Proceedings of Autocarto 12, D. Peuquet (Ed.), Charlotte, NC, February 1995.

On the Robustness of Qualitative Distance- and Direction-Reasoning*

Jung-Hong Hong
Department of Surveying Engineering
National Cheng-Kung University
1 University Road
Tainan, Taiwan, Republic of China
junghong@mail.ncku.edu.tw

Max J. Egenhofer
National Center for Geographic Information and Analysis
and
Department of Surveying Engineering
Department of Computer Science
University of Maine
Orono, ME 04469-5711, U.S.A.
max@mecan1.maine.edu

Andrew U. Frank
Department of Geo-Information E127
Technical University Vienna
Gusshausstr. 27-29
A-1040 Vienna, Austria
frank@geoinfo.tuwien.ac.at

Abstract

This paper focuses on spatial information derived from the composition of two pairs of cardinal directions (e.g., North and North-East) and approximate distances (e.g., near and far), i.e., given the approximate distances a1 (A, B) and a2 (B, C) and the cardinal directions c1 (A, B) and c2 (B, C), what are a3 (A, C) and c3 (A, C)? Spatial reasoning about cardinal directions and approximate distances is challenging because distance *and* direction will affect the composition. This paper investigates the dependency between qualitative and quantitative inference methods for reasoning about cardinal directions and approximate distances. Cardinal directions are based on a 4-sector model (North, East, South, West), while approximate distance correspond to a set of ordered intervals that provide a complete partition (non-overlapping and mutually exclusive) such that the following interval is greater than or equal to the previous one (for example, “far” would

* This work was partially supported through the NCGIA by NSF grant No. SES-8810917. Additionally, Max Egenhofer’s work is also supported by NSF grant No. IRI-9309230, a grant from Intergraph Corporation, and the Scientific Division of the North Atlantic Treaty Organization. Jung-Hong Hong’s work was partially supported by a University of Maine International Student Tuition Waiver. Andrew Frank’s work is partially supported by a grant from Intergraph Corporation.

extend over a distance that is at least as great as “medium.”) We ran comprehensive simulations of quantitative reasoning, and compared the results with the ones obtained from quantitative reasoning. The results indicate that the composition is robust if the ratio between two consecutive intervals of quantitative distances is greater than 3.

Introduction

The domain of this paper is the intelligent inference of spatial information in Geographic Information Systems (GISs), which record a variety of geographic data to aid human decision making about the real world. Spatial reasoning denotes the inference of new spatial information that is otherwise not available. People do spatial reasoning using knowledge they acquire from environment and learning experiences, so that they can make decisions even if the available spatial information is incomplete, uncertain, or inconsistent (Collins *et al.* 1975). For example, after living in an area for a period of time, people can usually find a path from one place to another, or draw a sketch map about the whole area—even if they would not know the exact relationship between any two objects. People have the flexibility to adapt to the environment and the reasoning processes do not necessarily follow human-designed models like mathematics or logic. Unlike humans, information systems with spatial reasoning capabilities must rely on an appropriately designed spatial models and their formalizations. Of course the result of such a reasoning must make sense to humans (Kieras 1990). From the perspective of data management and query processing, the design of spatial reasoning mechanisms for GISs must consider at least two factors: (1) it must have unambiguous definitions of spatial relations and operations to process queries and (2) it must have the capability to search for appropriate combinations of relations and derive reasonable answers promptly.

The domain of this paper is reasoning about qualitative distances and directions (e.g., near and North), also known as approximate distances and cardinal directions (Frank 1992). Since people use spatial relations to make sense of observations, spatial relations become an essential part of GISs. Qualitative spatial relations—topological relations, cardinal directions, and approximate distances—are important, because they are closer to human cognition in everyday lives than their quantitative counterparts. Qualitative relations are based on a small set of symbols and a set of inference rules of how to manipulate symbols. Although qualitative relations are often vague in their geometric meaning and have less resolution than their quantitative counterpart, people have little difficulty in processing them and using them to communicate with others. Dutta (1989) and Freksa (1992) even argued that most human spatial reasoning is qualitative rather than quantitative. Currently, human spatial behaviors are yet incompletely understood and consequently the design of spatial theory and its formalization for GIS is difficult. Future GISs should not only be mechanisms for the storage and retrieval of geographic information as most current database systems do, but also be intelligent knowledge-based systems capable of incorporating human expertise to mimic human behaviors in decision making (Abler 1987). Since qualitative spatial relations stored in databases are often incomplete, the deduction of new relations has to rely on built-in inference mechanisms (Kidner and Jones 1994; Sharma *et al.* 1994); however, such processing of qualitative spatial relations in computers is currently impeded by the lack of a better understanding of human spatial knowledge.

