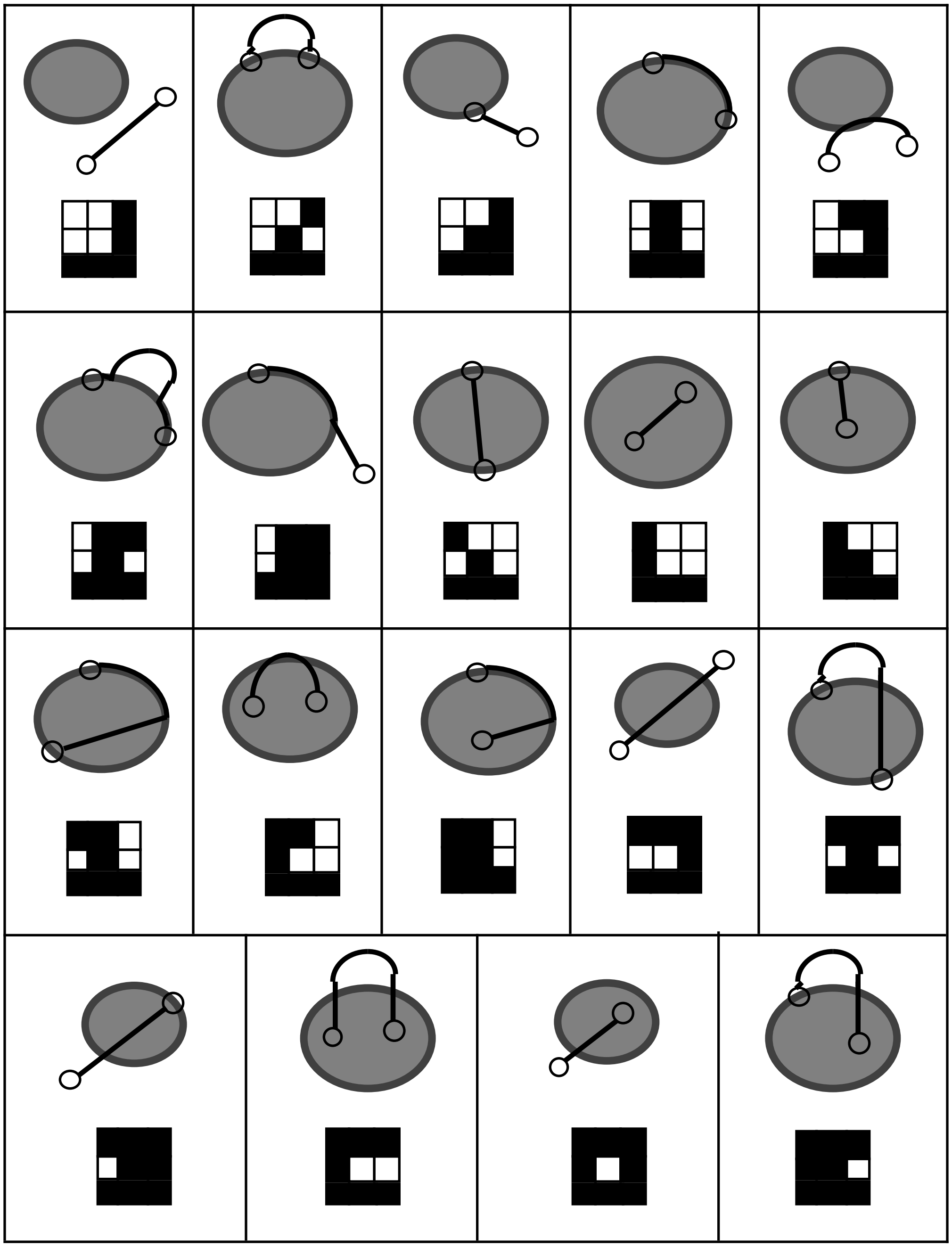

Figure 4. The 19 topological relations distinguished by the 9-intersection, together with their empty/non-empty bitmaps.

The 19 situations can be presented in a diagram that links cases where exactly one of the nine intersections is different but the others are the same; Egenhofer and Al-Taha (1992) presented such a diagram for spatial relations between two regions. This diagram (Figure 5) has a particular "symmetry" such that situations in the equivalent positions on the left and right sides differ only in the fact that the "interior" and "exterior" of the region have been interchanged. In most cases, it also is possible to transform between neighboring situations through smooth geometric transformations.

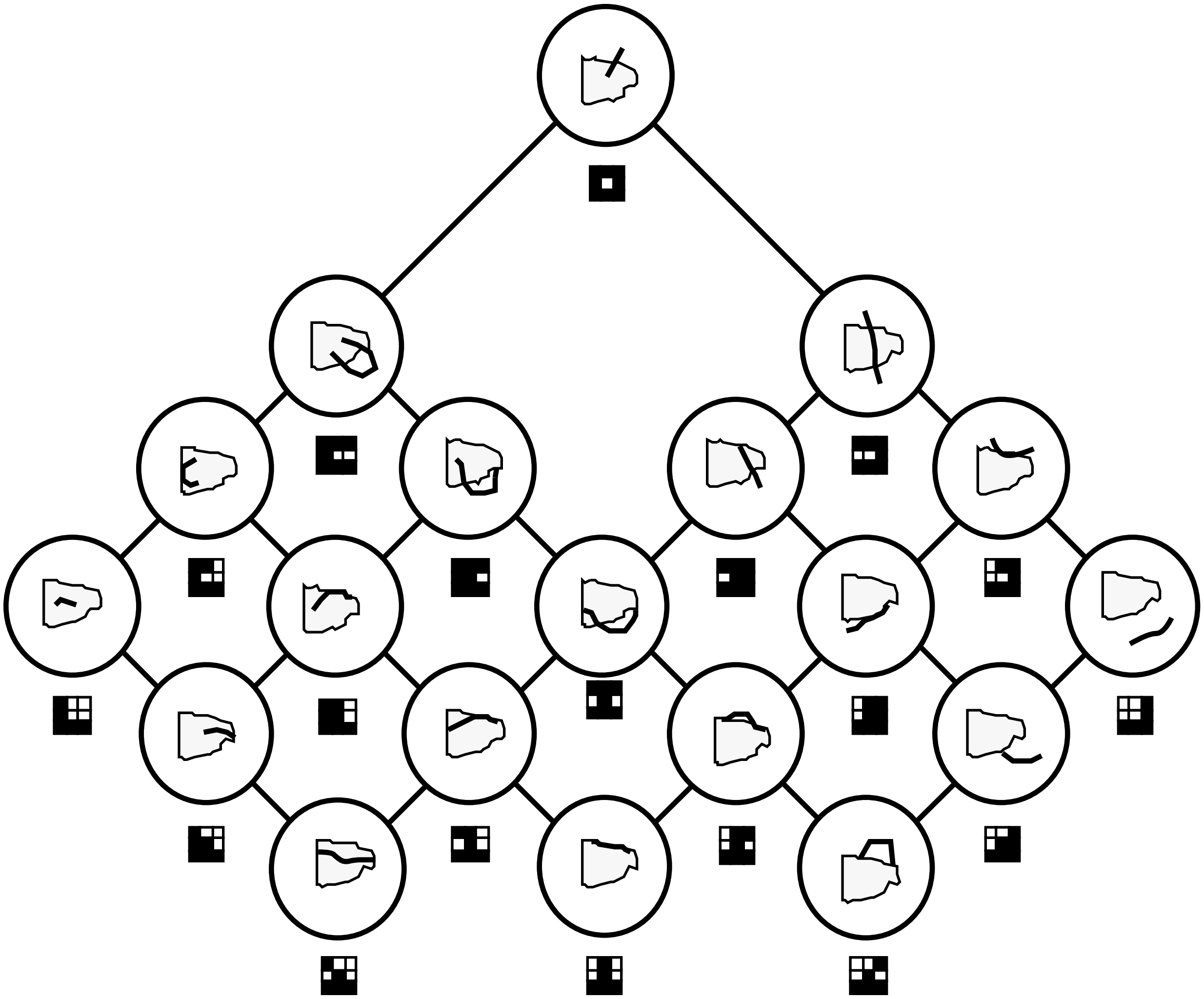

Figure 5. Diagram illustrating relationships among the 19 topologically-distinct spatial relations between a line feature and an region feature according to Egenhofer and Herring's "9-intersection" model. Situations are connected in the diagram if they differ for exactly one of the nine "intersections."

\Whereas the 9-intersection can be shown to be a correct and complete characterization of a system of spatial relations, mathematics alone cannot indicate whether the distinctions made are relevant, or whether relevant distinctions have been omitted. As noted above, additional distinctions can be made, and the number of such possible distinctions may be essentially unlimited. Thus any particular level of detail in topological distinctions may be viewed as a level of abstraction, and the question becomes: is the level of abstraction for spatial relations represented by the 9-intersection an appropriate level for GIS or for cognitive science? In particular, the 9-intersection model distinguishes 19 different line-region relationships. However, the intuition of at least some researchers in the field seems to indicate that most people would not distinguish that many different kinds of topological relations between a line and a region. To evaluate this intuition, we developed experiments to examine how people categorize spatial relations between lines and regions in a geographic context.

3. The First Experiment

3.1 Experimental Design

For the first experiment, we produced 40 drawings of a line and a region. The region was identical in each drawing, and was bounded by a thin solid line and filled with a gray tone. The line was drawn with a line weight twice that of the region boundary. The position of the line relative to the region was different in each case, and the lines were positioned so as to provide two (or, in two cases, three) geometrically distinct examples of each of the 19 topologically-distinct cases of line-region relations. Whenever possible, the line was straight in one example of each 9-intersection situation, and curved in the other(s). Subjects were told that the region was a "park" and that the line was a "road," although the representations of those features did not follow standard cartographic symbology. Some examples are shown in Figure 6, and others appear in subsequent diagrams in this paper.

