Neighborhood Relations between Fields with Applications to Cellular Networks

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Abstract. In some spatial applications the objects of interest are fields, caused by spatially distributed sources, and one of the central questions is to find neighborhood relations between these fields. The motivating example for this paper is a cellular network: base transceiver stations transmit signals with continuous distribution, the signal strength, in an urban environment. In order to avoid interference, *neighbored* transceivers must not use the same frequency, so that neighborhood knowledge is one key to frequency planning. In this paper we define a concept of neighborhood for fields, and we propose a vector-based model to determine neighborhoods between given fields. In contrast to this vector-based model, the commonly used raster-based models suffer in urban areas from their resolution as well as from the prediction of signal propagation.

1 Introduction

In this paper we focus on how to define and to determine neighborhood relations between fields. We deal with *fields* of phenomena having a source (or absolute maximum strength) and a distribution in space. The distribution needs neither to be continuously derivable nor to be monotonous. However, the distribution is expected to be (practically) finite; a threshold on the field strength value can realize this property. Then fields cover a finite coverage area wherein each point is characterized by a field strength, signal strength or *intensity* (Figure 1). The intensity outside the coverage area is neglected and *per definitionem* assumed to be zero. We deal with problems where several or even many fields cover the space, and where holes – i.e., uncovered areas of space – may exist. Coverage areas of different fields may overlap or even contain each other.

In order to define neighborhood relations between fields we use a discrete model of fields. This discrete model is a *polygon-model*: each field is represented by a *set of hierarchical polygons*, shortly called *polygon set*. The hierarchical polygons result from isoline mapping (Dent 1985); they form a planimetric (2D) representation of the 2.5D intensities of fields (Figure 2). We call the polygon set of a field hierarchical, because isolines of a field are partially ordered. Successive isolines lie in one another or may coincide, but they must not cross. Especially the coverage area is bound to the lowest isoline. The definition of neighborhood relations between fields will be based on such polygon sets.

The second goal is to derive rules that will allow us to calculate the neighborhood relations between any two polygon sets of the plane and subsequently to conclude a neighborhood relation between the two fields they represent. To facilitate the task of finding neighborhood relations we will use a graph algorithm.

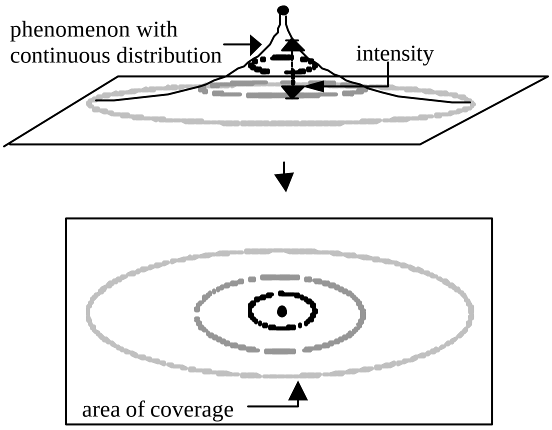

Figure 1: Planimetric representation of a field with isolines.

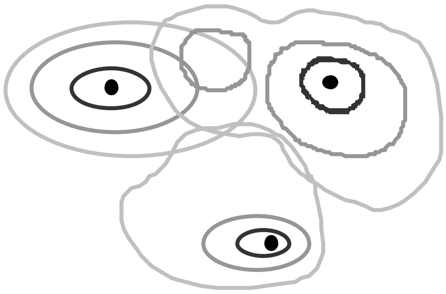

Figure 2: A hierarchical polygon-model of three fields; the sources are denoted as points.

We show that a discrete 2D-model of fields, consisting of sets of polygon sets, allows retrieving neighborhood information between fields. Neighborhood will be based on a set of topological constraints. These topological constraints will be formulated in terms of topological relations between mutual pairs of polygons. We transform the relations between polygons into a graph that is dual to the polygon sets. The dual graph will be the basis for concluding neighborhood relations without having to handle polygons or geometry anymore.

The motivation of this task can be found in a problem that occurs in cellular networks of telecommunication. Base transceiver stations (we will speak of transceivers in the following) transmit signals with continuous distribution, the signal strength. In free space the intensity of the signal would decrease monotonously with the distance from the transceiver, but often obstacles cut the distribution abruptly (the intensity is not necessarily continuously derivable) and reflections lead to local maximums or disconnected fields (the intensity is not monotonous). Within these circumstances, when a call from a cellular client is in progress, the mobility of the user may induce the need to change the serving transceiver, e.g., when transmission quality drops below a given threshold. The process of automatically transferring a call from one transceiver to another is called handover. The decision to trigger a handover and the choice of the target transceiver are based on a number of parameters, which have to be designed and adjusted very carefully. For each transceiver in the network a neighbor list, as possible targets of a handover process, has to be defined. The perfect planning of neighbor lists is essential to achieve high handover success rates. In order to avoid interference, neighbored transceivers must not use the same frequency, so the neighborhood knowledge is a prerequisite for the task of frequency planning.