Most commonly used reasoning mechanisms for qualitative spatial relations are purely quantitative. Examples are coordinate calculations, which derive from a number of quantitative spatial relations a new spatial relation, also in a quantitative format. These reasoning mechanisms, however, cannot be directly applied to qualitative spatial relations (Futch *et al.* 1992). Recent research focused on the individual types of qualitative spatial relations (Egenhofer and Franzosa 1991; Peuquet 1992; Cui *et al.* 1993), but only few researchers investigated spatial reasoning involving different types of spatial relations (Dutta 1991; Freksa and Zimmermann 1992; Hernández 1993). This paper investigates reasoning about the spatial relations of distance *and* direction, called *locational relation*. By considering these two types of relations simultaneously, stronger constraints between two objects can be established. We suggest to build a reasoning model on the basis of composition operators, which describes the behavior of the combination of two locational relations. The goal of this model is to derive approximate, reasonable, and qualitative reasoning results with the defined composition operators.

Although this paper deals with only the reasoning about qualitative locational relations, it does not suggest that qualitative reasoning should replace the widely used quantitative reasoning. Qualitative and quantitative representation are complementary approaches for human abstractions of the spatial relations in the real world, and one or the other should be used whenever it best serves users' needs.

Background

When qualitative spatial relations are explicitly stored in a database, there are two scenarios when processing a query (Sharma *et al.* 1994):

- If the queried relation is already stored in the database, the system retrieves this information and delivers it to the user.
- If the queried relation is not directly available, a reasoning mechanism must be invoked to infer the queried relation from those relations that exist in the database.

The reasoning mechanism must have two basic functions: First, it must be able to analyze if the available information is sufficient to derive the queried relation. If so, the reasoning model takes selected relations to derive the queried relations. The reasoning result is preferred to be conclusive, i.e., the number of possible answers should be as small as possible. The core of this reasoning model is a set of composition that define the composition behavior for the particular types of relations. A composition operator ";" takes two known relations, r1 (A, B) and r2 (B, C), to derive the relation r3 between A and C, i.e.,

$$r1\ (A, B)\ ;\ r2\ (B, C) \Rightarrow r3\ (A, C) \tag{1}$$

Vector addition is a good analogy to composition. The addition of vector (A, B) and vector (B, C) will yield vector (A, C). For locational relations, vector addition is actually what the quantitative-based reasoning approach is based on. Nevertheless, this concept is not directly applicable to the composition of qualitative locational relations.

Proceedings of Autocarto 12, D. Peuquet (Ed.), Charlotte, NC, February 1995.

Related Work

Most current spatial reasoning systems and GISs only store locational relations in a quantitative format and consequently only deal with spatial reasoning in a quantitative matter, e.g., through numerical calculations for distances and directions as in Kuipers' (1978) TOUR model. The last few years have seen a growing interest in qualitative inferences of spatial relations. The common part among different reasoning models suggested is that they start with the modeling of the spatial domain (geographic space or spatial relations) and then define their respective composition operators. Depending on the way the reasoning is conducted, we divide the different approaches into those that transform qualitative locational relations into a quantitative format and solve the reasoning problem quantitatively; and those that use operators to define the composition behavior of qualitative relations.

In the first class, the result can be either kept in a quantitative format or transformed back to a qualitative format. For example, the SPAM model (McDermott and Davis 1984) treats a qualitative locational relation as a fuzz box and calculates the possible range for the queried relation. On the other hand, Dutta (1989) used fuzzy set theory (Zadeh 1974) to model the approximate and uncertain nature of qualitative locational relations. Both models require a transformation between qualitative and quantitative representations to be established first. After such transformations, the newly inferred relations are calculated with a quantitative method.