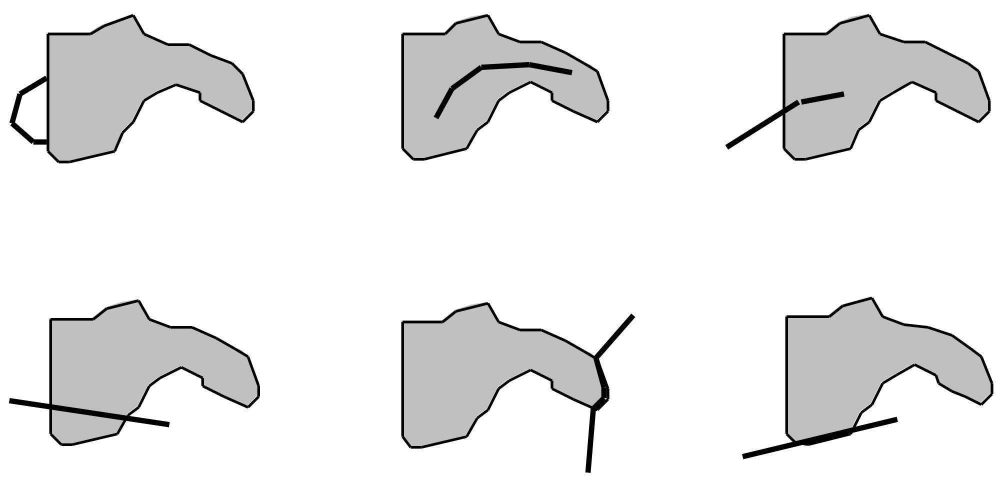

Figure 6. Some examples of the stimuli used in this research. The right and middle examples in the lower row are topologically identical but geometrically-distinct.

The 40 drawings were then printed on individual cards 7.0 by 10.8 cm (about the size of standard playing cards), and were shown to 12 native speakers of English, 12 native speakers of Chinese, 3 native speakers of German, and one native speaker of Hindi. With one exception[1], the instructions were given and responses were recorded in the native language of the subject. In English, the instructions were:

> "Here are 40 different sketches of a road and a state park. Please arrange the sketches into several groups, such that you would use the same verbal description for the spatial relationship between the road and the park for every sketch in each group."

1 The Hindi-speaking subject, was asked the question in English, and asked to use English-language phrases to describe the categories.

When the subject completed the task, the experimenter recorded the groups, and elicited the descriptive phrase or sentence for the spatial relation for each group. Lastly, most of the subjects were asked to select the "best example" from each group, as a prototype.

3.2 Results

3.2.1 Number of Groups

The numbers of groups identified by the subjects varied widely, from 4 to 20 (Figure 7). The median number of groups was somewhat higher for the Chinese subjects, but there is sufficient variation within each group that it is clear that there is no systematic difference in group sizes across the languages tested. Attention thus was turned to the actual groupings.

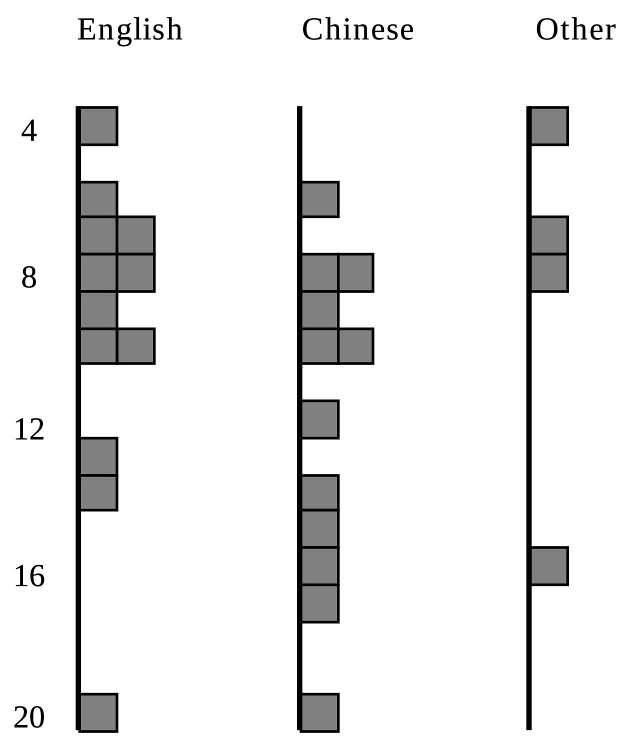

Figure 7. Histogram showing the number of groups defined by the subjects.

3.2.2 Groupings

Groupings by individual subjects can be examined visually after plotting them on the diagram introduced in Figure 5, above. We found that the groups normally appeared as connected subgraphs of this diagram. However, there was a great variation in the groupings by individual subjects (for examples, see Figures 8, 9, and 10). Even for individual subjects, the groups seemed at times to overlap. However, because we required each stimulus to be put into exactly one group, such overlaps could not be detected from the data.

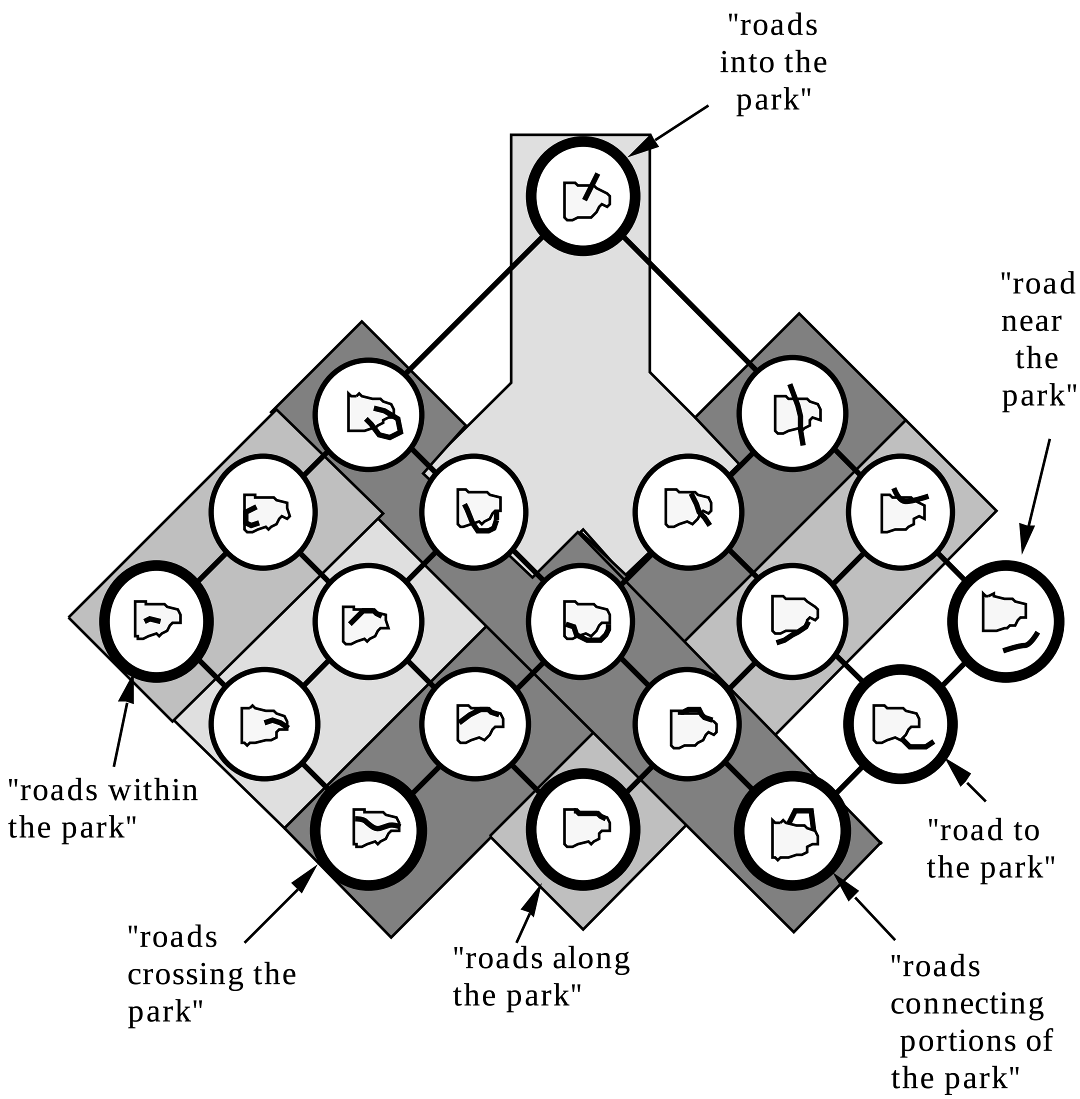

Figure 8. The responses of one of the English-language subjects, plotted on the diagram introduced in Figure 5. Each shaded polygon represents a group; the shadings are arbitrary and simply are intended to discriminate the groups within one diagram. If the group polygon covers just half of a circle representing one of the 9-intersection situations, that means that only one of the two diagrams representing that situation was placed in the group. A heavy circle boundary designates the group prototype; a heavy circle on the white general background indicates that the subject isolated the two or three diagrams representing that 9-intersection situation as a small group.