The derivation of neighborhood knowledge deals with detecting overlapping propagation areas, comparison of the intensities in the overlap (not all overlaps are relevant), and finally the qualification of the found neighborhood relations between transceivers. Commonly, raster-based intensity models of the signals of the transceivers are used for neighborhood planning. A disadvantage of commonly used raster-models is that the resolution of the raster is too low for effectively modeling the propagation area in urban environments (Siebe and Büning 1996). Furthermore the prediction of propagation in urban environment is extremely difficult. An innovative idea to overcome the inadequacy of commonly used raster-models in urban areas is to apply the polygon-model instead (Lang 1999).

In Section 2 we show how neighborhood in continuous and discrete models of fields can be characterized. In Section 3 topological relations between areas are reviewed. They are the basis to develop the theoretical model of this paper: Section 4 shows how topological relations between area objects can be used to conclude neighborhood relations between sets of hierarchic polygons. In Section 5 we show that a dual graph helps to identify neighborhood relations between fields. In Section 6 we show test results and in the last section we draw conclusions and look at future work.

2 Using 2D Information for Defining Neighborhood

In this section we explain how to characterize neighborhood relations by using the continuous (2.5D) model of fields. Then we show how to use the discrete 2D-model that uses areas to do the same. At the end we explain how we derive the polygon-model from fields.

2.1 Definition of Neighborhood in a 2.5D-Model

Fields may penetrate each other. Projected onto 2D their coverage areas overlap, but the field surface in 2.5D intersects. We call the line where two surfaces intersect the *border* between the two fields (Figure 3**).** Necessarily, along the border between two fields the intensity of one field equals the intensity of the other (Nowok 1995). We say that if there exists a border between two fields they are *related*. By this definition we exclude disjoint fields – fields are assumed to be finite always – from 'being related' as well as fields which are completely covered in one another so that the intensity surfaces nowhere intersect. Up to now, the relation between two fields is unspecified, because the existence of a border is not sufficient to establish a neighborhood relation. The border may be partially or totally covered by a stronger field (or several other fields). If a border is totally covered by stronger fields the relation established by the border is irrelevant with regard to neighborhood, as the border itself is. In all other cases the border is called a *relevant border*, and only relevant borders establish a *neighborhood relation* between two related fields.

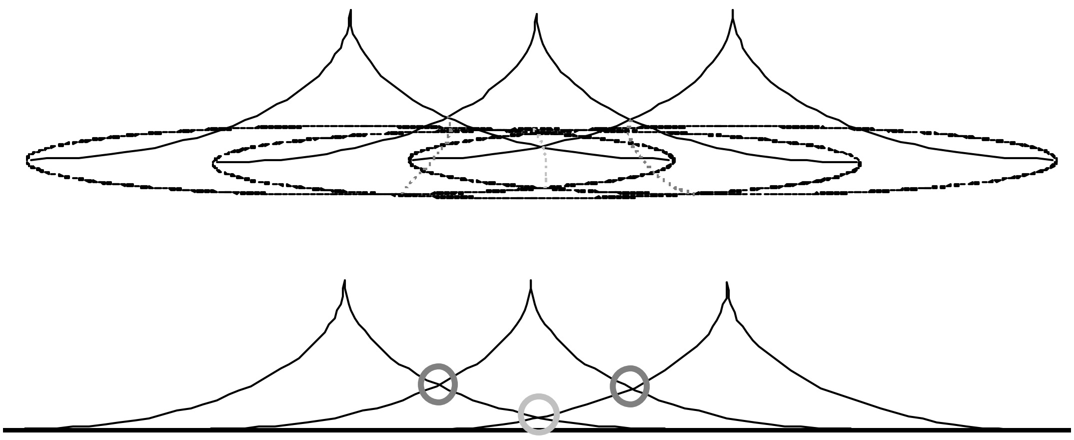

Figure 3: 2.5D visualization of penetrating fields: relevant borders are in dark gray, and an irrelevant border, covered by a stronger field, is in light gray.

The reason for ignoring irrelevant borders can be explained by the neighborhood problem in the cellular telecommunication network. In a cellular telecommunication network the mobile station (e.g. cellular phone) will always be served by the transceiver with the strongest field in an area. If the mobile station moves out of the area, it is handed over to the transceiver that is the strongest in the area of the new position (Mouly and Pautet 1992, Rappaport 1996). Therefore only relevant borders can establish a neighborhood relation between fields.

2.2 Discrete 2D Representation of 2.5D Fields: The Polygon-Model

First the 2.5D fields are transformed into a discrete 2D-model. Afterwards it is possible to derive information about intersecting areas of equal intensity (see following section).

In Section 1 we introduced roughly the polygon-model as a set of hierarchical polygon sets. Polygons are associated with isolines in a finite gradation. Every isoline has a *level* ***X***, which stands for a discrete intensity or signal strength. The area enclosed by isolines of a given level ***X*** is called *iso-polygon*. ***A(X)*** is an iso-polygon of the source *A* at the level ***X***. The iso-polygon ***A(X)*** contains all points of the field *A* with intensity or signal strength equal or higher than ***X***. Note that an iso-polygon may be disconnected, e.g., if the field has more than one peak (for an example see Figure 2).