Spatial reasoning models in the second class employ a purely qualitative reasoning process. For example, symbolic projections (Chang *et al.* 1987) were originally proposed to store the spatial knowledge in an image. This model records the relationships among objects in a symbolic image separately in horizontal or vertical directions. Several extensions have been suggested in the past years (Jungert 1988; Jungert 1992; Lee and Hsu 1992). Despite of these extensions, the symbolic projection model does not record distance information, that is, it cannot appropriately represent "how far" two separated objects are. Allen's (1983) temporal logic, originally proposed to solve the reasoning in one-dimensional domain, was expanded to reasoning in higher-dimensional spaces (Guesgen 1989). Papadias and Sellis (1992; 1993) suggest the use of symbolic arrays to store the spatial information hierarchically and take advantage of the array structure for reasoning. Freksa (1992) developed a reasoning model for qualitative directions based on an orientation grid and the conceptual neighbors of qualitative spatial relations. Zimmermann (1993) extended this model to include distance constraints, represented by comparing a distance value to a known distance (> di, = di, < di). Hernández (1991) suggested a model that deals with the reasoning of both directions and topological relationships. He separated these two types of reasoning and solve the reasoning problem individually. No model mentioned above investigates the reasoning of qualitative distances (e.g., near). Frank (1992) developed an algebra for the reasoning of qualitative distances and directions. Like Hernández's model, it also separates these two types of relations and solve the reasoning individually. This algebra can achieve satisfactory results under some restricted condition; for some cases, only approximate results can be derived, i.e., the queried relation is not conclusive.

Proceedings of Autocarto 12, D. Peuquet (Ed.), Charlotte, NC, February 1995.

A Model of Qualitative Distances and Directions

A locational relation includes a qualitative distance component and a qualitative direction component. To simplify the problem domain, only locational relations between point-like objects will be considered here. One property of qualitative distances and directions is their imprecise geometric meaning. Take distances for example, near is often interpreted as a range of quantitative distances rather than a specific value ("The church is about 50 meters away.") The same applies to qualitative directions. Although one may intuitively interpret East as a specific quantitative direction (i.e., azimuth of 90 degree), people often consider an object whose azimuth is 85 degrees to be East as well. Peuquet and Zhan (1987) adopted the cone-shaped concept to investigate the cardinal directions between two extended objects, which also treats a cardinal direction as a range of quantitative directions.

Qualitative relations will be represented by *symbolic values* (in comparison with numerical values for quantitative relations). We therefore have symbolic distance values and symbolic direction values; each one of them represents a specific locational constraint. The name of the symbolic values can be chosen arbitrarily as long as its semantic meaning is reasonable and will not cause confusion. For example, North and South are usually understood as two directions in the opposite direction and the name selection should not violate that. Theoretically the number of symbolic distance values is not limited, yet research in psychology and cognitive science has demonstrated that the number of categories humans can handle simultaneously has a limitation. In this paper, the number of symbolic distance values is chosen to be four. The proposed model can be expanded to include less or more symbolic distance values according to the application. The number of symbolic direction values depends on the applications. Two often used direction systems include either four or eight symbolic values. The following lists a model consisting of four symbolic distance values and eight symbolic direction values.

Distance: {very near, near, far, very far}
Direction: {North, Northeast, East, Southeast, South, Southwest, West, Northwest}

Certain order relationships exist among these symbolic values. The order among symbolic distance values describes distances from the nearest to the furthest. The order among symbolic direction values can be either clockwise or counter-clockwise. To simplify the model design, the following two criteria are introduced:

- complete coverage, i.e., the designed symbolic values describe any situation in its respected domain, and
- mutual exclusive, i.e., any situation in the domain can be described by one and only one symbolic value.

