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REPRESENTING AND REASONING WITH QUALITATIVE SPATIAL RELATIONS ABOUT REGIONS

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Abstract. This chapter surveys the work of the qualitative spatial reasoning group at the University of Leeds. The group has developed a number of logical calculi for representing and reasoning with qualitative spatial relations over regions. We motivate the use of regions as the primary spatial entity and show how a rich language can be built up from surprisingly few primitives. This language can distinguish between convex and a variety of concave shapes and there is also an extension which handles regions with uncertain boundaries. We also present a variety of reasoning techniques, both for static and dynamic situations. A number of possible application areas are briefly mentioned.

1. Introduction

Qualitative Reasoning (QR) has now become a mature subfield of AI as its tenth annual international workshop, several books (e.g. (Weld and De Kleer 1990, Faltings and Struss 1992)) and a wealth of conference and journal publications testify. QR tries to make explicit our everyday commonsense knowledge about the physical world and also the underlying abstractions used by scientists and engineers when they create models. Given this kind of knowledge and appropriate reasoning methods, a computer could make predictions and diagnoses and explain the behavior of physical systems in a qualitative manner, even when a precise quantitative description is not available or is computationally intractable. Note that a representation is not normally deemed to be qualitative by the QR community simply because it is symbolic and utilizes discrete quantity spaces but because the distinctions made in these discretizations are *relevant* to high-level descriptions of the system or behavior being modeled.

Most QR systems have reasoned about scalar quantities, whether they denote the height of a bouncing ball, the amount of fluid in a tank, the temperature of some body, or perhaps some more abstract quantity. Although there have been spatial aspects to the systems reasoned about, these have rarely been treated with any sophistication. In particular, the multidimensional nature of space has been ill addressed until recently, despite some important early forays such as (Hayes 1985a, Forbus, Nielson and Faltings 1987).

The neglect of this topic within AI may be due to the *poverty conjecture* promulgated by Forbus, Nielsen and Faltings (Weld and De Kleer 1990, page 562): "there is no purely qualitative, general purpose kinematics". Of course, qualitative kinematics is only a part of qualitative spatial reasoning (QSR), but it is worth noticing their third (and strongest) reason for putting forward the conjecture — "No total order: Quantity spaces don't work in more than one dimension, leaving little hope for concluding much about combining weak information about spatial properties." They point out that transitivity is a vital feature of a qualitative quantity space but doubt that this can be exploited much in higher dimensions and conclude: "we suspect the space of representations in higher dimensions is sparse; that for spatial reasoning almost nothing weaker than numbers will do."

However, there is now a growing body of research in the QR and, more generally, in the Knowledge Representation community and elsewhere that, at least partly, refutes this conjecture. A rich space of qualitative spatial representations is now being explored, and these can indeed exploit transitivity.

There are many possible applications of QSR; we have already mentioned reasoning about physical systems, the traditional domain of QR systems. Other workers are motivated by the necessity of giving a semantics to natural language spatial expressions, e.g., (Vieu 1991), which tend to be predominantly qualitative rather than quantitative (consider prepositions such as 'in', 'on' and 'through'). Another large and growing application area is Geographical Information Systems (GIS): there is a need for qualitative spatial query languages for example (Clementini, Sharma and Egenhofer 1994) and for navigation (Schlieder 1993). Other applications include specifying the syntax and semantics of Visual Programming languages (Gooday and Cohn 1995, Gooday and Cohn 1996b).

This chapter is devoted largely to presenting one particular formalism for QSR, the RCC[1] calculus which has been developed at the University of Leeds over the last few years in a series of papers including (Randell, Cui and Cohn 1992, Cui, Cohn and Randell 1992, Cohn, Randell, Cui and Bennett 1993, Cui, Cohn and Randell 1993, Bennett 1994b, Gotts 1994b, Cohn and Gotts 1996a, Gotts, Gooday and Cohn 1996, Cohn 1995), and indeed is still the subject of ongoing research. One interpretation of the acronym RCC is 'Region Connection Calculus': the fundamental approach of RCC is that extended spatial entities, i.e. *regions* of space, are primary rather than the traditional mathematical dimensionless point. The primitive relation between relations is that of connection, thus giving the language the ability to represent the structure of spatial entities.