Due to the fact that iso-polygons represent a continuous field, an iso-polygon at a given level of intensity has to be contained by or equal to an iso-polygon at a lower level of intensity:

$$\boldsymbol{A(Y)} \subseteq \boldsymbol{A(X)} \qquad \text{if } \boldsymbol{Y} > \boldsymbol{X} \qquad (1)$$

If Equation 1 holds for a set of iso-polygons we say the set is *hierarchic*. Polygon-models contain only hierarchic sets of iso-polygons.

2.3 Definition of a Border in the Polygon-Model

In this section the notion of a border between two continuous fields is transferred to the discrete 2D representation. In the continuous case, the border was defined as the intersection line of the intensity surfaces of two fields (Section 2.1). The existence of such an intersection of the continuous surfaces induces for the discrete polygon-model the intersection of two iso-polygons that have to belong to the same level (i.e., the intersection set is *not empty*). The exact position of the border is neither required nor in the discrete model to be determined.

Consider Figure 4. Let us assume that the two intersecting iso-polygons ***A(X)*** and ***B(X)*** are of the same level ***X***. The *intersection-polygon*, shortly called *int-polygon* (marked in Figure 4) is again of the level ***X***. We use the notation ***[A,B](X,X)*** for the int-polygon: in square brackets are given the names of the fields, and in parenthesis are given the intersecting levels. In this case, ***A(X)*** and ***B(X)*** have the same level thus a border between the fields *A* and *B* lies in ***[A,B](X,X)***. If the situation in Figure 4 is complete, the border is relevant, therefore *A* and *B* are neighbored. – If the level of the iso-polygons were different their intersection would not establish a neighborhood.

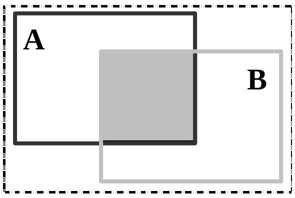

Figure 4: Two iso-polygons of different sources (*A*: black, *B*: bright), and their intersection set.

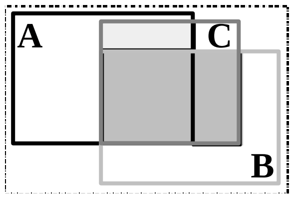

Figure 5: Three iso-polygons of different sources (*A*: black, *B*: bright, *C*: medium), and their intersection sets.

The situation is different if there are more than two fields penetrating. In such cases a border between two of the fields can be partially or completely covered by a stronger signal of another field. Consider Figure 5 now, which contains a third iso-polygon from a field *C* (Scheinert 1995). Let us first assume that all three iso-polygons are of the same level ***X***: ***A(X)***, ***B(X)***, ***C(X)***. By intersecting the three iso-polygons pair-wise, one finds the following three int-polygons (intersection is commutative)**:** ***[A,B](X,X) = A(X) Ç B(X)***, ***[A,C](X,X) = A(X) Ç C(X)***, ***[B,C](X,X) = B(X) Ç C(X)***. The fact that now ***A(X) Ç B(X)*** is completely covered by ***C(X)*** bears no significance since all iso-polygons have the same level. The result is that every field has a neighborhood relation with the others.

But let us now assume that in Figure 5 the level of ***C(Y)*** is higher than the level of ***A(X)*** and ***B(X)***: ***Y>X***. In this case the border between *A* and *B* – somewhere in the int-polygon ***[A,B]*** – is completely covered by an iso-polygon of higher intensity, i.e., the border is irrelevant. ***C(Y)*** *cancels* the neighborhood relation between ***A(X)*** and ***B(X)***. ***A(X)*** and ***C(Y)***, and ***B(X)*** and ***C(Y)*** overlap, but again this does not establish a neighborhood of *A* and *C* or *B* and *C*, because the iso-polygons do not have the same intensity in the area of intersection. Consider now especially the area ***A(X) Ç B(X) Ç C(Y)***. This area is never identified by an int-polygon, because int-polygons were defined as *binary* relations of two iso-polygons from different fields. The decisive statement is that $\boldsymbol{A(X) \cap B(X)}$ is *contained by* $\boldsymbol{C(Y)}$. One has to check (topological) containment relations in order to distinguish relevant and irrelevant borders.

3 Topological Relations between Iso-Polygons

Spatial reasoning attempts to solve problems dealing with objects in space (Kak 1988). It offers its users new spatial information which has not been explicitly recorded and which is otherwise not immediately available in the form of raw data (Egenhofer 1991). As a basis for spatial reasoning here the part of point-set topology is used that treats topological relations of area objects. Firstly previous work is reviewed. Then it is focused on the development of sets of topological relations between sets of hierarchical areas as bounded by iso-polygons. At the end of this section this knowledge is applied for reasoning neighborhood relations between fields.

3.1 Previous Work

The basics of general point-set theory and topology are presupposed. The task to solve is the determination of topological relations between areas, where areas are given through their bounding iso-polygons. The two prominent approaches were the point-set based model of the Egenhofer relations (Egenhofer 1989; Egenhofer and Franzosa 1991) and the logic based model of the region connection calculus (Randell et al. 1992; Cohn et al. 1997). In this paper, the way relations are determined (Section 3.2) is derived from the point-set based method. However, we use a boundary-less representation of iso-polygons. Excluding boundaries will leave out five relations. This resulting set of relations is identical to the five relations in RCC-5, a theory of region connection calculus (Randell et al. 1992).