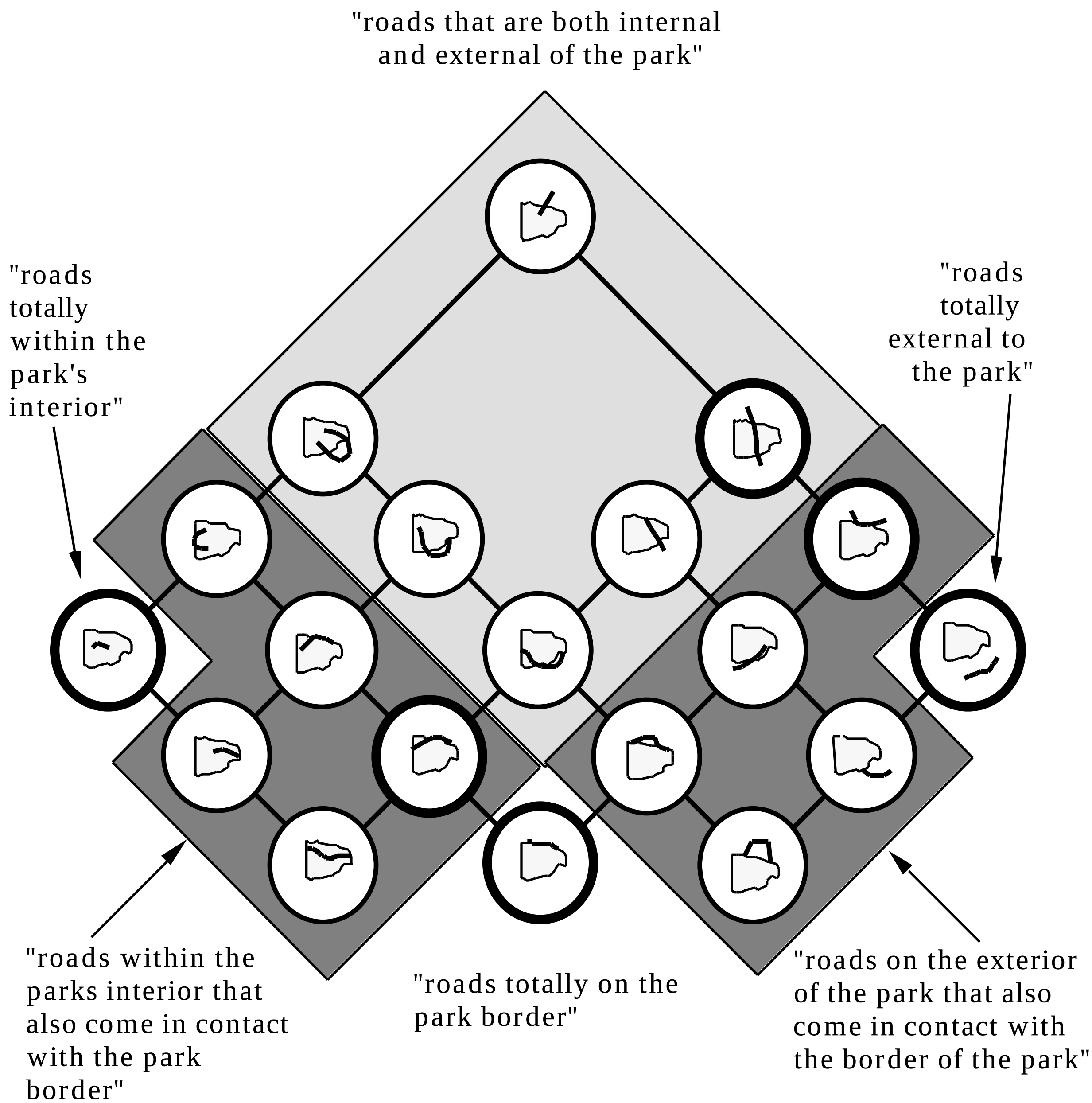

Figure 9. Diagram for another English-language subject. Note that these groups are much more compact than those produced by the previous subject (Figure 8), and that the diagram is perfectly symmetric.

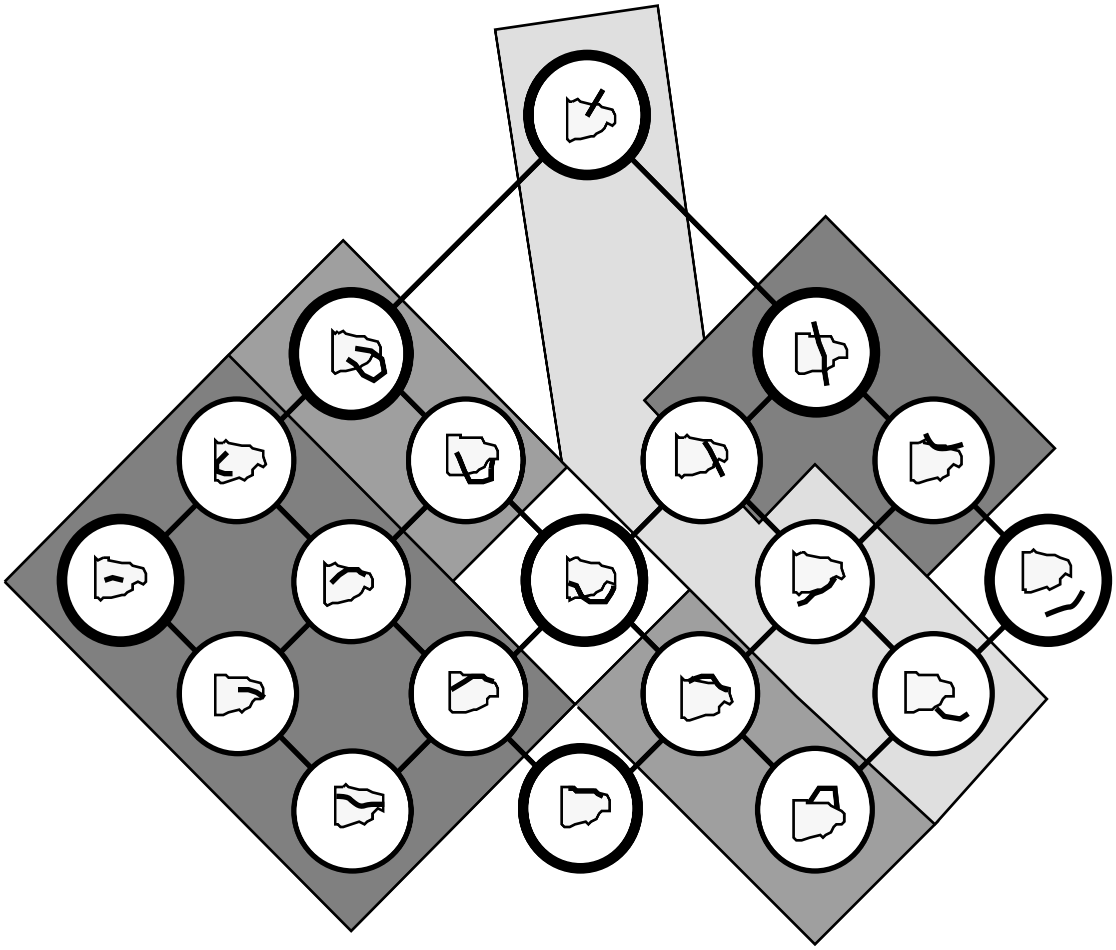

Figure 10. Diagram for one of the German-language subjects. This subject provides an example of the tendency of several subjects to aggregate much more on the left side of the diagram (where the road in within the park), but to differentiate more finely on the right side.

3.2.3 Validity of the 9-Intersection

With 40 stimuli, there are 760 distinct pairs of stimuli ($n(n-1)/2$). Each of these stimulus pairs was grouped together by between all (28) and none (0) of the 28 subjects. Since we did not observe any particular differences across languages, we considered all 28 subjects together, counting how often each pair of stimuli was aggregated and ranking the 760 possible pairs by their grouping frequencies. If we consider only the basic set of 38 stimuli (two for each situation), ***the 19 most frequently grouped pairs were exactly the 19 that were within the 9-intersection classes.*** This appears to be a strong confirmation of the fundamental nature of the 9-intersection model. Seven of these within-situation pairs were grouped by all 28 subjects, and 4 others were grouped by 27 subjects. Interestingly, all of these categories were around the margin of the diagram. The remaining eight 9-intersection situations, toward the middle of the diagram, were grouped by between 23 and 25 subjects, still more frequently than any between-situation pair. In fact, the most frequently grouped between-situation pair was grouped by 22 of 28 subjects, and only 5 such pairs were grouped by 20 or more subjects.

3.2.4 Prototype Effects

Part of the experiment involved asking each subject to designate one stimulus from each group as the best example of that group, to serve as a prototype. Figure 11 shows the frequencies of prototype choice for 23 of the subjects, the 12 Chinese speakers plus 11 of the 12 English-language subjects. Many of the prototypes for the line-to-region relation categories were at the ends of the categories, rather than in the centers. Initially, this was surprising, because for other prototype studies (for example, Berlin and Kay's 1969 study of color categories), prototypes are usually near the centers of categories. However, most of the prototypes for line-region relations were cases in which the body (interior) of the line fell entirely into one of the three parts (interior, boundary, exterior) of the region. To put this another way, the spatial relations in the prototypes seem more “simple.” The more complicated cases that fall toward the central part of the diagram (Figure 11) were seldom considered to be “best” examples of relationships, probably because they combined elements of several different relationships. When the prototypes for all categories for the 23 subjects are aggregated, situations around the edge of the diagram are selected far more often than those toward the middle.

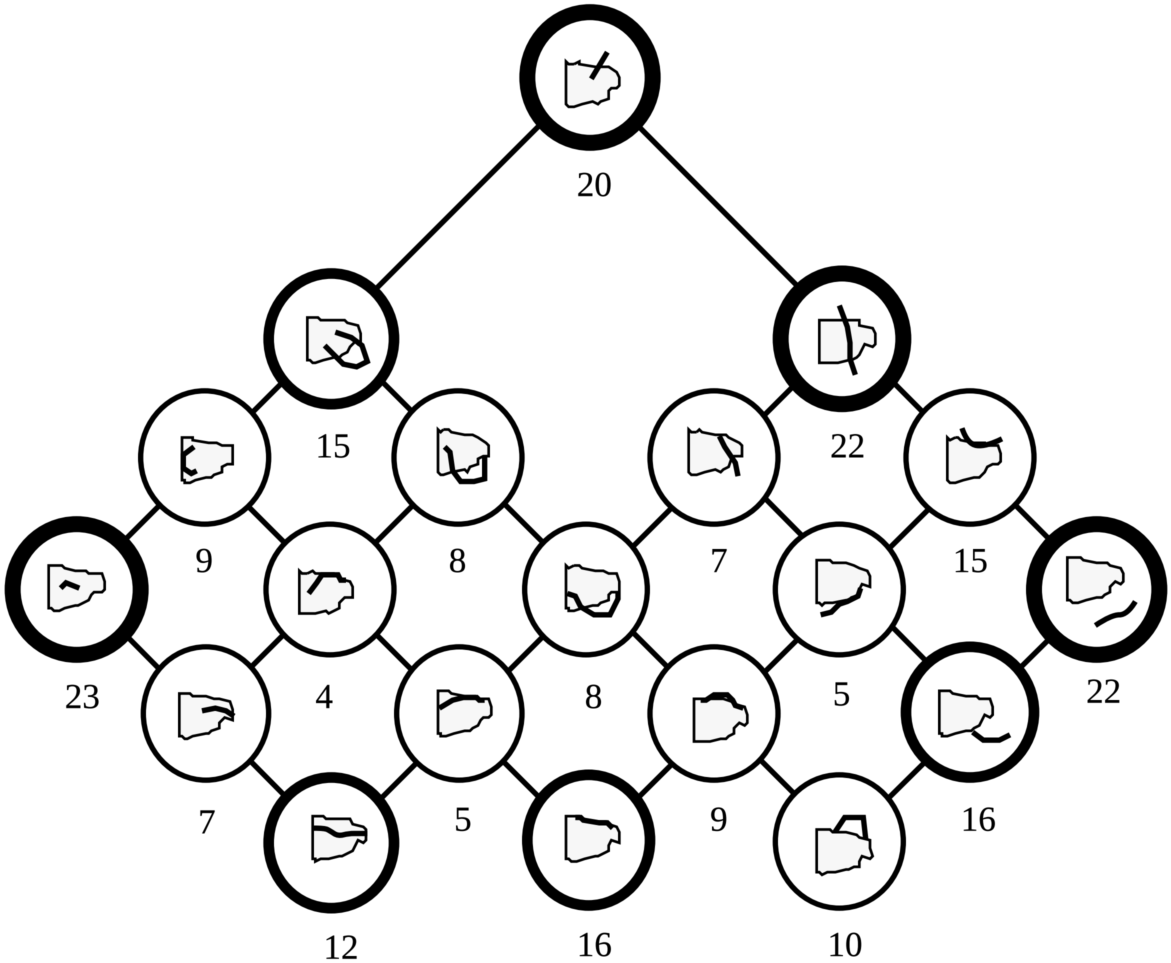

Figure 11. Frequency with which each situation was chosen as a group prototype by 23 subjects.