Mapping Qualitative onto Quantitative Locational Relations

Qualitative and quantitative representations describe the same domain, only the symbols used are different (symbolic values vs. numerical values). Since a relation between two objects can be represented qualitatively or quantitatively, it should be possible to transform between these two representations. Also, the number of symbolic values is smaller than its quantitative counterpart, but it has to describe the same domain, so it is reasonable to assume that a symbolic value should correspond to a range of quantitative values (an interval on a one-dimensional scale). On the other hand, a quantitative value should correspond to only one symbolic value. Because of the property of mutual exclusiveness, no gap or overlap will be allowed between two neighboring intervals. Therefore, a number of symbolic values correspond to the same number of intervals of the quantitative values; that is, there is an interval-based transformation between the qualitative and quantitative representations.

This interval-based transformation is context dependent. For example, near can be interpreted as an interval from 0 to 500 meters for walking, while also as an interval from 5 km to 10 km for driving. Such an interval-based transformation can be applied to both the domain of distances and directions. For distance systems, every symbolic value corresponds to an interval of quantitative distances. This divides a two-dimensional space into several tracks; each track represent a unique qualitative distance (Figure 1a). If the cone-shaped concept is chosen for direction systems, the direction domain is divided into a number of cones with the same resolution (Figure 1b). The basic property is that the geometric range of each cone increases with the increase of the distance (Peuquet and Zhan 1987).

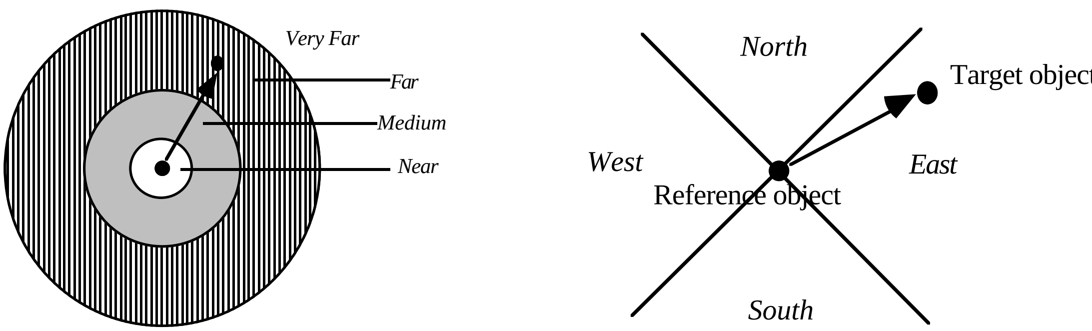

Figure 1: (a) Qualitative distances and (b) qualitative directions.

By considering distances and directions together, the locational relation system becomes a sector-based model (Figure 2), where each sector corresponds to a specific pair of symbolic distance and direction values. Objects in the same sector share the same qualitative locational relationship with respect to the origin of the system.

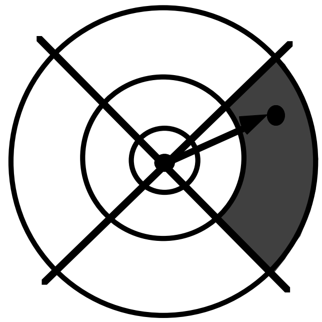

Figure 2: Illustration of the space divided by the distance and direction systems.

With this model, the composition of two qualitative locational relations can be numerically simulated by the compositions of their corresponding quantitative locational relations. Assuming that every sector can determine N quantitative locational relations, the total number of possible composition between these two sectors is N^2. Every quantitative composition can be mapped onto a qualitative locational relation, which represents a possible answer for the particular composition. The set of possible answers can be used to define the composition operators for qualitative locational relations. Figure 3 illustrates this transformation. Given two qualitative locational relations QL_1 and QL_2, determine the transformations $f(QL_i)$ and $f^{-1}(QN_j)$, and map with f QL_1 onto a set of quantitative relations QN_1 and QL_2 to a set of QN_2 , respectively. In the quantitative domain, apply quantitative reasoning methods to all the possible combinations between the sets of QN_1 and QN_2 to derive a set of results QN_3, which is then mapped onto a set of QL_3 using f^1.This process is based on the interval-based numeric simulation and well-developed quantitative inference methods.