Cohn and Gotts model *vague* regions by using two concentric sub-regions and call it the *egg-yolk approach* (Cohn and Gotts 1996). They describe also the relation between two egg-yolk pairs by using region connection calculus. In our context, the egg-yolk approach can be seen as a prototype of the hierarchical polygon set. However, the interpretation of the egg-yolk is different here, speaking of different levels of field intensity, and the hierarchical polygon set is not limited to two levels only. For that reason a more complex formalism is required (Section 4).

3.2 Topological Relations between Area Objects

In this section the formalism is developed to determine topological relations between two area objects. It is shown that five types of topological relations between area objects exist if the only distinction is made between object and non-object, or interior or exterior respectively.

For iso-polygons of fields it is natural to describe their location in the plane by a step function, instead of a bounded polygonal area. A location function *f(x,y)* defines an area object *A* by a membership rule for points in the plane, according to the point-set theory:

$$f(x,y) = \begin{cases} 0 & if \;\; f(x,y) \notin A \\ 1 & if \;\; f(x,y) \in A \end{cases} \tag{2}$$

where $x,y \in \mathbb{R}^2$. Iso-polygons, represented by this location function, are well-formed area objects: they contain no peaks and no degenerated holes (however, two-dimensional holes are allowed), but they may be multiply connected or even disconnected. These properties fit to the requirements of Section 2.

The location function distinguishes between two sets: the *interior* (f = 1) and the *exterior* (f = 0) of an area. No boundaries can be distinguished. Intersecting two areas *A* and *B* yields a set of four intersection sets in total. They are explained in Figure 6, where the two areas *A* and *B* overlap.

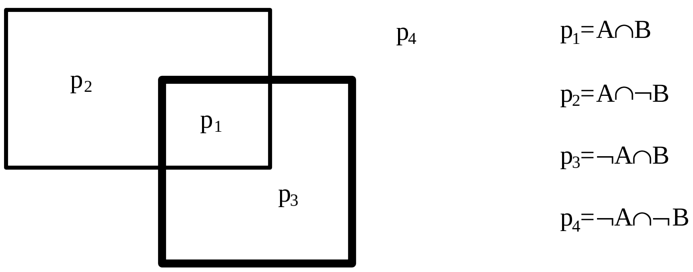

Figure 6: Areas *A* (thin rectangle) and *B* (thick rectangle). The respective intersection sets p_1 … p_4 form a partition of the plane. The background p_4 is assumed to be unlimited.

From the assumption that *A* and *B* are finite follows that p_4 is never empty. Thus the situation between the areas can be described qualitatively by considering the sets p_1, p_2 and p_3 only. Consider the (ordered) triple $\{p_1, p_2, p_3\}$. Each set p_x with $x \in$ 1,...,3 can either be empty ('0') or not empty (‘1’). That yields $2^3 = 8$ combinations that are theoretically possible (Winter to appear). Since neither A (=p_1Èp_2) nor B (=p_1Èp_3) are empty, we can exclude three of the eight possible triples: {0,0,0}, {0,0,1}, {0,1,0}. The remaining five triples correspond to the following *topological relations* (Table 1).

Table 1: The five topological relations to be distinguished between areas with no explicit boundaries. In the third column the corresponding Egenhofer relations are listed.

{0,1,1}	***DT [A,B]***	disjunct touching	*A* and *B* have nothing in common.
{1,1,1}	***OL [A,B]***	overlaps	*A* and *B* have some parts in common, some not.
{1,0,0}	***EQ [A,B]***	equals	All parts of *A* are parts of *B*, and vice versa.
{1,1,0}	***CS [A,B]***	covers contains	All parts of *B* are part of *A*, and *A* has additional parts.
{1,0,1}	***CB [A,B]***	coveredBy containedBy	All parts of *A* are part of *B*, and *B* has additional parts.

The five topological relations are jointly exhaustive and pair-wise disjoint. This means that always exactly one of these relations holds between two non-empty and finite areas.

4 Topological Relations in the Polygon-Model

With the topological relations at hand, we now analyze the intersection of sets of hierarchic areas and then focus on relations that are relevant for neighborhood relations of fields.

4.1 Intersections of Sets of Hierarchic Polygons

Considering the hierarchic polygon set that represents a field in the polygon-model, two fields *A* and *B* can have many int-polygons, even at equal levels. Furthermore, often more than two fields penetrate at a given location, so that the general situation is to be analyzed where many polygon sets intersect.

At the beginning, some definitions are required for different types of relations. On the one hand, the topological relation between two iso-polygons is called a *polygon relation*. On the other hand, the relation between two fields (or polygon sets) is called the *neighborhood relation*. There are two types of a polygon relation: if the considered iso-polygons are of the same level we call the relation a *balanced* polygon relation, otherwise an *unbalanced* polygon relation.

For polygon relations we introduce the following notation: *relation[field1,field2](level1,level2)* specifies the topological relation of a pair of polygons, where one polygon is the *level1* iso-polygon of the polygon set *field1*, the other polygon is the *level2* iso-polygon of the polygon set *field2*. For example, ***OL[A,B](X,X)*** means that the polygon sets *A* and *B* overlap with their two iso-polygons at level ***X***. This notation extends the notation for int-polygons; note that (only) in the special case of a disjoint- or touch-relation (***DT***) this int-polygon remains empty.