For several cases, a situation on the right side of the diagram was selected as a prototype considerably more often than the equivalent diagram on the left. One example is the case where the road is entirely outside the park, except for touching it at one end. That situation was a group prototype for 16 of the 23 subjects for whom prototype information was tabulated, whereas its inside-out equivalent was a prototype by only 7 subjects. In fact, the two stimuli in which the road ended at the park boundary from outside were isolated as a group of two by 13 subjects (making them automatically their own prototype); 2 additional subjects added just one other stimulus to the pair, and only 1 subject used this pair as the prototype for a larger group (10 stimuli). Of the 7 subjects who marked the situation where the road touched the boundary at one end but otherwise was inside as a prototype, 4 had the two stimuli as an isolated pair, and the other 3 had a member of that pair as a prototype of a larger group, or 4, 6, or 8 stimuli. The right-left asymmetry in prototype selection was somewhat more common among the English-language subjects (right-side prototypes outnumber left-side prototypes by 48-34) than among the Chinese-language subjects (right-side prototypes outnumber left-side prototypes by 58-49), but the cross-language difference is not strong.

3.2.5 Similarity Among Subjects

In order to examine similarities and differences among the subjects, an index of similarity between each pair of subjects was computed. First, for each subject, a 40 by 40 binary symmetric matrix was determined, in which a "1" in any position indicates that the subject placed the pair of stimuli denoted by the row and column in the same group, and a "0" otherwise. The fewer groups a subject made, the more within-group pairs there are, and the more "1"s there are in the binary matrix for that subject. Then, for each pair of subjects, we counted the number of places in their binary matrices that were identical (that is, the two subjects treated that pair of stimuli identically, either grouping them or not), and divided this count by 1,600 to get a similarity index that would be 1.0 if the two subjects came up with identical groupings. Two subjects did indeed have identical responses, so the maximum value of the similarity index was 1.0, and the minimum observed value for any pair of subjects was 0.736.

In addition to the human-subjects data, we created 4 "synthetic" subjects, one being the exact groupings of the 9-intersection model, and the other 3 from topological models that would result if certain distinctions made by the 9-intersection model were ignored. In two models, the boundary of the region is either merged with its interior ("region-closure model") or the exterior of the region ("open-region model"). A final model lumped the interior and boundary of both the line and the region ("line- and region-closure model"), thus letting the model degenerate to just the "contains," "overlaps," and "disjoint" relations that some previously-published categorizations of spatial relations had recognized. Some of these classifications had much lower similarities to the data from the human subjects, with indices as low as about 0.55.

In the analysis we emplyed multidimensional scaling (MDS), a technique for determining configurations of points given only a matrix of inter-point distances or similarities. Usually, no configuration would replicate the inter-point similarities exactly, and so MDS finds the configuration that best fits the data according to some goodness-of-fit criterion. The solution does not determine such factors as scale, rotation, or reflection, and so the axes of the output configuration are arbitrary. The similarity indices among all pairs of subjects and models, as discussed in the preceding paragraph, were entered into SPSS-X's multidimensional scaling procedure, and this produced a 2-dimensional configuration of points (Figure 12). Except for the fact that there are very few Chinese subjects appearing as outliers on the diagram, the subjects do not seem to cluster by language. This seems to confirm impressions gained from visual inspection of the subjects' groupings, that individual differences within languages are greater than distances between languages. Note that the "region-closure," "open-region," and "line- and region-closure" models fall outside the convex hull of the data for the 28 subjects, although data for several subjects plotted closer to the "region-closure" model than to the 9-intersection model itself.

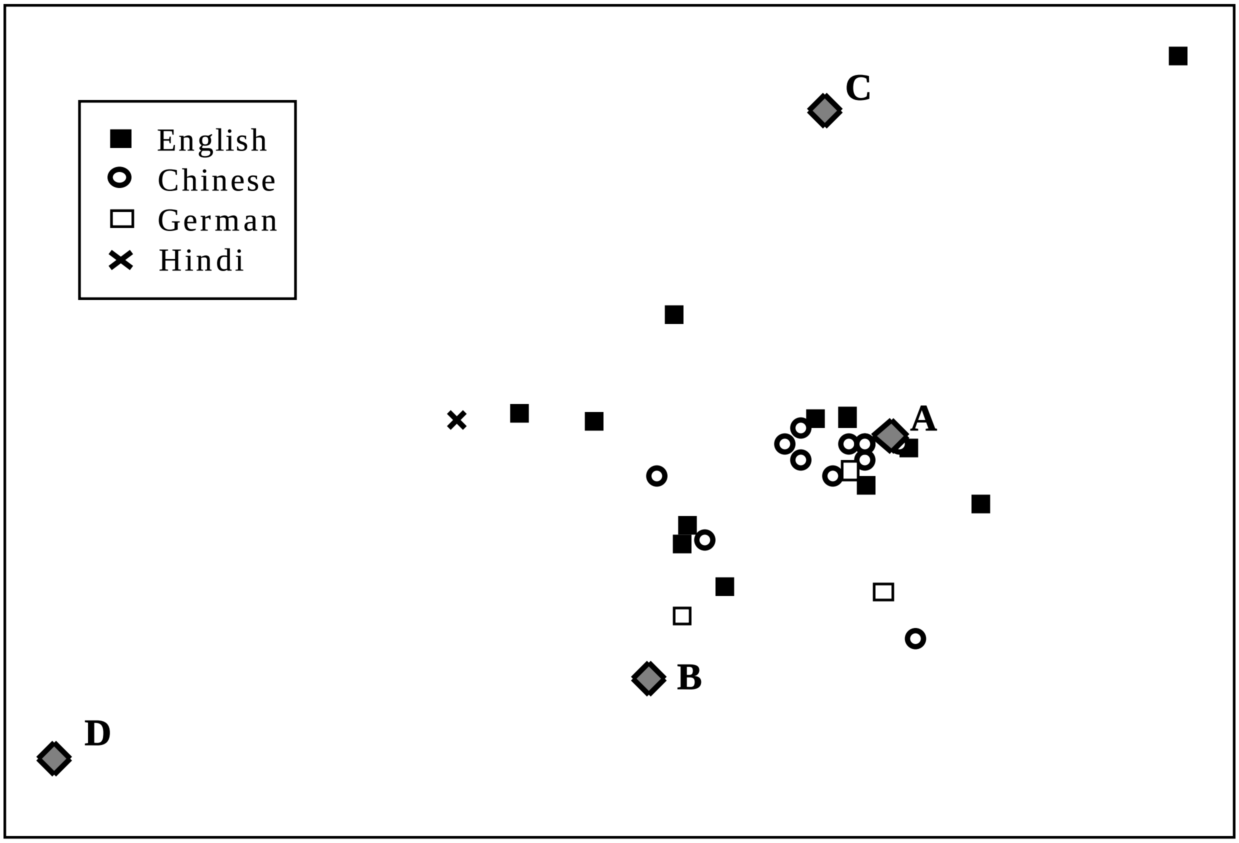

Figure 12. Configuration of the 28 subjects and 4 models of spatial relations, produced by multidimensional scaling. The models are indicated by shaded diamond shapes: A, the 9-intersection model with 19 groups; B, the region-closure model (combining the interior and boundary of the region); C, the open-region model (combining the boundary and exterior of the region); and D, the line- and region-closure model (boundaries of both the line and the region are merged with their interiors). The axes of the diagram are arbitrary dimensions.

3.2.6 A Rare Example Of Discrimination By Geometry

Whereas almost all subjects appeared to emphasize topological factors in their responses to the grouping task, one of the English-language subjects apparently used a geometric criterion to classify some of the stimuli. Figure 13 shows the stimuli involved in this exception. The particular geometry of the four cases on the upper right-hand side of Figure 13 caused them to be grouped together by this subject, who noticed that these had a straight segment in exactly the same position relative to the concavity on the lower ("southern") side of the park. However, none of the other 27 subjects grouped these 8 stimuli in this way, and the exception does not contradict the general tendency among subjects to classify the stimuli primarily according to topological criteria and to generally ignore geometric characteristics.