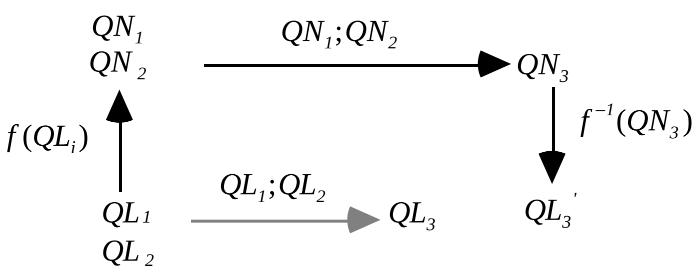

Figure 3: Framework of the reasoning model design.

Given any pair of locational relations and an interval-based transformation, their composition operator can thus be defined. For example,

$$\text{(very near, North) ; (very near, North)} \Rightarrow \text{(very near, North) or (near, North)} \qquad (2)$$

Theoretically there is an infinite number of interval-based transformation depending on the applications. To use composition operators as the basis for reasoning, it is important to investigate if the interval-based transformation will affect the definition of composition operators. If the composition operators are largely dependent on the interval-based transformation, it is not necessary to define composition operators, as the queried relation must be calculated anyway. If the interval-based transformation is insignificant to the definition of composition operators, i.e., composition operators are robust, then it is possible to build a qualitative reasoning mechanism on the basis of the composition operators, in which no calculation will be necessary for the inference.

The All-Answer Model

To ensure the correctness for the queried relation, most qualitative reasoning models define their composition operators in a way that all the possible answers will be found. This concept will be called the *all-answer model*. Since this model is based on numeric simulation, efficient sampling becomes very important. We find that it is only necessary to select samples on the boundary of the two sectors with which the two locational relations are corresponding to. This can largely reduce the number of samples, while still derive all the possible answers. Since the all-answer model only checks if a particular relation is a possible answer, every possible answer will be treated equally. That is, although some answers may turn out to be more probable, the all-answer model will not distinguish them. To simplify this answer selection process, Hong (1994) suggested two other models, the *likely-answer model*—eliminating compositions that have a low probability—and the single-answer model—selecting the composition with the highest probability.

The advantage of the all-answer model is that the actual answer is guaranteed to be one of the possible answers at the end of the reasoning process. However, the disadvantage is that sometimes the number of possible answers becomes too high and the reasoning process becomes very complicated. To solve this problem, more than one combination of locations must be found so that the queried relations can be better constrained. Furthermore, for better efficiency, the reasoning mechanism should have built-in intelligence to select appropriate combinations of relations with better locational constraints and discard those without.

Simulation of Test Data

The interval-based transformation is subjective to applications and individual experiences. To investigate the influence the interval-based transformation has on the definition of composition operators, thirteen interval sets were tested (Table 1). The interval sets are designed in a way that there is a constant ratio relationship between the lengths of two neighboring intervals. Of course these thirteen sets do not make a complete list for the intervals humans may use. The intention here is to observe the results of the defined composition operators to investigate their robustness and distribution. The finding is very important to the design and evaluation of qualitative reasoning mechanism.

Ratio	$dist_0$	$dist_1$	$dist_2$	$dist_3$
1	(0, 1]	(1, 2]	(2,3]	(3, 4]
2	(0, 1]	(1, 3]	(3 , 7]	(7, 15]
3	(0, 1]	(1, 4]	(4, 13]	(13, 40]
4	(0, 1]	(1, 5]	(5, 21]	(21, 85]

5	(0, 1]	(1, 6]	(6, 31]	(31, 156]
6	(0, 1]	(1, 7]	(7, 43]	(43, 259]
7	(0, 1]	(1, 8]	(8, 57]	(57, 400]
8	(0, 1]	(1, 9]	(9, 73]	(73, 585]
9	(0, 1]	(1, 10]	(10, 91]	(91, 820]
10	(0, 1]	(1, 11]	(11, 111]	(111, 1111]
20	(0, 1]	(1, 21]	(21, 421]	(421, 8421]
50	(0, 1]	(1, 51]	(51, 2551]	(2551, 127551]
100	(0, 1]	(1, 101]	(101, 10101]	(10101 , 1010101]

Table 1: Simulated intervals for four symbolic distance values.