There are a number of reasons for eschewing a point-based approach to qualitative spatial representation and indeed simply using the standard tools of mathematical topology. Firstly, regions give a natural way to represent a kind of indefiniteness that is germane to qualitative representations. Moreover the space occupied by any real physical body will always be a region rather a point. Even in natural language, the word "point" is not usually used to mean a mathematical point: a pencil with a sharp point still draws a line of finite thickness! It also turns out that it is possible to reconstruct a notion of mathematical point from a primitive notion of region (Biacino and Gerla 1991). The standard mathematical approaches to topology, general (point-set) topology and algebraic topology, take points as the fundamental, primitive entities and construct extended spatial entities as sets of points with additional structure imposed on them. However, these approaches generalize the concept of a 'space' far beyond its intuitive meaning; this is particularly true for point-set topology but even algebraic topology, which deals with spaces constructed from 'cells' equivalent to the n-dimensional analogues of a (2-dimensional) disc, concerns itself chiefly with rather abstract reasoning concerning the association of algebraic structures such as groups and rings with such spaces, rather than the kinds of topological reasoning required in everyday life, or those which might illuminate the metaphorical use of topological concepts such as 'connection' and 'boundary'. The case against using these standard point based mathematical techniques for QSR is made in rather more detail in (Gotts et al. 1996), where it is argued that the distinction between intuitive and counter-intuitive concepts is not easily captured and that the reasonable desire (for computational reasons) to avoid higher order logics does not mesh well with quantifying over sets of points.

Of course, it might be possible to adapt the conventional mathematical formalisms for our purposes, and indeed this strategy is sometimes adopted (see, for example (Egenhofer and Franzosa 1991, Egenhofer and Franzosa 1995, Worboys and Bofakos 1993)). However, because we take the view that much if not all reasoning about the spaces occupied by physical objects would not, *a priori*, seem to require points to appear in one's ontology, we do not follow this route but rather prefer to take regions as primitive and abandon the traditional mathematical approaches.

In fact there is a minority tradition in the philosophical and logical literature that rejects the treatment of space as consisting of an uncountably infinite set of points and prefers to take spatially extended entities as primitive. Works by logicians and philosophers who have investigated such alternative approaches ('mereology'[2] or 'calculus of individuals') include (Whitehead 1929, Leśniewski 1927-1931, Leonard and Goodman 1940, Tarski 1956, de Laguna 1922) and more recently (Clarke 1981, Clarke 1985) — Clarke developed the the immediate 'ancestor' of RCC — (Simons 1987, Casati and Varzi 1994, Smith 1994). Simons' book contains a review of much of the earlier work in this area.

Because RCC is closely based on Clarke's system, it is worth briefly presenting the main features of this system. Clarke (1981, 1985) presents an extended account of a logical axiomatization for a region-based spatial (in fact Clarke's intended interpretation was spatio-temporal) calculus; he gives many theorems as well to illustrate the important features of the theory. The basis of the system is one primitive dyadic relation $\mathsf{C}(x, y)$ read as "x connects with y."

If one thinks of regions as consisting of sets of points (although we have indicated above that this is not our preferred interpretation), then in terms of points incident in regions, $\mathsf{C}(x, y)$ holds when at least one point is incident in both x and y. There are various axioms which characterize the intended meaning of C (for example, two such axioms state that C is reflexive and that it is symmetric). In Clarke's system it is possible to distinguish regions having the properties of being (topologically) closed or open. A closed region is one that contains all its boundary points (more correctly all its limit points), whereas an object is open if it has no boundary points at all. Many topological relations (for example, regions touching or being a tangential or non tangential part) are defined in Clarke's system and many properties are proved of these relations. Clarke defines many other useful concepts including quasi-Boolean functions, topological functions (interior and closure), and in his second paper provides a construction for points in terms of regions following earlier work by Whitehead (1929). This, however, is faulty; a correction is provided by (Biacino and Gerla 1991).