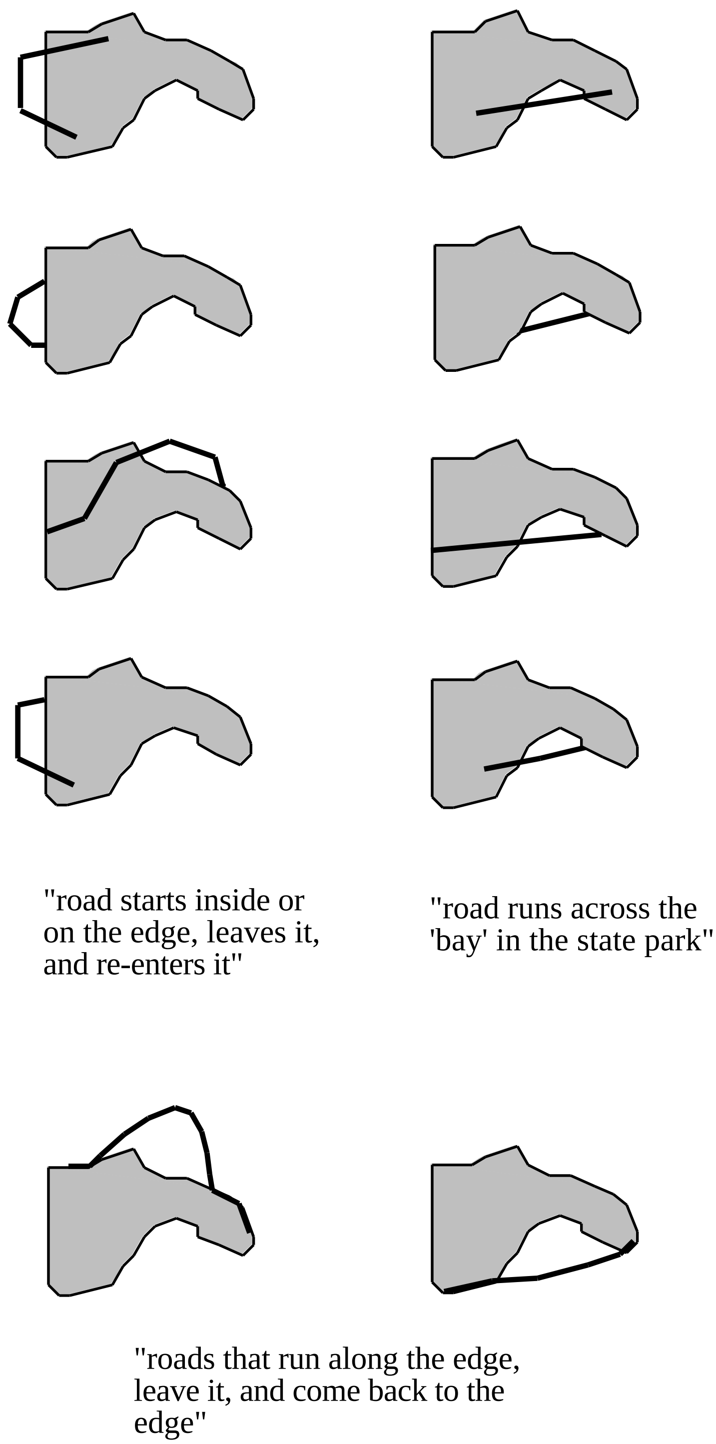

Figure 13. Three groups of stimuli according to one of the English-language subjects. The four stimuli in the upper left part of the diagram were described by the sentence "starts inside or on the edge, leaves it, and re-enters it," whereas the four in the upper right were grouped under "road runs across the "bay" in the state park." The lower two examples were grouped together and described by the phrase "roads that run along the edge, leave it, and come back to the edge." Each of the left-right pairs of park drawings above represents two realizations of the same 9-intersection relation.

3.3 Summary of the First Experiment

This first experiment has shown that there is a great deal of variation in the ways in which people classify spatial situations that involve roads (lines) and parks (regions). There are, however, underlying patterns. One of the strongest of these is the 9-intersection model. Whereas it is possible that the experiment contains biases that promote the recognition of the 9-intersection distinctions by the subjects[2], that model definitely emerged as an underlying structure. The 19 pairs of stimuli most often grouped by the subjects were exactly the 19 cases that the 9-intersection model does not distinguish. Also, since the differences between the two members of each topologically-similar pair of road-park examples were geometric—different orientation, shape, and length of the line, etc.—the outcome suggests that people often ignore such quantitativedifferences and are primarily concerned with qualitative (topological) differences. The results of the experiment suggest that many of the qualitative differences that people make regarding spatial relations are captured by the 9-intersection model.

The diagram introduced in Figure 5, constructed analytically on the basis of “least distinguishable differences” in the 9-intersection, reflected subjects’ judgments very well: the 37 most-frequent pairs, and 53 of the 58 most-frequent pairs, were either within 9-intersection classes, or were between adjacent classes on that diagram. Figure 14 shows a consensus diagram, which groups all pairs that were combined by 14 or more of the 28 subjects. The groups are somewhat larger on the left side of the diagram than on the right, which has a larger number of isolated 9-intersection classes.

---

2 One of the English-language subjects, who was making rather fine distinctions, noticed that identical topological relations usually came in pairs, and then used this as part of his classification, trying to find a “twin” for any apparently-isolated stimulus, and also looking for ways to divide any groups of three he had formed. In fact, the two “extra” examples that were added to the stimulus set created a problem for him at the end, as he worked on the two groups of 3 stimuli for quite a while before finally leaving one of them, and breaking the other into a group of 2 and a singleton. It is possible that other subjects used similar reasoning.

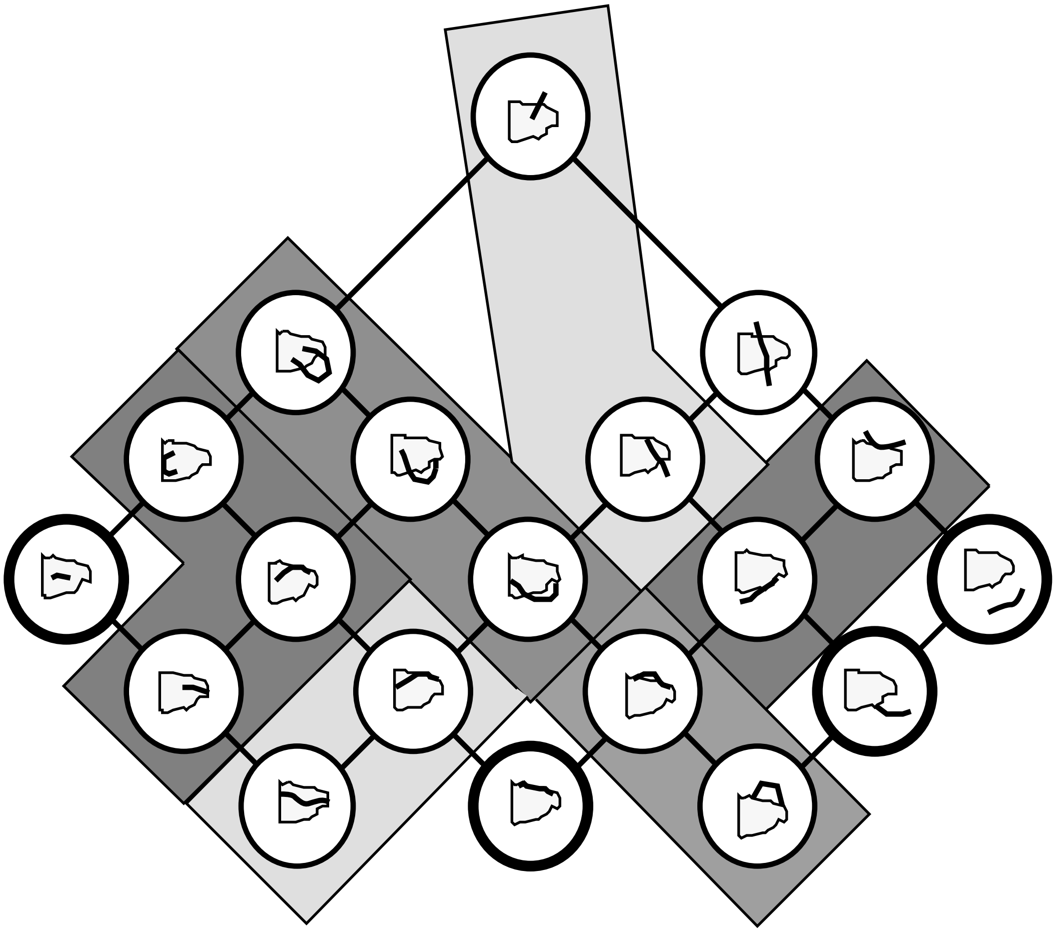

Figure 14. Consensus diagram for all 28 subjects. For the 19 9-intersection situations on the diagram, all within-situation stimulus pairs were grouped together by well over half of the subjects. Heavy circles surround those 9-intersection situations that were not grouped with any other stimuli by 14 or more subjects. The shaded zones surrounding the remaining situations indicate groups for which every within-group pair was combined by at least 14 subjects.

Although there are a few intriguing suggestions of language-based differences in the results, some of which have been reported above, the experiment described provides no solid evidence of differences in judgments about line-region spatial relations between speakers of the languages tested. It is quite possible that no such differences exist for roads and parks, or even more generally regarding lines and regions. If such differences exist between English and Chinese, they are probably subtle, at least with respect to this experimental design, and thus larger samples or more focused experiments (or both) will be needed to establish any differences that may exist. It is also possible that, by coincidence, Chinese and English happen to be very similar for the spatial situations included in the experiment; speakers of other languages will have to be tested before any generalizations about cross-linguistic universal principles can even be proposed, let alone substantiated.

The high individual differences across subjects, both in groupings and in the language used to describe those groups, make it difficult to examine the possible meanings of various locative phrases, or to relate the results to more practical issues of queries in a GIS context. Therefore, a second, more specific experiment was designed.

4. The Second Experiment

4.1 Experimental Design

To further evaluate the model described above, we designed a more specific test, in which subjects were presented with sentences in English that described a spatial relation between a “road” and a “park.” Potential spatial predicates to be tested were drawn from the subjects’ responses in the first experiment. From these, we selected “the road crosses the park” and “the road goes into the park” for testing. The test instrument consisted of six pages. The first page (for the test of “cross”) presented the following instructions:

> “Each of the accompanying 40 diagrams represents a State park and a road. Please examine each map, decide how strongly you agree or disagree with the statement that in that case, ‘the road crosses the park,’ and mark your response on the scale from 1 to 5 under each diagram.”

This instruction page was followed by five pages, each with 8 road-park diagrams. The top half of the first page, containing stimuli 1, 2, 5, and 6, is shown in Figure 15.

