Qualitative distances

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Abstract

A framework for the representation of qualitative distances is developed inspired by previous work on qualitative orientation. It is based on the concept of "distance systems" consisting of a list of distance relations and a set of structure relations that describe how the distance relations in turn relate to each other. The framework is characterized by making the role of the "frame of reference" explicit, which captures contextual information essential for the representation of distances. The composition of distance relations as main inference mechanism to reason about distances within a given frame of reference is explained, in particular under "homogeneous structural restrictions". Finally, we introduce articulation rules as a way to mediate between different frames of reference.

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1 Introduction

The qualitative approach to the representation of spatial knowledge has gained considerable popularity in recent years. Qualitative representations are characterized by making only as many distinctions in the domain of discourse as necessary in a given context.

In previous work, the qualitative description of space has been mostly restricted to topological relations (Randell and Cohn 1989; Egenhofer and Herring 1991; Egenhofer and Franzosa 1991; Clementini, Di Felice, and van Oosterom 1993; Clementini and Di Felice 1995) and orientation relations (Hernández 1991; Freksa 1992; Latecki and Röhrig 1993; Zimmermann 1993). The combination of topological and orientation relations provides a restricted form of positional information that is mainly useful in small-scale environments such as "the objects in a room" (Hernández 1994).

In this paper we develop a model for the qualitative representation of distances as they are needed in the context of geographic space. People's concepts of space and therefore distance are dependent upon both culture and experience (Lowe and Moryadas 1975). What it means for A to be near B depends not only on their absolute positions (and the metric distance between them), but also on their relative sizes and shapes, the position of other objects, the frame of reference, and "what it takes to go from A to B". With other words, distance concepts are context dependent. Quantitative approaches try to avoid contextual issues by reducing all distance information to an absolute metric scale. However, this is not always feasible or desirable. Qualitative approaches must deal with contextual dependencies since, by definition, only those distinctions relevant in a given context are made. In our approach, we capture contextual information by using frames of reference to qualify distance relations.

We will restrict our attention here to two-dimensional space, which is commonly used as a projection of the three-dimensional physical space. Thus, we can consider a scene to be made up of geometric objects (points, lines, and areas) variously arranged in the plane and possibly overlapping. Furthermore, we assume an isotropic space, in which the effort to move is the same in all directions, and thus the isolines connecting all points at the same distance are concentric circles. Most anisotropic spaces can be translated into isotropic ones by appropriate transformations.

In what follows we will put our contribution in the context of existing literature by briefly reviewing some related work. We will then introduce various levels of distance distinctions and their domain structure (section 3) as well as the composition of distance relations as main inference mechanism (section 4). While those sections assume for simplicity a uniform reference frame, we discuss in section 5 how to mediate between different frames of reference.

2 Related Work

There is a considerable amount of recent work in the area of qualitative spatial reasoning to which the model presented in this paper relates. We shall focus here only on the most closely related papers and refer to Hernández (1994) and Freksa and Röhrig (1993) for a more general review of that literature.[1]

Most related work has concentrated on the qualitative description of size, which being a linear quantity has some similarity to distance. In particular, Allen (1983) briefly describes an extension of his temporal interval reasoner to handle duration, the temporal equivalent of size. Mukerjee and Joe (1990) who extend Allen's approach to multi-dimensional spaces (essentially by maintaining tuples of 1-dimensional relations), base their representation of relative size on the "flush translation operator ϕ". The idea is to observe the relations between two intervals as they move along what the authors call "relation continuum" and deduce the relative size from them. Zimmermann (1991) develops a representation for object sizes based on differences and a partial ordering. The relation $A(>, d_1)B$ denotes the fact that "A is higher/larger than B by the amount d_1", since $|A| = |B| + |d_1|$. In Zimmermann (1993) this "delta calculus" is combined with orientations. However, only a restricted set of distance distinctions is possible in that model. While representations based on direct comparisons can handle moderately different sizes, other calculi concentrate on differences in the order of magnitude (Raiman 1986; Mavrovouniotis and Stephanopoulus 1988). We shall use order-of-magnitude relations below to express the structure of distance systems.