While on the subject of related work it is certainly worth mentioning the work done on interval temporal logics for two reasons; first, because the style of much of the work on QSR closely mirrors this work on inter-

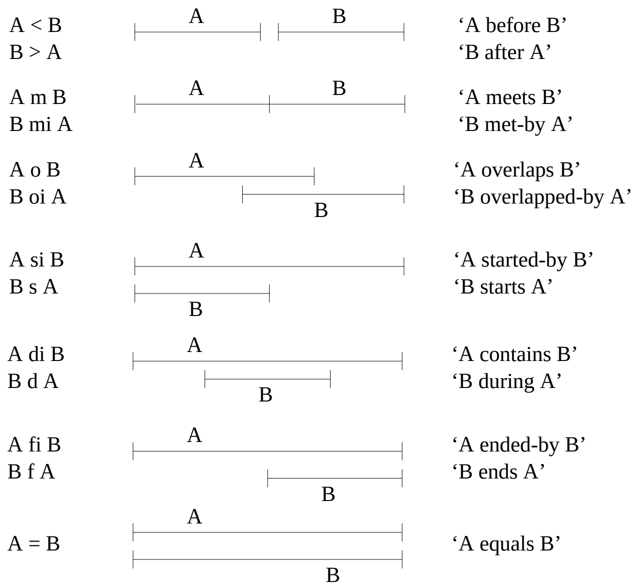

Figure 1. Allen's thirteen interval-interval relations

val temporal logic and indeed can be naturally seen as the extension of these ideas from the temporal to the spatial domain; secondly, of course it is possible to use this work directly by reinterpreting the calculus as a one-dimensional spatial calculus, though there are problems with doing so. Allen's interval calculus (Allen 1983) is best known within AI; however, the credit for inventing such calculi is not due to him; Van Benthem (1983) describes an interval calculus, while (Nicod 1924, chapter 2) is probably the earliest such system. Allen's logic defines thirteen Jointly Exhaustive and Pairwise Disjoint (JEPD) relations for convex (one-piece)[3] temporal intervals (see Fig.1). Various authors including Mukerjee and Joe (1990) have used Allen's system for spatial reasoning, using a copy of the calculus for each dimension; however, although attractive in many ways, this has the fundamental limitation that it forces rectangular objects, and is thus not very expressive: consider the configuration in Fig.2.

The structure of the rest of this chapter is as follows. First we present the basic topological part of the calculus in some detail, although space precludes a full exposition. Then we turn to presenting some basic reasoning techniques including a qualitative spatial simulator. Following this we then extend the calculus with an additional primitive to allow a much finer-grained representation than a purely topological representation allows. Up to this point the representation is in a first-order logic; we then show how much of the calculus can be re-expressed in a zero-order logic to a com-

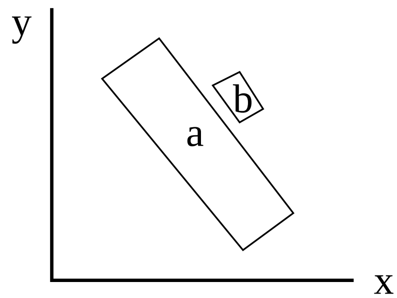

Figure 2. Allen's calculus can be used for reasoning about each spatial dimension, but it forces rectangular objects aligned to the chosen axes. In the diagram above, the two rectangles are not so aligned, and although the smaller one is part of the larger one when projected to each axis individually, this is not so in two dimensions; but this cannot be detected by comparing the one-dimensional projects.

putational benefit. A few possible application areas are then mentioned followed by an extension to handle regions with uncertain boundaries. We conclude by mentioning some current and future research and summarizing our work.

2. An Introduction to the Region Connection Calculus (RCC)

The original motivation for this work was an essay in Naïve Physics (Hayes 1985b, Hayes 1985a), We were interested in developing a theory for representing and ultimately reasoning about spatial entities; the theory should be expressed in a language with a clean well-understood semantics. Our desire was principally to create an epistemologically adequate formal theory (rather than necessarily a cognitively valid naïve theory).

We should make precise exactly what counts as a region. In our intended interpretation the regions may be of arbitrary dimension, but they must all be the same dimension and must not be of mixed dimension (for example, a region with a lower dimensional spike missing or sticking out is not intended). Such regions are termed *regular*. Normally, of course our intended interpretation will be 3D, though in many of the figures in this chapter, for ease of drawing, we will assume a 2D world (as is also usual in GIS applications). We will deal with the question of whether regions may be open, closed or both below. We also intend regions to *really* be spatially extended, i.e. we rule out the possibility of a region being null. Other than these restrictions, we will allow any kind of regions, in particular they may be multipiece regions, have interior holes and tunnels.