Relations between sets cannot be described in such a simple way as the relation between two iso-polygons. The relation between sets is characterized by the topological relation of each single iso-polygon of one set towards all iso-polygons of the other set (Lang 1999). If n_A is the number of iso-polygons of a polygon set *A*, and n_B the number of iso-polygons of a polygon set *B*, the total number of topological relations between *A* and *B* is $n = n_A * n_B$. Practically, given a set of fields most of the (finite) fields will be disjoint, which excludes from further consideration for being neighbored directly. If one only is interested in a specific subset of relations (e.g., the non-disjoint relations), *n* yields at least the upper limit of the number of relations to be stored.

4.2 Polygon Relations for the Neighborhood between Fields

In this section it is shown how to find out which of the polygon relations are relevant for the neighborhood relation between fields.

Looking for (non-empty) int-polygons where different fields have the same intensity (as discussed in Section 2.3), the primary interest is on balanced polygon relations. Furthermore, the fact that the pair of considered iso-polygons shall intersect excludes the balanced disjoint relation ***DT[A,B](X,X)*** from consideration. Only the existence of one of the relations ***OL[A,B](X,X)***, ***EQ [A,B](X,X)***, ***CS[A,B](X,X)***, or ***CB[A,B](X,X)*** between *A* and *B* might result in a neighborhood relation. These four topological relations are called the *neighborhood enabling relations* (NER).

Consider an int-polygon ***[A,B](X,X)*** establishing a NER. In Section 2.3 it was shown that if ***[A,B]*** is covered totally by an iso-polygon of higher level from a third polygon set, ***C(Y)*** with ***Y>X***, the evidence for neighborhood between *A* and *B* is cancelled, and the NER is no longer relevant. More precisely, the covering area may consist of any set of iso-polygons of any level higher than ***X*** (we denote ***X+i***: *i* levels higher then *X*). Because of the hierarchic structure of the polygon set, it is sufficient to check whether the union of all iso-polygons at level ***X+1*** – including those of *A* and *B* – cancels neighborhood. We call such a union the *uni-polygon* ***U(X+1)***. Thus, if *A* and *B* have a NER at level ***X***, one must check whether the unbalanced polygon relation ***CB([A,B](X,X),U(X+1))*** exists. If this is true, then the NER at ***X*** is canceled.

4.3 Determining Neighborhood Between Fields

The goal of this section is to use the knowledge gained till now to find definitive rules to derive neighborhood relations between fields. For this reason we take polygon sets of several fields, and analyze systematically the different relations. For simplicity, each set shall consist of the same number of iso-polygons, and the equidistance between the levels shall be constant.

The following notation will facilitate the explanations: ***LMIN***: lowest level in a set; ***LMAX***: highest level in a set; ***X-i***: *i* levels lower than ***X***, where ***X-i*** must be $\geq$ ***LMIN***; ***X+i***: *i* levels higher than ***X***, with ***X+i*** $\leq$ ***LMAX***; $X_{>}$: all levels higher than ***X***; $X_{<}$: all levels lower than ***X***.

One way to get new spatial information which has not been explicitly recorded and which is otherwise not immediately available in the form of raw data is by concluding relations via other relations (Frank 1996). The composition of two topological relations over a common object is of particular interest in spatial reasoning since it allows for the derivation of new spatial information. The derivation of the composition of topological relations is based upon the transitive property of the subset relationship ($\subseteq$). Transitivity can be applied in a hierarchy of iso-polygons.

Again, the primary interest is to conclude balanced polygon relations from already known balanced polygon relations. Transitivity can be applied to iso-polygons of different hierarchies too. Applied to our problem we found two rules.

Rule 1: If the iso-polygons of level ***X*** of two polygon sets are ***DT***, all iso-polygons at the levels $X_{>}$ are ***DT*** as well.

Following formula backs Rule 1: if the intersection of ***A(X)*** with ***B(X)*** is empty and ***A(X+1)*** is a subset of ***A(X)*** and ***B(X+1)*** is a subset of ***B(X)*** then the intersection of ***A(X+1)*** and ***B(X+1)*** is empty as well.

$$A(X) \cap B(X) = \emptyset \wedge A(X+1) \subseteq A(X) \wedge B(X+1) \subseteq B(X) \quad \Rightarrow \quad A(X+1) \cap B(X+1) = \emptyset$$

Rule 2: If iso-polygons of the level ***X*** have a NER, all iso-polygons at the levels $X_<$ have a NER as well.

Following formula backs Rule 2: if the intersection of ***A(X)*** with ***B(X)*** is not empty and ***A(X)*** is a subset of ***A(X-1)*** and ***B(X)*** is a subset of ***B(X-1)*** then the intersection of ***A(X-1)*** and ***B(X-1)*** is not empty as well.