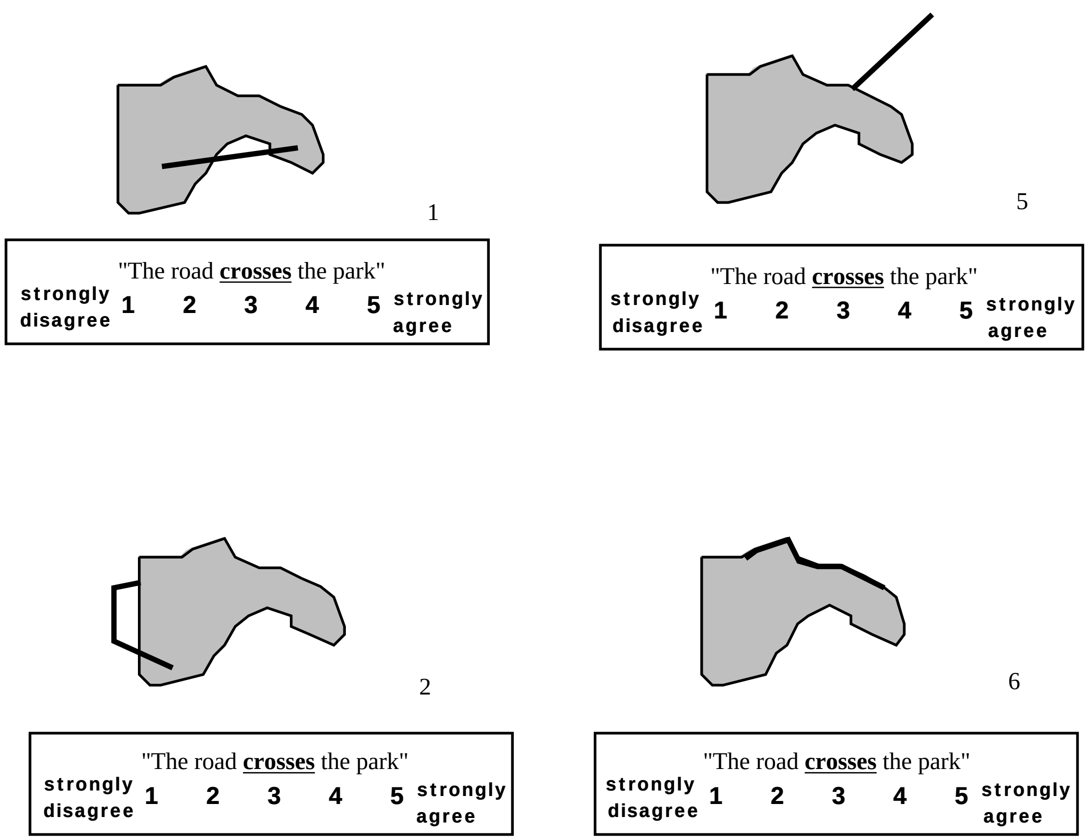

Figure 15. Four of the stimuli (the top half of the first page of the test instrument) used in the experiment designed to evaluate the concept of a linear feature “crossing” an area feature.

Each subject was asked to compare the sentence to each of the 40 diagrams that were used in the first experiment, and to evaluate on a scale of 1 to 5 the strength with which they disagreed (1) or agreed (5) that the sentence described the situation portrayed in that diagram. Then the average rating for all subjects was obtained for each diagram, and this average was rescaled so that 0.0 would represent "strongly disagree" and 1.0 would indicate "strongly agree." Since these ratings were quite similar for the 2 or 3 examples of each of the 19 relations distinguished by the 9-intersection model, we further averaged the results across the stimuli for each of those 19 relations[3]. These summary ratings are empirical estimates of the probability that a subject would consider that a drawing illustrating that topological relation represents the concept to which the sentence refers.

4.2 The Road "Crosses" the Park

The first spatial relation that we tested was the concept of a line "crossing" a region. Some *apriori* analysis suggests that in order for a line to "cross" a region, it should satisfy two topological constraints: the line must have some intersection with the interior of the region, but also should not terminate within the region. As linguist Leonard Talmy has discussed (Talmy 1983), the prototypical meaning of "cross" involves completion of a side-to-side traversal of a two-dimensional entity, and thus there is a possibility that metrical properties will be important. Figure 16 illustrates the two subsets of the 9-intersection diagram that are excluded by the restrictions noted above, and the five remaining relations, which should correspond to "cross."

3 The first 38 stimui (two for each 9-intersection class) were drawn with the road in an "ordinary" relation to the park. Stimuli #39 and #40, however, had specific geometries designed to examine specific aspects of road-park relations. These two special stimuli were excluded from the averages calculated for the 9-intersection relations, and results for #39 are reported separately below.

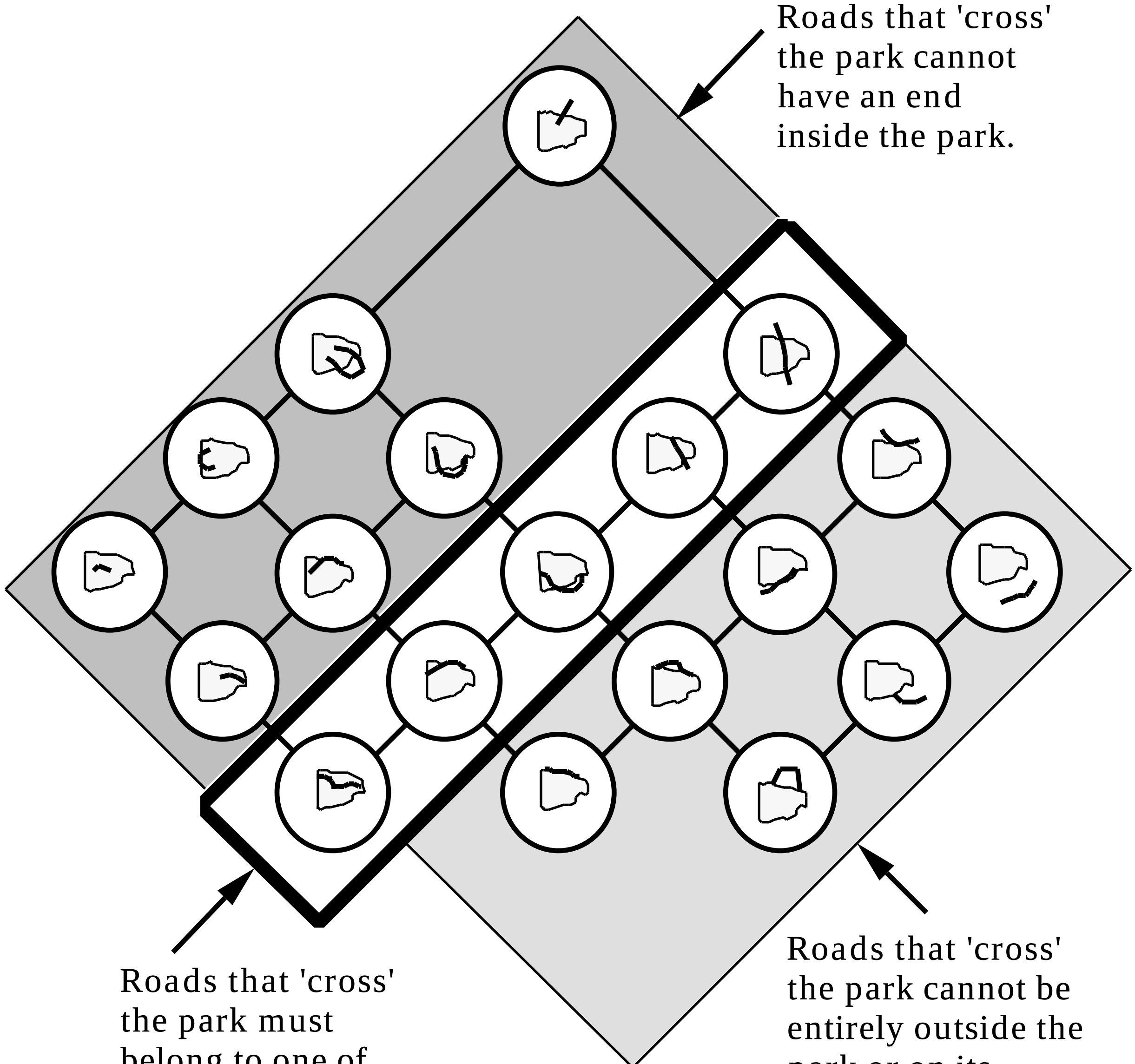

Figure 16. *A priori* analysis of the probable constraints on the concept of a linear feature "crossing" a region feature.

We collected agreement ratings for "the road crosses the park" from 13 native English-speakers and for three other subjects[4]. The results are illustrated in Figure 17, and confirm the conceptual model outlined above. The stimuli fall into 3 groups. The 5 spatial relations that were predicted to "cross" the park had the highest mean ratings, 0.68 or above. It is interesting to note that the highest agreement ratings are for the two situations at the ends of the "crosses" class; this supports

4 These three additional subjects, two who were native speakers of Chinese and one of Hindi, were all fluent in English and were tested in English.

the generalization presented in section 3.2.4 above, that best examples (prototypes) tend to be at the ends of categories. The 7 relations for which the road does not enter the park's interior at all had the lowest ratings, 0.14 or lower. The 7 cases in which the road enters the park but ends inside it had intermediate mean ratings, between 0.21 and 0.36. Evidently, ending inside the park does not exclude a road from "crossing" a park as strongly as not entering the park at all.