Our initial system was reported in (Randell and Cohn 1989), which followed Clarke's system closely. However, in (Randell, Cui and Cohn 1992) we presented a revised theory that deviates from Clarke's theory in one important respect, which has far-reaching implications. The change is to the interpretation of $\mathsf{C}(x, y)$: Clarke's interpretation was that the two regions x and y share at least one point whereas our new interpretation is that the topological closures of the two regions share at least one point. Because we consider two regions to be identical if they are conected to exactly the same set of regions, so we could regard regions as equivalence classes of point-sets whose closures are identical. We also, require regions to be of uniform dimension and in terms of point-set topology this means that all the sets in these equivalence classes should have *regular* closures. From within the RCC theory it is not possible to distinguish between regions that are open, closed or neither but have the same closure, and we argue that these distinctions are not necessary for qualitative spatial reasoning. Such regions occupy the same amount of space and, moreover, there seems to be no reason to believe that some physical objects occupy closed regions and others open, so why introduce these distinctions as properties of regions? Moreover, Clarke's system has the odd result that if a body maps to a closed region of space then its complement is open and the two are disconnected and not touching! Another peculiarity is that, if a body is broken into two parts, then we must decide how to split the regions so formed: one will have to have be open (at least along the boundary where the split occurred) whilst the other must be closed and there seems to be no principled reason for this assymetry.[4] Thus we argue that, from the standpoint of our naïve understanding of the world, the topological structure of Clarke's system is too rich for our purposes, and in any case appearing in this formal theory, it poses some deep conceptual problems. Furthermore, is it necessary to understand sophisticated topological notions such as interior and closure to create a theory of 'commonsense' qualitative space?

It should be noted that the absence of the open/closed distinction from our theory does not make it incompatible with interpretations in terms of standard topology. A particularly straightforward model is that the regions of our theory are the (non-null) elements of the *regular open Boolean algebra* over the usual topology on $\Re^n$. In such an algebra the Boolean product operation is simply set intersection, while Boolean complement corresponds to the interor of the set complement (hence, by DeMorgan, the (regular open) Boolean sum of two (open) sets is the interior of the sum of their closures). Thus all regions are identified with regular open sets.[5] We can then say that two regions are connected if the closures of the (regular open) sets identified with the regions share a point. So, although openness and closure figure in the model theoretic interpretation of the theory, they are not properties of regions and indeed have no meaning within the theory itself.

Hard-line critics of point-based theories of space might still argue that giving a point-set-theoretic semantics for our theory of regions is unsatisfactory. An alternative interpretation of C might be given informally by saying the distance between the two regions is zero. To do this formally would obviously require some (weak) notion of metric space definable on regions but we have not yet attempted to formally specify a semantics of this kind.

Insofar as openness and closedness are not properties of our regions, our theory is simpler than theories such as Clarke's, and hence, we believe that it will also prove to be more suitable for computational reasoning. Furthermore, we believe that the loss of expressive power resulting from our simplification does not restrict the utility of our theory as a language for commonsense reasoning about spatial information. It might be argued that without the open/closed distinction, certain important types of relation between regions cannot be differentiated. For example, Asher and Vieu (1995) have distinguished 'strong' and 'weak' contact between regions. In the former case the regions share a point, whereas in the latter they are disjoint but their closures share a point. Two bodies may then said to be 'joined' if the regions they occupy are in strong contact but merely 'touching' if their regions are in weak contact. Whilst we acknowledge that the distinction between bodies being joined and merely touching is important, we believe that these relations are not essentially spatial and therefore should not be embodied in a theory of spatial regions. They should rather be modeled within a more general theory of relationships among material substances, objects and the regions they occupy.