The most closely related work that explicitly describes a method for qualitative reasoning about distances (*far*, *close*) and cardinal directions (N, E, S, and W) in geographic space is Frank (1992). It is based on an algebra of paths on which the two operations of *inversion* and *composition* are defined. Frank discusses two direction systems, one based on triangular areas and one based on projections, and presents alternatives for the combination of distance and direction, some of which produce only 'Euclidean approximate' results. Our qualitative distance model is superior to Frank's in that not only equally spaced distance intervals but also regions of varying sizes are dealt with.

[1]There is also a large amount of relevant cognitive and linguistic work which we are excluding here for brevity.

3 Modeling Distances Qualitatively

We propose a qualitative framework where three elements are needed to establish a distance relation: the *primary object* (PO), the *reference object* (RO), and the *frame of reference* (FofR). The distance between the reference object A and the primary object B is expressed by $d_{AB} = d(A, B)$. We will defer discussion of frames of reference to section 5, while this section and Section 4 treat aspects that are independent or the reference frame.

A distinction has to be made between *comparing* the magnitudes of distances and *naming* distances. For comparing distances, the obvious set of predicates is $<$, $=$, $>$, which characterize the result of direct comparison. With respect to naming, the types of objects involved and the context in which they are embedded are decisive factors for establishing the set of relations to be used. The first level of granularity that comes to mind distinguishes between *close* and *far*. Those two relations subdivide the plane into two regions centered around the reference object, where the outer region goes to infinity. Characteristic of the semantic of qualitative distance relations is that they partition the physical space into regions of different sizes (where the difference can be even in the order of magnitude).

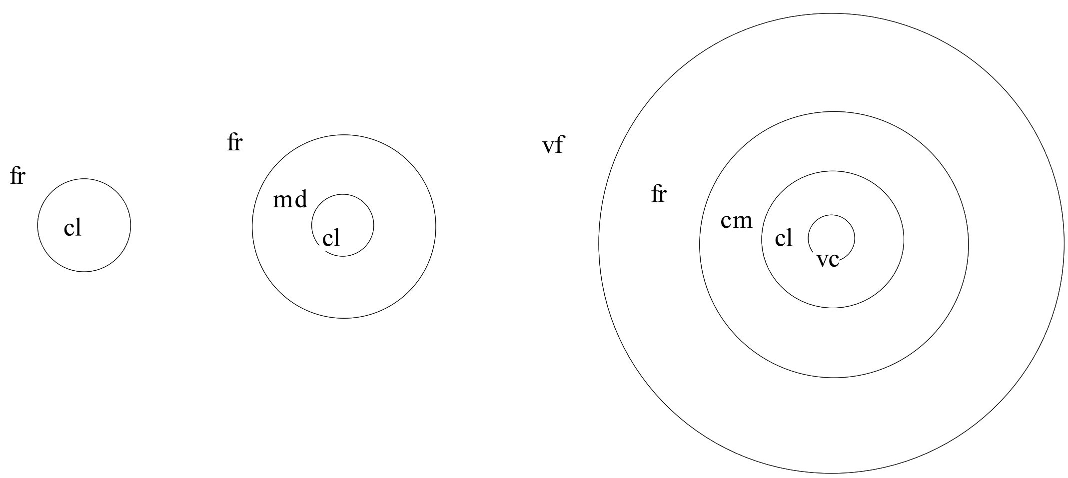

Figure 1: Various levels of distance distinctions

Cognitive considerations suggest the need for systems of distance relations organized along various levels of granularity, for e xample: a level with three distinctions *close*, *medium*, and *far*, a level with four distinctions *very close*, *close*, *far*, and *very far*, a level with five distinctions *very close*, *close*, *commensurate*, *far*, and *very far*, and so on. Notice that the names given to relations are arbitrary, since we do not discuss linguistic reasons to associate a meaning to a given term. The relations partition the plane in circular regions (see Figure 1).

The qualitative approach deals implicitly with uncertainty in that the next coarser level of distinctions is chosen whenever no decision can be made about the appropriate relation at a finer level. Most of the time this is better than coming up with fuzzy membership numbers, which can be quite arbitrary. However, the general framework presented here is independent of the kind of boundary (sharp, fuzzy, overlapping) between the regions.