Although tests on different numbers of symbolic distance and direction values were conducted, only the group of four symbolic distance values and eight symbolic direction values will be discussed here. Detailed discussions about other groups (e.g., three distances and eight directions) can be found in (Hong 1994).

Robustness

Given two locational relations, if their composition operator remains the same despite the changes of interval-based transformation, the composition operator is *robust*. If so, it is unnecessary to define composition operators for every interval-based transformation and a group of transformation can share the same set of composition operators.

To measure the robustness, a quantitative measure, called *robustness measure* (*RM*), is introduced. It is the ratio between the numbers of answers in the robust set (the intersection of the qualitative and quantitative sector), and the union set (the set union of the two sectors). The domain of *RM* is $\{0 \leq RM \leq 1\}$. Two compositions are compatible if *RM* is equal to 1. If there is no common answer between the two sets of answers, *RM* is equal to 0. Two compositions are therefore incompatible if their robustness measure is less than 1. The advantage of this method is that if most of the selected answers between these two sets are similar, their robustness measure will be close to 1.

Table 2 gives the robustness measures based on the direction differences. The first variable shows the number of incompatible composition in a group and the second variable shows the robustness measure for the group. Between ratio 1 and ratio 2, thirteen compositions do not have the same set of selected answers. The average robustness measure of this group is 0.95, which indicates that the selected answers between ratio 1 and ratio 2 are either identical or very similar. All compositions are robust if the ratio is greater than 2.

	Δdir = 0	Δdir = 1	Δdir = 2	Δdir = 3	Δdir = 4	Total
ratio (1, 2)	(2, 0.92)	(1, 0.98)	(1, 0.99)	(3, 0.94)	(6, 0.92)	(13, 0.95)
ratio (2, 3)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)
ratio (2, 100)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)

Table 2: Robustness measures.
ratio (a, b): a = number of incompatible compositions; b = *RM* for the group.
Δdir = direction difference.

The majority of the compositions for the thirteen groups are robust, with the first group (ratio = 1) as the only exception. When we specifically compare the two groups of compositions where ratio is 1 and 2, there are 13 cases (out of 74) between which the possible answers are different. This result indicates that even if the composition operators are not rigorously robust, most of them are robust. From a cognitive and linguistic point of view, such an interval set—ratio = 1, length (near) = length (far)—rarely exists. It is therefore possible to build a model on the basis of the composition operators, such that the context-dependent nature of qualitative distances will not affect the definition of composition operators except for some extreme cases (e.g., ratio = 1).

Distribution

Distribution is the analysis of the selected answers for a particular composition. To make the reasoning process easier, it is important to keep the number of selected answers as few as possible. The following lists some important finding regarding to the distribution of the selected answers using the all-answer model. Through such an analysis, we wish to identify some "preferred" compositions (i.e., fewer possible answers) that can provide better constraints on the queried relation. This can certainly be used as the basis for the design of an intelligent reasoning mechanism.

- The number of possible answers increases with the direction differences; therefore, the further apart the two directions are, the less-constrained their composition is.
- A composition usually extends over 2 or 3 distance values.
- The number of possible directions for each composition largely depends on the direction differences.
- Compositions of relations in the same direction provide the best constraint for both distances and directions.
- Compositions of two relations in opposite directions provide the least constraints, especially for directions.
- Compositions of relations with different distance values usually provide better constraints than compositions of relations with the same distance value.
- If the distance values of the two relations are different, the composition largely depend on the relation with the greater distance value.

Conclusions

This paper demonstrated the first results for investigations into reasoning about qualitative distances and directions. We proposed to build a qualitative reasoning model that takes two qualitative locational relations to derive a new locational relation also in a qualitative format. An important finding is that the composition operator of two qualitative locational relations are robust for the majority of the cases tested. Since the context-dependent nature of the qualitative distances does not seem to be a significant factor, we can build a reasoning mechanism on the basis of the composition operators.