To formalize our theory we use a sorted first-order logic based on the logic LLAMA (Cohn 1987), but the details of the logic need not concern us here. The principal sorts we will use are Region, NULL, and PhysOb. Notice that with this sort structure we distinguish the space occupied by a physical object from the physical object itself, partly because it may vary over time which we represent via a function $\mathsf{space}(x,t)$.[6] The sort NULL is true of regions that are not spatially extended and is used to model the intersections of disoint regions or the spatial extent of physical objects that do not exist at a particular time for example.

In fact, the axiomatic theory we have developed so far deals only with relationships between entities of sorts Region and NULL. Axiomatization of relations involving physical objects would be part of the more general theory of material substances in space, which was mentioned above. So, at present, the sort PhysOb and the $\mathsf{space}(x,t)$ merely serve to indicate how our theory would be incorporated into this much broader theory.

2.1. AXIOMS FOR C

Since our interpretation of C has changed, we need to re-axiomatize it and redefine many of the relations Clarke defined which we still want to use. The two main axioms expressing the reflexivity and symmetry of C in fact remain unchanged:

(1) $$\forall x[\mathsf{C}(x,x)]$$

(2) $$\forall x \forall y[\mathsf{C}(x,y) \rightarrow \mathsf{C}(y,x)]$$

Using $\mathsf{C}(x,y)$, a basic set of dyadic relations are defined (Randell, Cui and Cohn 1992, section 4). Definitions and intended meanings of those used here are given in table 1. Unless otherwise specified, the all arguments to the functions and predicates we define are of sort Region. The relations P, PP, TPP and NTPP being non-symmetrical support inverses. For the inverses we use the notation Φi, where $\Phi \in \{\mathsf{P}, \mathsf{PP}, \mathsf{TPP}, \mathsf{NTPP}\}$, for example, TPPi.

TABLE 1. Some relations definable in terms of C

	Relation	*interpretation*	*Definition of* $R(x,y)$
(3)	$\mathsf{DC}(x,y)$	x is disconnected from y	$\neg\mathsf{C}(x,y)$
(4)	$\mathsf{P}(x,y)$	x is a part of y	$\forall z[\mathsf{C}(z,x) \rightarrow \mathsf{C}(z,y)]$
(5)	$\mathsf{PP}(x,y)$	x is a proper part of y	$\mathsf{P}(x,y) \wedge \neg\mathsf{P}(y,x)$
(6)	$\mathsf{EQ}(x,y)$	x is identical with y	$\mathsf{P}(x,y) \wedge \mathsf{P}(y,x)$
(7)	$\mathsf{O}(x,y)$	x overlaps[7] y	$\exists z[\mathsf{P}(z,x) \wedge \mathsf{P}(z,y)]$
(8)	$\mathsf{DR}(x,y)$	x is discrete from y	$\neg\mathsf{O}(x,y)$
(9)	$\mathsf{PO}(x,y)$	x partially overlaps y	$\mathsf{O}(x,y) \wedge \neg\mathsf{P}(x,y) \wedge \neg\mathsf{P}(y,x)$
(10)	$\mathsf{EC}(x,y)$	x is externally connected to y	$\mathsf{C}(x,y) \wedge \neg\mathsf{O}(x,y)$
(11)	$\mathsf{TPP}(x,y)$	x is a tangential proper part of y	$\mathsf{PP}(x,y) \wedge \exists z[\mathsf{EC}(z,x) \wedge \mathsf{EC}(z,y)]$
(12)	$\mathsf{NTPP}(x,y)$	x is a nontang'l proper part of y	$\mathsf{PP}(x,y) \wedge \neg\exists z[\mathsf{EC}(z,x) \wedge \mathsf{EC}(z,y)]$

Of the defined relations, those in the set {DC, EC, PO, EQ, TPP, NTPP, TPPi and NTPPi} (illustrated in Fig.3) are provably JEPD (Jointly Exhaustive and Pairwise Disjoint). We refer to the theory comprising this set of eight relations (and the quasi-Boolean functions to be defined below) as RCC8[8]. The complete set of relations described above can be embedded in a relational lattice. This is given in Fig.4. The symbol $\top$ is interpreted as tautology and the symbol $\perp$ as contradiction. The ordering of these relations is one of subsumption with the weakest (most general) relations connected directly to top and the strongest (most specific) to bottom. For example, TPP implies PP, and PP implies either TPP or NTPP. A greatest lower bound of bottom indicates that the relations are mutually disjoint. For example with TPP and NTPP, and P and DR. This lattice corresponds