In general, at a given granularity level space surrounding a reference object is partitioned according to a number of totally ordered distance distinctions $Q = \{q_0, q_1, q_2, \ldots, q_n\}$, where q_0 is the distance closest to the reference object and q_n is the one farthest away (going to infinity). Distance relations are organized in *distance systems* (D) consisting of:

- a list of *distance relations* (i.e., the set of qualitative distinctions being made and their increasing distance order);
- a set of *structure relations* describing how the distance relations in turn relate to each other (e.g., order-of-magnitude relations between the various named distance ranges).

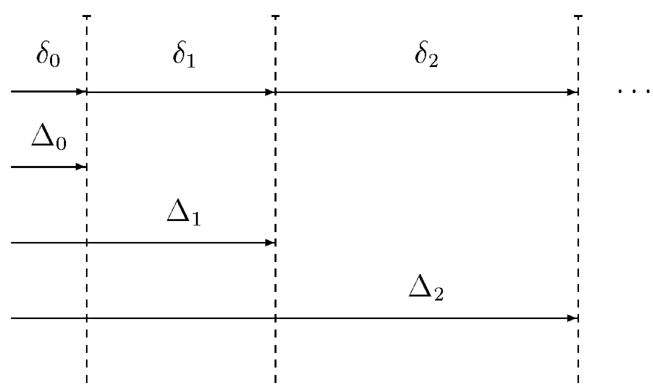

Figure 2: Distance ranges vs. distance from origin

In this paper we will only consider *homogeneous* distance systems, in which all distance relations are related to each other by the same type of property. In order to describe those properties, we distinguish between δ_i being the "distance range i", and Δ_i being the "distance range from the origin up to and including the distance range δ_i" (Figure 2). (Note that the distance symbol q_i labels all distances starting from the origin and falling in the range Δ_i.)

A very common restriction is that of *monotonically* increasing ranges:

$$\delta_0 \leq \delta_1 \leq \delta_2 \leq \ldots \leq \delta_n \tag{1}$$

An additional useful *range restriction* is that a given distance range be bigger than the sum of the previous ones:

$$\delta_i \geq \Delta_{i-1}, \forall i > 0 \tag{2}$$

Finally, if a distance range δ_j is much bigger than a previous one δ_i ($\delta_j \gg \delta_i$), then δ_j will absorb δ_i in the composition:

$$\delta_j \pm \delta_i \simeq \delta_j \tag{3}$$

These restrictions constrain the resulting sets in the composition of relations as we will see in the next section. The restrictions imposed on homogeneous distance systems correspond to the most common types of distance concepts used. The general case of heterogeneous distance systems is dealt with in (Clementini, Di Felice, and Hernández 1995).

4 Composition of Distance Relations

Given the distance $d_{AB} = d(A, B)$ between the reference object A and the primary object B and the distance $d_{BC} = d(B, C)$ between B and C, the composition of distances gives us the distance d_{AC} between A and C. This resulting distance will in general be a range of possible distances, for which we will find a lower and an upper bound.

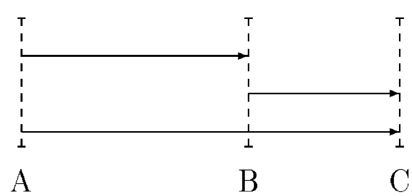

Figure 3: Composition of distances for same orientation

In the case in which the orientation of B with respect to A is the same as the orientation of C with respect to B (see Figure 3), the composition amounts to adding two "positive quantities", so that the lower bound cannot be less than the bigger of the two distances:

$$\mathrm{LB}(d_{AC}) = d_{AB} \oplus d_{BC} = \max(d_{AB}, d_{BC}). \tag{4}$$

Without structural restrictions, the upper bound for the composition is q_n. Assuming monotonically increasing distance ranges (restriction 1 above), however, we obtain:[2]

$$\mathrm{UB}(d_{AC}) = \mathrm{ord}^{-1}(\mathrm{ord}(d_{AB}) + \mathrm{ord}(d_{BC})). \tag{5}$$

Considering five possible distance symbols, the resulting composition table

$\oplus$	q_0	q_1	q_2	q_3	q_4
q_0	q_0,q_1	q_1,q_2	q_2,q_3	q_3,q_4	q_4
q_1	q_1,q_2	q_1,q_2,q_3	q_2,q_3,q_4	q_3,q_4	q_4
q_2	q_2,q_3	q_2,q_3,q_4	q_2,q_3,q_4	q_3,q_4	q_4
q_3	q_3,q_4	q_3,q_4	q_3,q_4	q_3,q_4	q_4
q_4	q_4	q_4	q_4	q_4	q_4

Table 1: Composition of distances for same orientation (monotonicity)

is given in Table 1.