$A(X) \cap B(X) = \neg\emptyset \wedge B(X) \subseteq B(X\text{-}1) \wedge A(X) \subseteq A(X\text{-}1) \quad \Rightarrow \quad A(X\text{-}1) \cap B(X\text{-}1)= \neg\emptyset$

Let us discuss the consequences of Rule 1 and 2: to get the set of polygon relations, successively all iso-polygons belonging to one level have to be intersected. This has to be done for all levels. Starting with intersecting the iso-polygons at ***LMIN***, we call it the *bottom-up-procedure*. Starting at ***LMAX***, we call it *top-down-procedure*. During the bottom-up-procedure Rule 1 reduces the effort: finding the first relation ***DT*** between two iso-polygons, it is known that all following polygon relations on higher levels are ***DT*** as well. During the top-down-procedure Rule 2 reduces the effort: finding the first NER between two iso-polygons, it is known that all following polygon relations on lower levels are a NER as well. This knowledge enables us to optimize the algorithm to determine neighborhood.

Each NER at a level ***X*** between two polygon sets *A* and *B* may be canceled by a covering ***U(X+1)***. The covering polygon sets separate *A* and *B*, i.e., a canceled NER is semantically equivalent to a ***DT*** relation. Any occurring NER requires checking whether this NER has to be canceled or not. If it has to be canceled, the relation is renamed into a ***DT*** relation. In a bottom-up-procedure, a canceled NER at level ***X*** requires that all relations at $X_>$ have to be ***DT*** as well. In a top-down-procedure canceling a NER at level ***X*** allows no clue to ***X-1***, so one has to intersect the next lower level.

The result of intersecting polygon sets is a set of polygon relations consisting of NERs, canceled NERs and ***DT*** relations. As a ***DT*** relation and its equivalent, a canceled NER, are not relevant for a neighborhood relation, we call them *irrelevant neighborhood relations* (INR). The relations are associated with the levels where they occur. Assume that a NER exists between two polygon sets. Then in the hierarchy of levels, there exists a NER at a level ***X*** that has a NER at ***X-1*** but an INR at ***X+1***: this special NER is called the *key relation*. A key relation establishes a neighborhood relation between the two polygon sets. Two special cases exist: if a

NER exists at ***X=XMAX*** this relation is always a key relation, and if a NER exists at ***X=XMIN*** it is a key relation if an INR exists at ***X+1***.

5 Transformation of Topological Relations into a Graph

Here a graph-based representation is introduced to find the key relations in the set of all NERs. Graphs are used to store and manage all relevant (= not canceled) NERs between all polygon sets in a polygon-model. These graphs are dual to the polygon sets (Wilson and Beineke 1979): vertices represent fields (or polygon sets), and edges represent the NERs between pairs of polygons of the fields. The graphs are then contracted to the key relations that establish neighborhood between fields.

5.1 The Level Graphs

We introduce the concept of a level graph, which allows treating separately NERs at each level.

Consider a polygon-model of several polygon sets representing fields. Let us assume the iso-polygons in all sets are of the same classification schema, i.e., the intensity levels in the single polygon sets are the same. Selecting one of the levels available, each field is represented by exactly one iso-polygon, as long as the field reaches that level somewhere. Analysis starts with finding the intersections of all pairs of iso-polygons, and passing the check of relevance for the found polygon relations (Section 4.2). Because in this case the intersecting iso-polygons belong all to the same level, the found polygon relations are balanced always. Balanced and checked int-polygons indicate NERs between the intersecting fields.

While the topological polygon relations (Section 3.2) are only partly symmetric – ***CS*** and ***CB*** are not symmetric – the existence of an int-polygon (in the case of a NER) is symmetric: if *A* intersects with *B*, then *B* intersects with *A* as well, or ***[A,B](X,X)=[B,A](X,X)***. Therefore, the intersection relation between two iso-polygons (or two fields, respectively) can be represented by an undirected edge. Start and end vertex of such an edge must be an iso-polygon (or a field, respectively). The *level graph* is set up straightforward. At the beginning there is the graph of the vertices of the fields in the polygon-model. This graph contains no edges yet; it is called a null graph. Basis for the neighborhood between fields at a certain level is the intersection of the iso-polygons of that level: the iso-polygons are pair wise intersected and checked for relevance. Relevant intersection relations are stored by an edge between the field vertices.

In terms of graph theory this process starts with a partition created by the overlay of all iso-polygons of one level. The partition graph can be represented alternatively by a null graph, where each vertex represents a part of the partition (Figure 7).

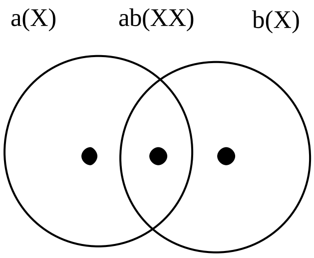

Figure 7: The intersection of two iso-polygons: each region can be represented by a vertex.

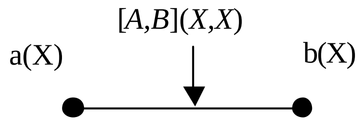

Figure 8: The region of the int-polygon can be contracted to a boundary edge, and the dual of the boundary is an edge representing a polygon relation.

Saying that the (checked) int-polygon creates a NER between *A* and *B*, temporarily the int-polygon ***[A,B]*** is contracted to a boundary edge between *A* and *B*. In this intermediate partition of the plane only neighbored fields share a common boundary. The dual of such a boundary edge is the edge in the level graph (Figure 8). The level graph is the dual graph to the intermediate (contracted) partition graph, with exception of the relations that have to be canceled.