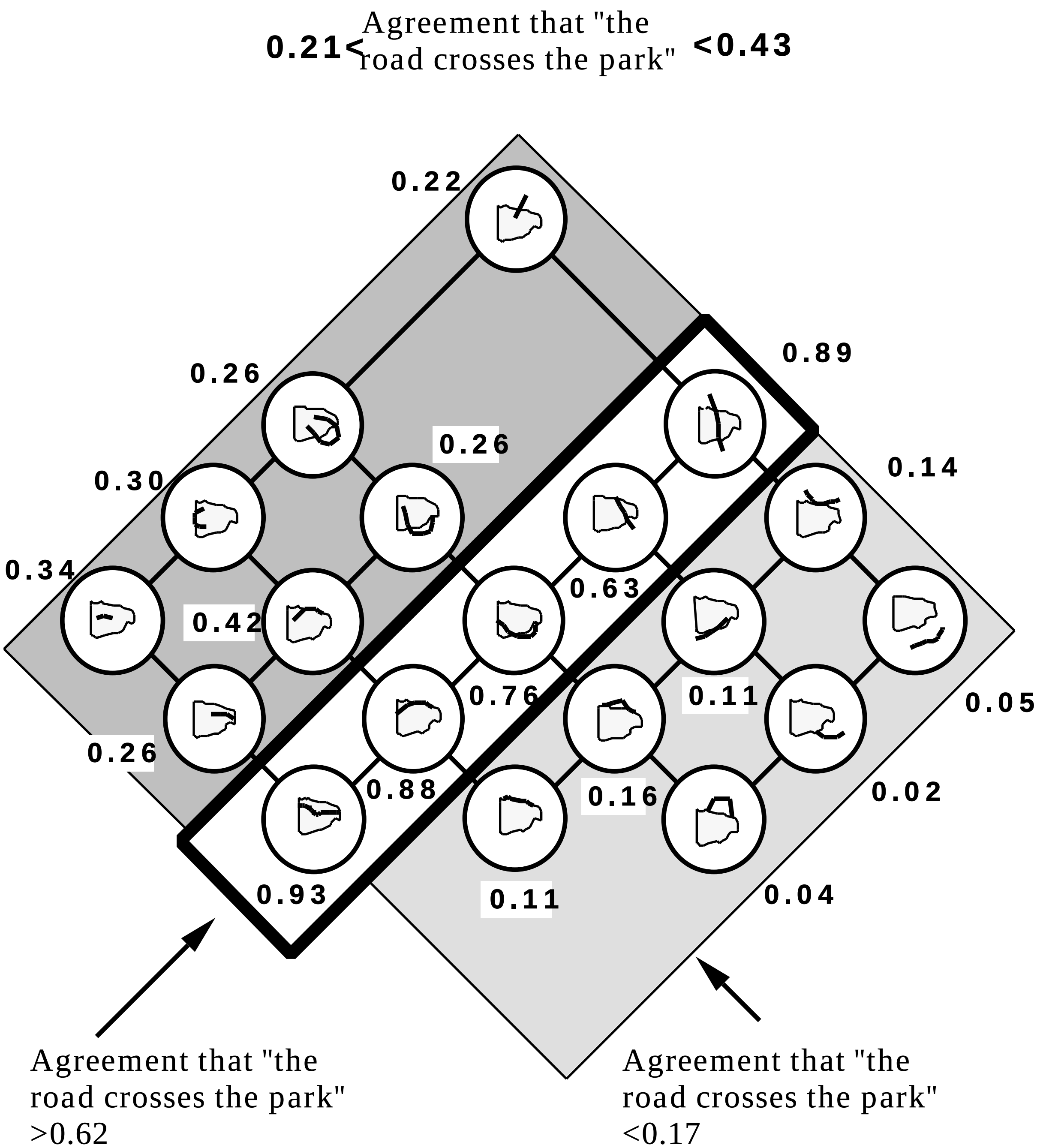

Figure 17. Strength of agreement that "the road crosses the park" by 9-intersection relation, averaged across 16 subjects and 2 or 3 cases per relation.

Talmy's (1983) emphasis on a side-to-side traversal suggests a further restriction might exist regarding geometry. Talmy presented the following example (in Mark *et al*. 1989a): if a person walks from one end of a pier to the other, straight down the middle, it would not be appropriate to say in English that the person crossed the pier, even though the walk had completed a traverse from one part of the pier's boundary to another. Thus, some situations that meet the topological restrictions noted above might still be excluded from the class of "roads crossing the park" by geometric properties. In fact, our stimulus #39, in which the road comes in one end, curves, and goes out through that same end, had a "cross" rating of 0.45, whereas the two topologically-identical stimuli in which the road traversed the park between two opposing sides had mean ratings of 0.84 and 1.00. Influences of geometry will be a focus of some testing in our further research.

4.3 The Road "Goes Into" the Park

The same set of stimuli and instructions were run with the phrase "the road goes into the park," which was a spatial-relation category that several of the English-language subjects in the classification experiment (first experiment) came up with, and which usually had as its prototype the situation with one end of the road outside the park, and the other end inside (the situation at the top of the 9-intersection diagram; see Figure 18).

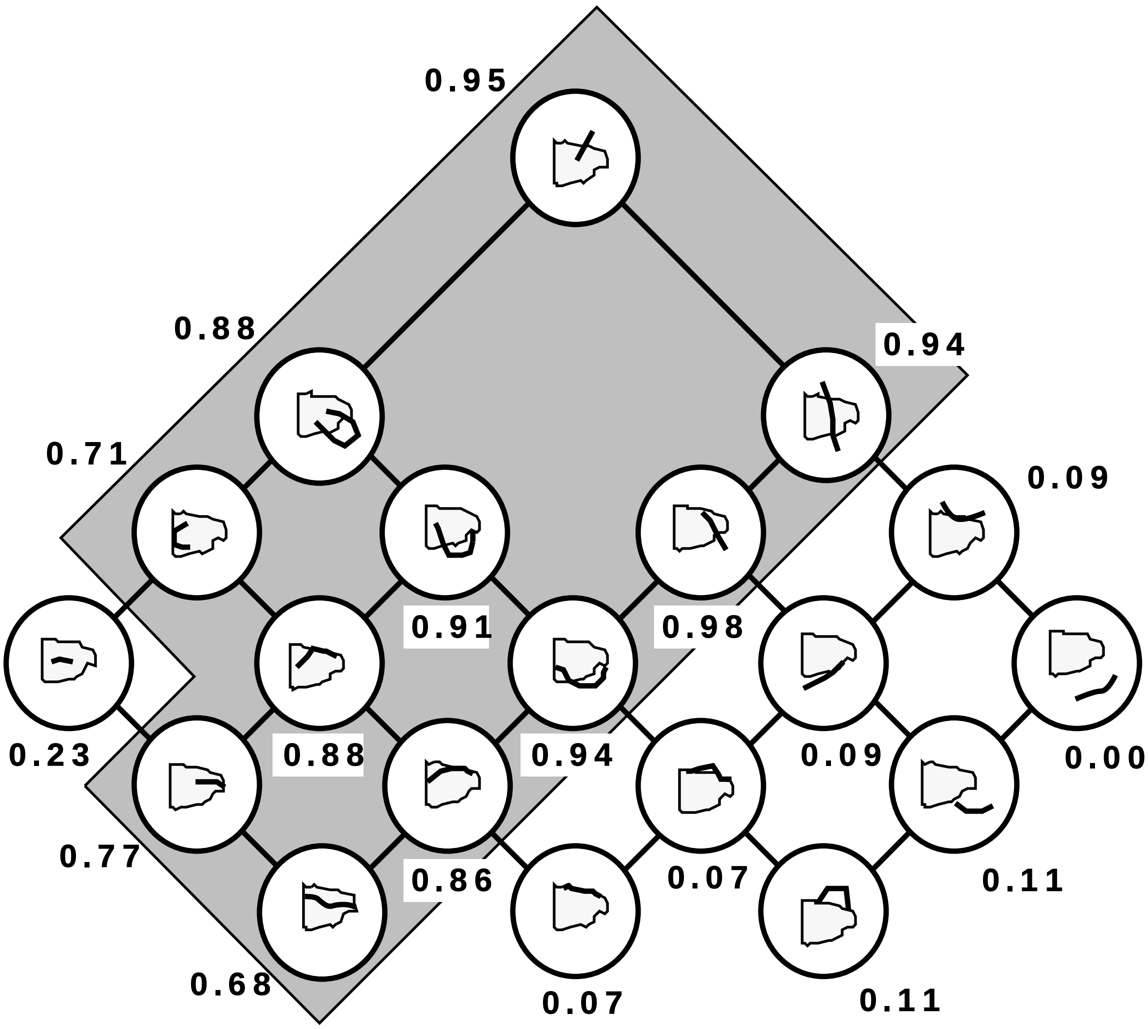

Figure 18. Strength of agreement that “the road goes into the park” by 9-intersection relation, averaged across 7 English-speaking subjects and 2 or 3 cases per relation. Situations with values above 0.5 are surrounded by shaded box.

Data on agreement with the phrase “the road goes into the park” were collected for 7 subjects. Again, there was considerable consensus within 9-intersection relations, across both subjects and stimuli. The results, presented in Figure 18, show high agreement for all cases in which the body of the road intersects the interior of the park, as long as at least one end of the road is outside or on the boundary. Furthermore, the relation at the top of the diagram, which as just noted was often the category prototype in the classification experiment, had a rating of 0.95, second highest of all the stimuli.

4.4 Summary of the Second Experiment

Comparing the results of this experiment for the two sentences tested, we find that some situations were strongly confirmed as belonging both to “the road crosses the park” and “the road goes into the park.” Other situations belong to one concept and not the other, and still others fit neither of these descriptions. This supports the idea that no single set of mutually-exclusive and collectively-exhaustive spatial predicates could satisfy all queries or natural language descriptions. On the other hand, the results give us further confidence that the 19 line-region relations distinguished by the 9-

intersection model have promise as a set of primitives, to be used as building blocks in developing a potentially large number of higher-level spatial concepts.

5. Future Research

Future research is indicated in many directions. Clearly, the experiments described in this paper should be repeated with larger samples. The cross-linguistic dimension of the problem also is worth pursuing, because of the implications for GIS user interfaces, query languages, and cross-linguistic technology transfer (Mark *et al.*, 1989b; Frank and Mark, 1991; Gould *et al.*, 1991). There also is potential to contribute to our understanding of the differences by which different languages express spatial concepts (Talmy, 1983), and thus subjects should be tested in other languages. The possible influence of the hypothetical phenomena in the drawings also is worth investigating. Would the results be significantly different if the test drawings for the line-region relation were described as a storm track and an island or peninsula? Or a road and a gas cloud? And does scale (scope) matter, that is, would the categorization be different if the line and region were things on a table-top, or were at continental scales?

It also would be interesting and potentially valuable to perform human-subjects experiments regarding the acceptability of hypothetical GIS responses to hypothetical quasi-natural-language queries regarding spatial relations between line features and region features. The second experiment was designed to test this aspect of the problem, but probably would be more clearly applicable to GIS if the queries were from a GIS context and if the test were performed on a computer rather than on paper. Also, we feel that the model described herein would provide a good basis for analyzing line-region queries provided in GIS software, or in testing spatial relations defined in the literature.

In addition to the specific results obtained, we feel that the studies reported in this paper demonstrate the value of human subjects testing and empirical evidence in the development and evaluation of formal models for spatial relations. The 9-intersection model for lines and regions can be understood more fully in light of data from human subjects. We hope that more researchers from the GIS and cartographic communities will combine experimentation and mathematical rigor to determine the strengths and limitations of the infinity of possible spatial relations that could be formally defined.

6. Acknowledgments

This paper is a part of Research Initiative 10, “Spatio-Temporal Reasoning in GIS,” of the U.S. National Center for Geographic Information and Analysis (NCGIA), supported by a grant from the National Science Foundation (SES-88-10917); support by NSF is gratefully acknowledged. Max Egenhofer’s research is also supported by a grant from Intergraph Corporation. The experiments described herein were approved by the human subjects review procedures of the Faculty of Social Sciences, SUNY Buffalo. Feibing Zhan administered the test to the Chinese subjects and assisted with the multidimensional scaling, and Hsueh-cheng Chou suggested some useful references. Daniel Montello, Ann Deakin, and Catherine Dibble provided useful comments on an earlier version of this paper. Thanks are especially due to the subjects who participated in the experiment.

7. References

Abler, Ronald F., 1987. The National Science Foundation National Center for Geographic Information and Analysis. *International Journal of Geographical Information Systems*, vol. 1, no. 4, pp. 303-326.

Bennis, K., B. David, I. Morize-Quilio, J. Thévenin, and Y. Viémon, 1991. GéoGraph: A Topological Storage Model for Extensible GIS. *Proceedings, Auto Carto 10*, pp. 349-367.