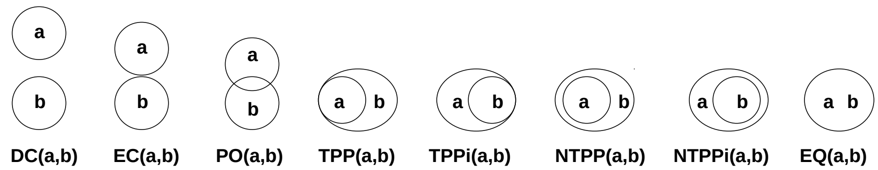

Figure 3. Illustrations of eight JEPD relations

to a set of theorems (such as $\forall xy[\mathsf{PP}(x,y) \leftrightarrow [\mathsf{TPP}(x,y) \vee \mathsf{NTPP}(x,y)]]$) which we have verified.

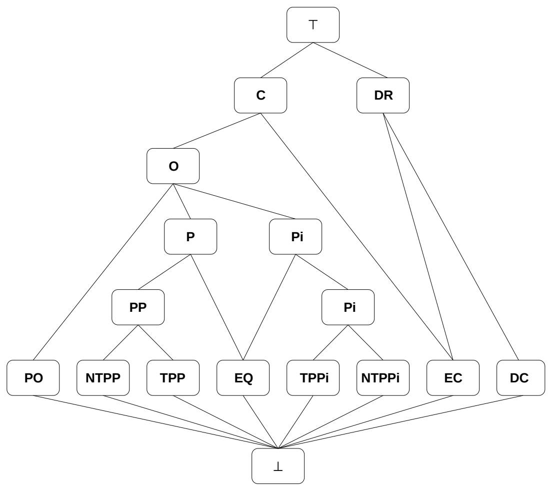

Figure 4. A subsumption lattice of dyadic relations defined in terms of C

Clarke axiomatized a set of function symbols in terms of C; the topological ones (interior, exterior, closure) we omit since (as already discussed) we do not wish to make these distinctions. However, he also defined a set of quasi-Boolean[9] functions which we will also require, though our definitions differ. The Boolean functions are: $\mathsf{sum}(x,y)$, the sum of x and y; u, the universal region; $\mathsf{compl}(x)$, the complement of x; $\mathsf{prod}(x,y)$, the product (intersection) of x and y; and $\mathsf{diff}(x,y)$, the difference of x and y (that is the part of x that does not overlap y). For brevity we will often use $*$, $+$ and $-$ rather than **prod**, **sum** and **diff**. The functions: $\mathsf{compl}(x)$, $\mathsf{prod}(x,y)$ and $\mathsf{diff}(x,y)$ are partial but are made total in the sorted logic by specifying sort restrictions and by letting the result sort of the partial functions be $\mathsf{REGION} \sqcup \mathsf{NULL}$. Our functions obey the following axioms:

(13) $\forall x \mathsf{C}(x,\mathsf{u})$

(14) $\forall x,y,z[\mathsf{C}(z,\mathsf{sum}(x,y)) \equiv \mathsf{C}(z,x) \vee \mathsf{C}(z,y)]$

(15) $\forall x, y[[\mathsf{C}(y, \mathsf{compl}(x)) \equiv \neg\mathsf{NTPP}(y, x)] \wedge [\mathsf{O}(y, \mathsf{compl}(x)) \equiv \neg\mathsf{P}(y, x)]]$

(16) $\forall x, y, z[\mathsf{C}(z, \mathsf{prod}(x, y)) \equiv \exists w[\mathsf{P}(w, x) \wedge \mathsf{P}(w, y) \wedge \mathsf{C}(z, w)]]$

(17) $\forall x, y[\mathsf{NULL}(\mathsf{prod}(x, y)) \equiv \mathsf{DR}(x, y)]$

(18) $\forall x, y, z[\mathsf{C}(z, \mathsf{diff}(x, y)) \equiv \mathsf{C}(z, \mathsf{prod}(x, \mathsf{compl}(y)))]$