The level graph LMIN has the maximum size, in terms of the number of edges. This is due to the spatial extension of the iso-polygons at ***LMIN***: they are bigger and therefore they intersect more often than iso-polygons at higher levels. The least number of edges will be found for the level graph at ***LMAX*** where the extension of the iso-polygons is smallest. The edges in the level graphs have the property to be hierarchically ordered, so that the set of edges in ***LMAX*** is a subset of the set of edges in ***LMIN***.

5.2 The Neighborhood Relation Graph

Given all level graphs, we now construct the *neighborhood relation graph*. In this graph, only the key relations are preserved from all NERs in the level graphs. We use a graph algorithm that automatically finds all existing key relations out of the total number of polygon relations. The graph algorithm exactly follows the rules we concluded in Section 4.3.

At the beginning again there is the null graph containing a vertex for each field, and the set of level graphs. Each of these graphs contains the same set of vertices. The hierarchical order of the edges in the level graphs allows to search

- top-down: all edges of the level graph ***LMAX*** represent key relations and have to be added to the neighborhood relation graph. At the next lower level, all edges between vertices that appear additionally to the edges already known represent key relations of level ***LMAX-1***. This process is recursive down to the level graph ***LMIN***.

- bottom-up: all edges of the level graph ***LMIN*** are candidates for key relations They can be added to the neighborhood relation graph. At the next higher level, some of the candidates occur again, so the only attribute of the edges, their level, has to be replaced. This process is recursive up the level graph ***LMAX***.

The number of the neighborhood relations (= number of edges of the neighborhood relation graph) is equal to the number of edges in the level graph ***LMIN*** necessarily. The only difference between the level graph ***LMIN*** and the neighborhood relation graph is the attribute of the neighborhood graph edges, giving the level of the key relation.

With the neighborhood relation graph at hand, the neighbors of each field can be determined simply by looking for the edges adjacent to the vertex representing the field, and collecting their opposite vertices as well as their level. This is a basic graph algorithm (Turau 1996). The result of this calculation is a list of neighbors for each field.

6 Test of Concept

The developed concepts and algorithms were tested with a data set of a real-world situation. We selected a part of an existing real cellular network, determined the neighborhood relations with our algorithm, and compared the results with existing neighbor lists. In this section we report about the experiences with the test.

For the test case we selected a region with ten transceivers in the city center of Vienna. The polygon-model consisted of thirty iso-polygons, i.e., three iso-polygons per transceiver (high, middle, low intensity). For each transceiver we used its neighbor list as reference data. Neighbor lists show the neighborhood relations of transceivers taking into account the whole network at a certain time. Due to the urban situation these lists were generated manually in a time consuming procedure based on the experience of experts. In a second step these lists were improved by the statistics of real handovers. This improvement removes for instance neighborhood relations that were originally in the list but which did not to exist in reality. Sometimes experts refuse a neighborhood relation from the neighbor list for sake of better communication performance. Thus we divide the neighborhood relations of a single transceiver into two kinds:

$\boldsymbol{R_l}$: neighborhood relations that exist and are listed in the neighbor list.

$\boldsymbol{R_r}$: neighborhood relations that exist but are not listed in the neighbor list.

Thus the sum of all listed and refused neighborhood relations is our reference. We call it $\boldsymbol{R_c}$:

$$\boldsymbol{R_c} = \boldsymbol{R_l} \cup \boldsymbol{R_r}$$

The results from our proposed algorithm for a single transceiver, the set of relations $\boldsymbol{T_c}$, will show a similar structure:

$$\boldsymbol{T_c} = \boldsymbol{T_l} \cup \boldsymbol{T_r} \cup \boldsymbol{T_f}$$

$\boldsymbol{T_c}$ is the set of all calculated neighborhood relations. It consists of $\boldsymbol{T_l}$, the set of found relations that exist in $\boldsymbol{R_l}$ also, plus $\boldsymbol{T_r}$, the set of found relations that exist in $\boldsymbol{R_r}$ also, plus $\boldsymbol{T_f}$, the set of fictitious neighborhood relations. A fictitious neighborhood relation is a surplus neighborhood relation that would have been canceled if the whole network had been modeled. Yet because the transceiver generating the field that would have canceled the fictitious neighborhood relations is not part of our polygon-model, the relation remains in our results. We expect that $\boldsymbol{T_l}$ equals $\boldsymbol{R_l}$ and $\boldsymbol{T_r}$ equals $\boldsymbol{R_r}$, which means our algorithm should find all listed and refused existing relations. With no possibility to distinguish $\boldsymbol{T_r}$ and $\boldsymbol{T_f}$ automatically, we expect at least that the found neighborhood relations contain the listed neighborhood relations completely: $\boldsymbol{T_l}$ =! $\boldsymbol{R_l}$.

In Table 2 we see the result of the calculation of $\boldsymbol{T_c}$ for each transceiver. We found all relations of $\boldsymbol{R_l}$ in $\boldsymbol{T_c}$ for all ten receivers, i.e., $\boldsymbol{T_l} = \boldsymbol{R_l}$. All surplus detected relations belong to $\boldsymbol{T_r}$ or $\boldsymbol{T_f}$, which was confirmed by the experts managing the neighbor lists. The computing time was in an acceptable range of less than three minutes; yet we do not claim that the algorithm is optimized.