From the above discussion, we can conclude that the distance constraint is usually poor, no matter what kind of combination is used, because it cannot be narrowed down to one single answer. In most situations, two relations with greater distance differences will provide better constraints. Compared with human reasoning, this finding is reasonable. For example, when asked about the relationship between San Francisco and Washington D.C., people are likely to select the relation San Francisco-Baltimore and Baltimore-Washington D.C. for reasoning rather than using the relation San Francisco-New Orleans and Washington D.C.-New Orleans. On the other hand, it is clear that the direction is much better constrained if the two locational relations are in the same direction. This is again not surprising when compared to human reasoning.

Although the all-answer model provides all the possible answers, the number of selected answers is often high and the reasoning process is expected to be tedious and inefficient. For the worst case, there is probably no conclusive result at the end of the reasoning. Further modification on the reasoning model that only track more likely answers is an alternative (Hong 1994).

To simplify the problem domain, we enforced some assumptions (e.g., mutual exclusiveness) on the transformation between qualitative and quantitative representations. It is not clear if humans do possess such a fine line to distinguish between two symbolic distance or direction values. It is therefore of interests to further investigate if the robustness still exists provided some parameters are changed (e.g., if one allows that two neighboring intervals overlap). Also, the discussion in this paper was restricted to point-like objects; to be used in GISs, this model must be further expanded to higher-dimensional domains, or integrated into a hierarchical spatial inference model.

References

R. Abler (1987) The National Science Foundation National Center for Geographic Information and Analysis. *International Journal of Geographical Information Systems* 1(4): 303-326.

J. F. Allen (1983) Maintaining Knowledge about Temporal Intervals. *Communications of the ACM* 26(11): 832-843.

S. K. Chang, Q. Y. Shi, and C. W. Yan (1987) Iconic Indexing by 2-D Strings. *IEEE Transactions on Pattern Analysis and Machine Intelligence* PAMI-9(6): 413-428.

A. Collins, E. Warnock, N. Aiello, and M. Miller (1975) Reasoning From Incomplete Knowledge. in: D. Bobrow and A. Collins (Eds.), *Representation and Understanding*. pp. 383-415, Academic Press, New York, NY.

Z. Cui, A. Cohn, and D. Randell (1993) Qualitative and Topological Relationships in Spatial Databases. in: D. Abel and B. Ooi (Eds.), *Third International Symposium on Large Spatial Databases*. *Lecture Notes in Computer Science* 692, pp. 296-315, Springer-Verlag, New York, NY.

S. Dutta (1989) Qualitative Spatial Reasoning: A Semi-Quantitative Approach Using Fuzzy Logic. in: A. Buchmann, O. Günther, T. Smith, and Y. Wang (Eds.), *Symposium on the Design and Implementation of Large Spatial Databases*. *Lecture Notes in Computer Science* 409, pp. 345-364, Springer-Verlag, New York, NY.

S. Dutta (1991) Topological Constraints: A Representational Framework for Approximate Spatial and Temporal Reasoning. in: O. Günther and H.-J. Schek (Eds.), *Advances in Spatial Databases—Second Symposium, SSD '91*. *Lecture Notes in Computer Science* 525, pp. 161-180, Springer-Verlag, New York, NY.

M. Egenhofer and R. Franzosa (1991) Point-Set Topological Spatial Relations. *International Journal of Geographical Information Systems* 5(2): 161-174.

A. Frank (1992) Qualitative Spatial Reasoning about Distances and Directions in Geographic Space. *Journal of Visual Languages and Computing* 3(4): 343-371.

C. Freksa (1992) Using Orientation Information for Qualitative Spatial Reasoning. in: A. Frank, I. Campari, and U. Formentini (Eds.), *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*. *Lecture Notes in Computer Science* 639, pp. 162-178, Springer-Verlag, New York, NY.

C. Freksa and K. Zimmermann (1992) On the Utilization of Spatial Structures for Cognitively Plausible and Efficient Reasoning. in: *IEEE International Conference on Systems, Man, and Cybernetics*, Chicago, IL, pp.

S. Futch, D. Chin, M. McGranaghan, and J.-G. Lay (1992) Spatial-Linguistic Reasoning in LEI (Locality and Elevation Interpreter). in: A. Frank, I. Campari, and U. Formentini (Eds.), *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*. *Lecture Notes in Computer Science* 639, pp. 318-327, Springer-Verlag, New York, NY.