Berlin, B., and P. Kay, 1969, *Basic Color Terms: Their Universality and Evolution*. Berkeley: University of California Press.

Boyle, A. R., Dangermond, J., Marble, D. F., Simonett, D. S., Smith, L. K., and Tomlinson, R. F., 1983. *Final Report of a Conference on the Review and Synthesis of Problems and Directions for Large Scale Geographic Information System Development*, National Aeronautics and Space Administration, Contract NAS2-11246. (Report available from ESRI, Redlands, CA.)

Claire, R., and S. Guptill, 1982, Spatial Operators for Selected Data Structures. *Proceedings, Auto-Carto 5*, Crystal City, VI, pp. 189-200.

Clementini, E., Di Felice, P., and van Oosterom, P., 1992, A small set of formal topological relationships suitable for end-user interaction. Technical Report, University of L'Aquilla, Italy, submitted for publication.

Cox, N. J., B. K. Aldred, and D. W. Rhind, 1980. A relational data base system and a proposal for a geographical data type. *Geo-Processing*, vol. 1, no. 3, pp. 217-229.

Egenhofer, M., 1989a. A formal definition of binary topological relationships. In W. Litwin and H. Schek (eds.) *3rd International Conference on Foundations of Data Organization and Algorithms (FODO)*, Paris, France Lecture Notes in Computer Science 367, Springer-Verlag, Berlin & New York: 457-472.

Egenhofer, M., 1989b. Spatial Query Languages. Ph.D. Thesis, University of Maine, Orono, May 1989.

Egenhofer, M., 1991. Extending SQL for Cartographic Display. *Cartography and Geographic Information Systems*, vol. 18, no. 4, pp. 230-245.

Egenhofer and Al-Taha, 1992. Reasoning About Gradual Changes of Topological Relationships. In Frank, A. U., Campari, I., and Formentini, U., editors, 1992. *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space* (Lecture Notes in Computer Science vol. 639). Berlin: Springer-Verlag., pp. 196-219.

Egenhofer, M., and R. Franzosa, 1991. Point-Set Topological Spatial Relations. *International Journal of Geographical Information Systems*, vol. 5, no. 2, pp. 161-174.

Egenhofer, M. and Herring, J., 1990. A mathematical framework for the definition of topological relationships. *Proceedings, Fourth International Symposium on Spatial Data Handling*, Zurich, Switzerland, 2: 803-813.

Egenhofer, M., and J. Herring, 1991. Categorizing Topological Spatial Relations Between Point, Line, and Area Objects. Technical Report, University of Maine, Submitted for Publication.

Fegeas, R. G., Cascio, J. L., and Lazar, R. A., 1992. An Overview of FIPS 173, The Spatial Data Transfer Standard. *Cartography and Geographic Information Systems* vol. 19, no. 5, pp. 278-293.

Frank, A., 1982, Mapquery—Database Query Language for Retrieval of Geometric Data and its Graphical Representation. *ACM Computer Graphics*, vol. 16, no. 3, pp. 199-207.

Frank, A. U., 1987. Towards a spatial theory. *Proceedings, International Symposium on Geographic Information Systems: The Research Agenda*, November, 1987, Crystal City, Virginia, 2: 215-227.

Frank, A. U., 1991. Qualitative spatial reasoning about cardinal directions. *Proceedings, Auto Carto 10,* pp. 153-172.

Frank, A. and D. M. Mark, 1991. Language Issues for GIS, in: D. Maguire, M. Goodchild, and D. Rhind, editors, *Geographical Information Systems: Principles and Applications*, Longman, London, vol. 1, pp. 147-163.

Freeman, J. 1975. The Modeling of Spatial Relations. *Computer Graphics and Image Processing*, vol. 4, pp. 156-171.

Freundschuh, S.M., 1989. Does anybody really want or need vehicle navigation aids? *Proceedings, VNIS'89, IEEE Conference on Vehicle Navigation and Information Systems*, Toronto, September 12-14, 1989: 439-442.

Freundschuh, S.M., 1991. The effect of the pattern of the environment on spatial knowledge acquisition. In D.M. Mark and A.U. Frank, editors, *Cognitive and Linguistic Aspects of Geographic Space*. Kluwer, Dordrecht.

Freundschuh, S.M., Gopal, S., Gould, M.D., Mark, D.M. and Couclelis, H., 1990. Verbal directions for wayfinding: implications for navigation and geographic information and analysis systems. *Proceedings, Fourth International Symposium on Spatial Data Handling*, Zurich, Switzerland 1: 478-487.

Gopal, S. and Smith, T.R., 1990. Human way-finding in an urban environment: a performance analysis of a computation process model. *Environment and Planning A* vol. 22, no. 2, pp. 169-191.

Gould, M.D., 1989. Considering individual cognitive ability in the provision of usable navigation assistance. *Proceedings, VNIS'89, IEEE Conference on Vehicle Navigation and Information Systems*, Toronto, September 12-14, 1989: 443-447.

Gould, M. D., Mark, D. M., and Gavidia Gadea, C., 1991. Resolucio de problemas geograficos en igles y español: Implicaciones para el diseño de los sistemas de información geográfica. *Proceedings, III Conferencia Latinoamericana sobre Sistemas de Informacion Geografico,* Viña del Mar, Chile, October 21-25 1991, v. 1, pp. 32-41.

Güting, R. H., 1988. Geo-relational algebra: a model and query language for geometric database systems. In Noltemeier, H., editor, *Computational Geometry and Its Applications*. New York: Springer-Verlag.

Hadzilacos, T., and Tryfona, N., 1992. A Model for Expressing Topological Integrity Constraints in Geographic Databases. In Frank, A. U., Campari, I., and Formentini, U., editors. *Theories* *and Methods of Spatio-Temporal Reasoning in Geographic Space* (Lecture Notes in Computer Science vol. 639). Berlin: Springer-Verlag., pp. 252-268.

Hazelton, N.W., L. Bennett, and J. Masel, 1992. Topoplogical Structures for 4-Dimensional Geographic Information Systems. *Computers, Environment, and Urban Systems*, vol 16, no. 3, pp. 227-237.

Hernández, D., 1991. Relative Representation of Spatial Knowledge: The 2-D Case, in: D. Mark and A. Frank, editors, *Cognitive and Linguistic Aspects of Geographic Space*, Kluwer Academic Publishers, Dordrecht, pp. 373-386.

Herring, J., R. Larsen, and J. Shivakumar, 1988. Extensions to the SQL language to support spatial analysis in a topological database. *Proceedings, GIS/LIS '88*, San Antonio, TX. pp. 741-750.

Herskovits, A., 1986. *Language and Spatial Cognition: A Interdisciplinary Study of the Prepositions in English*. Cambridge, England: Cambridge University Press.

Lakoff, George, 1987. *Women, Fire, and Dangerous Things: What Categories Reveal About the Mind.* Chicago: University of Chicago Press.

Mark, D.M., 1989. A conceptual model for vehicle navigation systems. Proceedings, VNIS'89, *IEEE Conference on Vehicle Navigation and Information Systems*, Toronto, September 12-14, 1989 448-453.

Mark, D. M., Frank, A. U., Egenhofer, M. J., Freundschuh, S. M., McGranaghan, M., and White, R. M., 1989a. Languages of spatial relations: Initiative two specialist meeting report. Santa Barbara, CA: National Center for Geographic Information and Analysis, Report 89-2.

Mark, D.M. and Gould, M.D., 1992. Wayfinding as discourse: A comparison of verbal directions in English and Spanish. *Multilingua*, vol. 11, no. 3, pp. 267-291.

Mark, D. M., Gould, M. D., and Nunes, J., 1989b. Spatial language and geographic information systems: cross-linguistic issues. *Proceedings, 2nd Latin American Conference on Applications of Geographic Information Systems*, Merida, Venezuela, 105-130.

Menon, S., and T. R. Smith, 1989. A declarative spatial query processor for geographic information systems. *Photogrammetric Engineering and Remote Sensing*, vol. 55, no. 11, pp. 1593-1600.

NCGIA (National Center for Geographic Information and Analysis), 1989. The research plan of the National Center for Geographic Information and Analysis. *International Journal of Geographical Information Systems*, vol. 3, no. 2, pp. 117-136.

Peuquet, D. J., 1988. Representations of geographic space: toward a conceptual synthesis. *Annals of the Association of American Geographers*, vol. 78, no. 3, pp. 375-394.

Peuquet, D. J., and Zhan C.-X., 1987. An algorithm to determine the directional relationship between arbitrarily-shaped polygons in the plane. *Pattern Recognition*, vol. 20, no. 1, pp. 65-74.

Pigot, S., 1990. Topological Models for 3D Spatial Information Systems. *Proceedings, Auto Carto 10*, pp. 368-392.

Pullar, D., and M. Egenhofer, 1988. Towards Formal Definitions of Topological Relations Among Spatial Objects, *Third International Symposium on Spatial Data Handling*, Sydney, Australia, pp. 225-243.

Raper, J., and M. Bundock, 1991. UGIX: A Layer-based Model for a GIS User Interface, in: D. Mark and A. Frank, editors, *Cognitive and Linguistic Aspects of Geographic Space*, Kluwer Academic Publishers, Dordrecht, pp. 449-476

Retz-Schmidt, G., 1986. *Deictic and intrinsic use of spatial prepositions: A multidisciplinary comparison.* Sonderforschungsberiech 314, Kunstliche Intelligenz - Wissensbasierte Systeme, KI - Labor am Lehrstuhl for Informatik IV, Universitat des Saarlandes, Memo Nr. 13.

Roussopoulos, N., C. Faloutsos, and T. Sellis, 1988, An Efficient Pictorial Database System for PSQL. *IEEE Transactions on Software Engineering*, vol. 14, no. 5, pp. 630-638.

Svensson, P. and H. Zhexue, 1991. Geo-SAL: A Query Language for Spatial Data Analysis, in O. Günther and H.-J. Schek, editors. *Advances in Spatial Databases—Second Symposium, SSD '91* (Lecture Notes in Computer Science vol. 525). Berlin: Springer-Verlag., pp. 119-140.

Talmy, L., 1983. How language structures space. In H. Pick and L. Acredolo (editors) *Spatial Orientation: Theory, Research and Application.* Plenum Press.

Worboys, M., and S. Deen, 1991. Semantic Heterogeneity in Geographic Databases, *SIGMOD RECORD*, vol. 20, no. 4, pp. 30-34.