Some of the entries in Table 1 are possible only in the case of equally spaced distance ranges. By considering the range restriction (2), the composition of two distances can at most be one step bigger than the maximum of the two distances. The upper bound becomes:[3]

$$\mathrm{UB}(d_{AC}) = \mathrm{succ}(\max(d_{AB}, d_{BC})). \tag{6}$$

Therefore, some of the resulting distances in Table 1 can be excluded obtaining the composition table in Table 2.

The upper bound can be further lowered with the absorption rule (restriction 3), which allows us to disregard the effect of the smaller relation (for example, for a difference between distances of two steps, i.e., $p = 2$, we obtain the results in Table 3):

$$|\mathrm{ord}(d_{AB}) - \mathrm{ord}(d_{BC})| \geq p \Rightarrow \mathrm{UB}(d_{AC}) = \max(d_{AB}, d_{BC}). \tag{7}$$

In the case of the composition of distances with opposite orientations—see Figure 4—the upper bound is given by the maximum of the two distances since this corresponds to the difference between two "positive quantities":

$$\mathrm{UB}(d_{AC}) = d_{AB} \ominus d_{BC} = \max(d_{AB}, d_{BC}). \tag{8}$$

[2]The function *ordinal* is defined as: $\mathrm{ord} : Q \rightarrow \{1 \ldots n+1\}$, such that $\mathrm{ord}(q_i) = i + 1$. Note that $\mathrm{ord}^{-1}(i) = q_n$ for $i > n$.

[3]The function *successor* gives the next symbol in the list, that is: $\mathrm{succ}(q_i) = q_{i+1}$ for each $i < n$ and $\mathrm{succ}(q_n) = q_n$.

$\oplus$	q_0	q_1	q_2	q_3	q_4
q_0	q_0,q_1	q_1,q_2	q_2,q_3	q_3,q_4	q_4
q_1	q_1,q_2	q_1,q_2	q_2,q_3	q_3,q_4	q_4
q_2	q_2,q_3	q_2,q_3	q_2,q_3	q_3,q_4	q_4
q_3	q_3,q_4	q_3,q_4	q_3,q_4	q_3,q_4	q_4
q_4	q_4	q_4	q_4	q_4	q_4

Table 2: Composition of distances for same orientation (range restriction)

$\oplus$	q_0	q_1	q_2	q_3	q_4
q_0	q_0,q_1	q_1,q_2	q_2	q_3	q_4
q_1	q_1,q_2	q_1,q_2	q_2,q_3	q_3	q_4
q_2	q_2	q_2,q_3	q_2,q_3	q_3,q_4	q_4
q_3	q_3	q_3	q_3,q_4	q_3,q_4	q_4
q_4	q_4	q_4	q_4	q_4	q_4

Table 3: Composition of distances for same orientation (absorption rule)

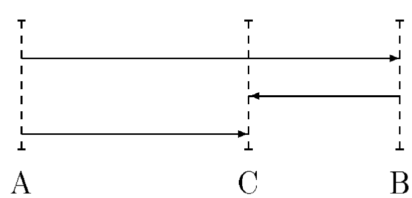

Figure 4: Composition of distances for opposite orientation

Without restrictions, the lower bound is $\mathrm{LB}(d_{AC}) = q_0$, while, applying a similar strategy as in the 'same orientation' case, we can increasingly restrict the lower bound. Imposing restriction 1, the lower bound becomes:

$$\mathrm{LB}(d_{AC}) = \mathrm{ord}^{-1}(|\mathrm{ord}(d_{AB}) - \mathrm{ord}(d_{BC})|). \tag{9}$$