H. W. Guesgen (1989) *Spatial Reasoning Based on Allen's Temporal Logic*. International Computer Science Institute, Technical Report TR-89-049.

D. Hernández (1991) Relative Representation of Spatial Knowledge: The 2-D Case. in: D. Mark and A. Frank (Eds.), *Cognitive and Linguistic Aspects of Geographic Space*. pp. 373-385, Kluwer Academic Publishers, Dordrecht.

D. Hernández (1993) Maintaining Qualitative Spatial Knowledge. in: A. Frank and I. Campari (Eds.), *Spatial Information Theory, European Conference COSIT '93, Marciana Marina, Elba Island, Italy*. 716, pp. 36-53, Springer-Verlag, New York, NY.

J.-H. Hong (1994) *Qualitative Distance and Direction Reasoning in Geographic Space*. Ph.D. Thesis, University of Maine.

E. Jungert (1988) Extended Symbolic Projections as a Knowledge Structure for Spatial Reasoning. in: J. Kittler (Ed.), *4th International Conference on Pattern Recognition*. *Lecture Notes in Computer Science* 301, pp. 343-351, Springer-Verlag, New York, NY.

E. Jungert (1992) The Observer's Point of View: An Extension of Symbolic Projections. in: A. Frank, I. Campari, and U. Formentini (Eds.), *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*. *Lecture Notes in Computer Science* 639, pp. 179-195, Springer-Verlag, New York, NY.

D. Kidner and C. Jones (1994) A Deductive Object-Oriented GIS for Handling Multiple Representations. in: T. Waugh and R. Healey (Eds.), *Sixth International Symposium on Spatial Data Handling*, Edinburgh, Scotland, pp. 882-900.

D. Kieras (1990) Cognitive Modeling. in: S. Shapiro (Ed.), *Encyclopedia of Artificial Intelligence*. pp. 111-115, Wiley-Interscience, New York, NY.

B. Kuipers (1978) Modeling Spatial Knowledge. *Cognitive Science* 2: 129-153.

S.-Y. Lee and F.-J. Hsu (1992) Spatial Reasoning and Similarity Retrieval of Images Using 2D C-String Knowledge Representation. *Pattern Recognition* 25(3): 305-318.

D. McDermott and E. Davis (1984) Planning Routes through Uncertain Territory. *Artificial Intelligence* 22: 107-156.

D. Papadias and T. Sellis (1992) Spatial Reasoning Using Symbolic Arrays. in: A. Frank, I. Campari, and U. Formentini (Eds.), *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*. *Lecture Notes in Computer Science* 639, pp. 153-161, Springer-Verlag, New York, NY.

D. Papadias and T. Sellis (1993) The Semantics of Relations in 2D Space Using Representative Points: Spatial Indexes. in: A. Frank and I. Campari (Eds.), *Spatial Information Theory, European Conference COSIT '93, Marciana Marina, Elba Island, Italy*. 716, pp. 234-247, Springer-Verlag, New York, NY.

D. Peuquet (1992) An Algorithm for Calculating Minimum Euclidean Distance Between Two Geographic Features. *Computers and Geosciences* 18(8): 989-1001.

D. Peuquet and C.-X. Zhan (1987) An Algorithm to Determine the Directional Relationship Between Arbitrarily-Shaped Polygons in the Plane. *Pattern Recognition* 20(1): 65-74.

J. Sharma, D. Flewelling, and M. Egenhofer (1994) A Qualitative Spatial Reasoner. in: T. Waugh and R. Healey (Eds.), *Sixth International Symposium on Spatial Data Handling*, Edinburgh, Scotland, pp. 665-681.

L. Zadeh (1974) *Fuzzy Logic and Its Application to Approximate Reasoning*. North-Holland Publishing Company.

K. Zimmermann (1993) Enhancing Qualitative Spatial Reasoning—Combining Orientation and Distance. in: A. Frank and I. Campari (Eds.), *Spatial Information Theory, European Conference COSIT '93, Marciana Marina, Elba Island, Italy*. *Lecture Notes in Computer Science* 716, pp. 69-76, Springer-Verlag, New York, NY.