However, the computation of neighborhood relations is an off-line process and thus not time critical.

B		D		G		L		L2	
R_l	T_c	R_l	T_c	R_l	T_c	R_l	T_c	R_l	T_c
S2	✓	S3	✓	S2	✓	B	✓	B	✓
G	✓	W	✓	M	✓	L	✓	L	✓
M	✓	S	✓	B	✓	G	✓	S2	✓
L2	✓		S2	L	✓	M	✓	W	✓
L	✓						W		S3
C	✓						S2		S
	W								
	S								

M		S		S2		S3		W	
R_l	T_c	R_l	T_c	R_l	T_c	R_l	T_c	R_l	T_c
G	✓	W	✓	B	✓	S2	✓	S	✓
B	✓	S3	✓	S3	✓	S	✓	S2	✓
L	✓	D	✓	G	✓	B	✓	L2	✓
	S2		S2	L2	✓	D	✓	S3	✓
			B	W	✓		L2	D	✓
			L2		S		W		B
					L				L
					M				
					D				

Table 2: Test result from a real-world situation of ten transceivers in the city of Vienna. The header lines contain the names of the transceivers (B-W). For each transceiver, the left column shows the listed neighborhood relations (***R_l***), and the right column shows the listed neighborhood relations found by our algorithm also (***T_l***), and relations found additionally (***T_r*** or ***T_f***).

In summary, the test with real-world data demonstrates:

- The theoretical concept of a polygon-model is correct.
- The polygon-model is a practical basis for calculating neighborhood relations.
- The proposed algorithm work satisfactory in real-world situations.

7 Results and Future Work

7.1 Summary

In this paper neighborhood for fields is defined, and the determination is described by a formal analysis of the problem. This concept is based on a discretization of the continuous fields into hierarchic polygon sets. We show that it is possible to use topological relations between hierarchic sets of polygons to deduce the neighborhood relations between fields.

Neighborhood manifests itself by the way in which fields penetrate. In the polygon-model, neighborhood manifests itself by the way in which sets of hierarchic polygons overlap. Hence we use topological rules that allow us to derive neighborhood relations on the basis of overlapping areas. We translate the criterion for continuous fields to be neighbored into these topological rules valid for the discrete polygon-model. We show that between sets of polygons always sets of topological relations exist, and we show how to reason from these relations to conclude relations between polygons without explicitly intersecting them. To handle all existing relations between polygons we use a dual graph. Not all polygon relations between sets are relevant for neighborhood. We develop a graph algorithm that reduces all existing relations between polygons, until only the relevant ones remain that lead to neighborhood between fields. The result is a graph that contains all relations between all modeled fields. Exploiting information from the graph we can name the neighbors of each single field.

7.2 Conclusion and Future Work

The most important and interesting part of the whole work was the translation of rules for fields into terms valid for the discrete model. We found clear translations of the situation between fields into a discrete geometric formulation. The model was tested successfully in a practical application, the neighborhood determination in a cellular communication network. We expect that other application areas exist with similar problems. Common ground is the idea of stationary fields that penetrate each other, where the fields can be of any type: noise, illumination, pressure, velocity, and so on. Neighborhood knowledge can support environmental planning, planning of public utilities, or optimize some problems of route planning or traffic planning, to name just a few.

Apart from practical applications, there is the theoretical contribution. The presented model extends the egg-yolk model (Randell et al. 1992), which is in principle a model for vague regions (Worboys 1998; Bittner 1999), by a multi-level model that is still discrete. We clearly define neighborhood qualitatively here. The various levels are not used to invent gradations for relations, except that the found neighborhood is characterized by the field intensity level of the key relation. That is different, e.g., from fuzzy concepts of neighborhood (Molenaar 1999; Papadias et al. 1999). The advantage – in the context of our example – is the clear semantic of a binary neighborhood relation (true or false), whereas a fuzzy membership value (for the applicability of "being neighbors") is difficult to interpret. For example, frequency planning of a telecom company is based on binary neighborhood of base transceiver stations.

However, it is an open question whether weights, e.g., for the significance of a found neighborhood relation, extend the model in a useful manner. Such a measure could be based on the size or the form of the intersection polygon of the key relation. Not all mathematically correct assignments of neighborhood are practically relevant, and low measures could motivate a planner to review the situation in the urban environment.

This problem is related to the observation of the fields, and especially to the (spatial) uncertainty in the observation. The discrete method applied on uncertain data yields some errors in the resulting neighborhood statements; there will occur first order errors – existing neighborhood relations that are not detected –, as well as second order errors – assignment of neighborhood to non-neighbored fields (in the case of frequency planning only the first order error causes conflicts, whereas the second order error induces too conservative decisions). If information about the spatial uncertainty is available, it could be introduced in a Monte Carlo simulation of neighborhood determination, which again attaches weights to neighborhood relations, corresponding to the robustness in the simulation.

Another open question is the optimization of the proposed procedure. The formal description is correct, and a test implementation demonstrated the applicability, but it is to expect that a consequent exploitation of the redundant topological relations can increase the efficiency.

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