The results for the five distance symbols are shown in Table 4. In Table 5 the

$\ominus$	q_0	q_1	q_2	q_3	q_4
q_0	q_0	q_0,q_1	q_1,q_2	q_2,q_3	q_3,q_4
q_1	q_0,q_1	q_0,q_1	q_0,q_1,q_2	q_1,q_2,q_3	q_2,q_3,q_4
q_2	q_1,q_2	q_0,q_1,q_2	q_0,q_1,q_2	q_0,q_1,q_2,q_3	q_1,q_2,q_3,q_4
q_3	q_2,q_3	q_1,q_2,q_3	q_0,q_1,q_2,q_3	q_0,q_1,q_2,q_3	q_0,q_1,q_2,q_3,q_4
q_4	q_3,q_4	q_2,q_3,q_4	q_1,q_2,q_3,q_4	q_0,q_1,q_2,q_3,q_4	q_0,q_1,q_2,q_3,q_4

Table 4: Composition of distances for opposite orientation (monotonicity)

$\ominus$	q_0	q_1	q_2	q_3	q_4
q_0	q_0	q_0,q_1	q_1,q_2	q_2,q_3	q_3,q_4
q_1	q_0,q_1	q_0,q_1	q_0,q_1,q_2	q_2,q_3	q_3,q_4
q_2	q_1,q_2	q_0,q_1,q_2	q_0,q_1,q_2	q_0,q_1,q_2,q_3	q_3,q_4
q_3	q_2,q_3	q_2,q_3	q_0,q_1,q_2,q_3	q_0,q_1,q_2,q_3	q_0,q_1,q_2,q_3,q_4
q_4	q_3,q_4	q_3,q_4	q_3,q_4	q_0,q_1,q_2,q_3,q_4	q_0,q_1,q_2,q_3,q_4

Table 5: Composition of distances for opposite orientation (range restriction)

results that may happen for equally spaced distance ranges (restriction 2) are removed. The results affected by this rule are those for which the difference between the distances is at least two steps, that is:[4]

$$|\mathrm{ord}(d_{AB}) - \mathrm{ord}(d_{BC})| \geq 2 \Rightarrow \mathrm{LB}(d_{AC}) = \mathrm{pred}(\max(d_{AB}, d_{BC})). \tag{10}$$

Eventually, by applying restriction 3, the lower bound becomes:

$$|\mathrm{ord}(d_{AB}) - \mathrm{ord}(d_{BC})| \geq p \Rightarrow \mathrm{LB}(d_{AC}) = \max(d_{AB}, d_{BC}). \tag{11}$$

With a difference $p = 2$, we have the results in Table 6.

[4]The function *predecessor* gives the previous symbol in the list, that is: $\mathrm{pred}(q_i) = q_{i-1}$ for each $i > 0$ and $\mathrm{pred}(q_0) = q_0$.

$\ominus$	q_0	q_1	q_2	q_3	q_4
q_0	q_0	q_0,q_1	q_2	q_3	q_4
q_1	q_0,q_1	q_0,q_1	q_0,q_1,q_2	q_3	q_4
q_2	q_2	q_0,q_1,q_2	q_0,q_1,q_2	q_0,q_1,q_2,q_3	q_4
q_3	q_3	q_3	q_0,q_1,q_2,q_3	q_0,q_1,q_2,q_3	q_0,q_1,q_2,q_3,q_4
q_4	q_4	q_4	q_4	q_0,q_1,q_2,q_3,q_4	q_0,q_1,q_2,q_3,q_4

Table 6: Composition of distances for opposite orientation (absorption rule)

In the general case, the composition of two distance relations must take into account any of the possible orientations and not just the two cases of same and opposite orientation (see Clementini et al. 1995). However, these two cases correspond to the two extremes of the range of resulting distances when an arbitrary orientation is taken into consideration. The case of opposite orientations gives the lower bound and the case of same orientation gives the upper bound of the range.

In what has been said up to now, we have ignored the role of frames of reference, which will be addressed in the following section.

5 Frames of Reference and Articulation Rules

Thus far, we have implicitly assumed that the scale in which the distance distinctions apply is known. The scale of a distance system is determined by the context in which the distinctions are made. Using an analogy to the qualitative representation of orientation, we distinguish among three different types of contexts or frames of reference (Hernández 1994):

- **Intrinsic frame of reference**
  The distance is determined by some inherent characteristics of the reference object, like its topology, size or shape. An object like a house, for example can implicitly determine what is *close* and *far* with respect to itself, without the need of any external factors.

- **Extrinsic frame of reference**
  The distance is determined by some external factor, like the arrangement of objects, the traveling time, or the costs involved.

- **Deictic frame of reference**
  The distance is determined by an external point of view. The most

immediate case is the one of objects that are visually perceived from an observer standing at the point of view. Deictic frames of reference include also cases in which the point of view is used figuratively, i.e., not in the sense of sight. Often the point of view is related to how an individual builds a mental map of space.

Thus, a frame of reference has to take into account all contextual information. For a given type, we have to establish the criteria fixing the scale and choose an appropriate distance system. Therefore, three components make up a frame of reference:

$$\text{FofR} = (T, S, D) \tag{12}$$

The type T is either intrinsic, extrinsic or deictic. The scale S is, depending on the type: a function of inherent characteristics of the reference object, e.g., $f(\text{size(RO)})$ (for the intrinsic type), a reference unit given by external factors (for the extrinsic type), or a function of the distance between the point of view and the reference object $f(d(\text{PV}, \text{RO}))$ (for the deictic type). The distance system D is a structure as defined in Section 3 (made up of distance relations and structure relations).

In the general case, the qualitative description of distance among a set of objects is reasonably assumed to be given according to different frames of reference. A "basic" type of qualitative reasoning is therefore to relate the distances to each other and be able to infer new information. Ideally, we would like to transform all distance descriptions to the same ("canonical") frame of reference. However, different distance frames of reference refer to different granularities or scales, thus making a transformation into an implicit frame difficult. We rather must restrict ourselves to giving articulation rules (cf. Hobbs 1985) that state how two particular frames of reference compare. This comparative information, which consists mainly of order information between reference magnitudes, must be explicitly maintained in the knowledge base. It can further constrain the relations maintained in the constraint network, and suggests deferring naming to those cases where it can be done in a disambiguating context. From the definition of frames of reference given above it follows that articulation rules must relate the scales and distance systems of the frames involved. The frame type does not need to be explicitly related, since it already determines the scale factor and is thus contained in it. The distance systems must be compared as to the sets of relations involved and their structure (i.e., the order-of-magnitude relations between the distances). In general, only similar distance systems might be successfully related to each other, and some might be incomparable to each other.

6 Conclusion

In spite the fact of distance being an important cognitive spatial concept, and the increased research activity in the area of qualitative spatial reasoning, no satisfying qualitative model for distances had been developed up to now. This paper's major contribution is providing such a model with the characteristic advantages of the qualitative approach: a flexible set of distance distinctions at various levels of granularity, an implicit way of handling uncertainty, and the corresponding reasoning mechanisms.

This, however, is only the beginning of a longer term collaborative research effort. Several of the explicitly stated assumptions and restrictions in the paper point to further research directions (some of which already have been pursued as reported elsewhere):

- Distance is only one component (the other one being orientation) of positional information. Further work will have to deal with the combination of distance and orientation, which we expect to constrain each other in a way that actually simplifies the reasoning process.

- Here we have only considered the case of homogeneous distance systems, in which all distance relations are related to each other by the same type of property. The more general case of heterogeneous distance ranges, which is likely to correspond to cognitive distance concepts without *a priori* restrictions, still needs to be investigated.

- We have ignored the case of extended objects: The extension of objects, however, influences the concepts of distance. As a first classification, we will consider three different scenarios. If the distances involved at a given scale are such that the extension of the objects can be disregarded, we use the point abstraction as in this paper. If the extension of the objects is of the same order of magnitude of the distances among them, then the distance will be computed between the boundaries. If the objects connect, the distances will be computed between the centroids of the objects.

- The articulation rules mechanism sketched in the previous section needs further study. This will be done in the context of an application in the domain of vehicle navigation systems. In that context, information at various scales and granularities needs to be dealt with to guide vehicles at the single road level (small-scale environment), at the city level (urban scale), and at the region level (geographic scale).

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