As already mentioned, and will be clear from the fact that we have introduced the sum function, regions may consist of disconnected parts. We can easily define a predicate to test for one-pieceness:[10]

(19) $$\mathsf{CON}(x) \equiv_{\mathrm{def}} \forall yz[\mathsf{sum}(y, z) = x \rightarrow \mathsf{C}(y, z)]$$

A rather deep theorem of the theory is given by the formula $\forall x \exists y[\mathsf{NTPP}(y, x)]$ which was demonstrated by informal argument in (Randell, Cui and Cohn 1992). Because we have so far not been able to give a fully formal proof of this theorem we often regard the formula as an additional axiom of the theory. This formula mirrors a formal property of Clarke's theory, where he stipulates that every region has a nontangential part, and thus an interior (remembering that in Clarke's theory a topological interpretation is assumed).

2.2. THEOREMS OF RCC8

In (Randell, Cui and Cohn 1992) we cite a number of important theorems which distinguish RCC8 from Clarke's system. First, note that for Clarke, two regions x and y are identical iff any region connecting with x connects with y and vice-versa (which in effect is an axiom of extensionality for $=$ in terms of C), that is

(20) $$\forall xy[x = y \leftrightarrow \forall z[\mathsf{C}(z, x) \leftrightarrow \mathsf{C}(z, y)]] .$$

In the new theory, an additional theorem concerning identity,

(21) $$\forall xy[x = y \leftrightarrow \forall z[\mathsf{O}(z, x) \leftrightarrow \mathsf{O}(z, y)]] ,$$

($=$ is extensional in terms of O) becomes provable, which is not a theorem in Clarke's theory: any region z which overlaps a closed region x will also overlap its open interior (and vice versa), thus making them identical according to this axiom, but Clarke distinguishes open and closed regions so they cannot be identical, thus providing a counterexample.

Perhaps the most compelling reason that led us to abandon Clarke's semantics for C is the following theorem which expresses an everyday intuition about space, that, given one proper part of a region, then there is another, discrete from the first:

(22) $$\forall xy[\mathsf{PP}(x, y) \rightarrow \exists z[\mathsf{P}(z, y) \wedge \neg\mathsf{O}(z, x)]] .$$

This is provable in the new theory, but not in Clarke's: the interior of a closed region is a proper part of it, but there is no remaining proper part, since in Clarke's (and our) system the boundary of a region is not a region. A related theorem is the following:

$$(23)\quad \forall xy[\mathsf{PO}(x,y) \rightarrow [\exists z[\mathsf{P}(z,y) \wedge \neg\mathsf{O}(z,x)] \wedge \exists w[\mathsf{P}(w,x) \wedge \neg\mathsf{O}(w,y)]]],$$

which again is a theorem in the new theory but not in Clarke's. A counter-example arises in Clarke's theory where we have two semi-open spherical regions, x and y (with identical radii), such that the northern hemisphere of x is open and the southern hemisphere is closed, and the northern hemisphere of y is closed and the southern hemisphere open. If x and y are superimposed so that their centers and equators coincide, then x and y will partially overlap, but no part of x is discrete from y, and vice-versa.

Another key distinction between our theory and Clarke's concerns the connection between a region and its complement. In the new theory, $\forall x[\mathsf{EC}(x, \mathsf{compl}(x))]$ holds, that is regions are connected with their complements, which seems a very intuitive result, while in Clarke, a region is disconnected from its complement: $\forall x[\mathsf{DC}(x, \mathsf{compl}(x))]$.

Some further theorems expressing other interesting and important properties of RCC can be found in (Randell, Cui and Cohn 1992) as can a discussion about how to introduce atomic regions into RCC. In the calculus as presented here, they are, of course, excluded because every region has a non-tangential proper part.

3. Expressing Topological Shape in Terms of C

So far, we have principally concentrated on binary predicates relating pairs of regions. Of course, there are also properties of a single region we would like to express, all of which, in some sense at least, characterize the *shape* of the region. Although we have only developed topological notions there is still quite a bit that can be said about the topological shape of a region. For example we have already introduced the predicate $\mathsf{CON}(x)$ which expresses whether a region is one-piece or not. We can do much more than this however, as (Gotts 1994a, Gotts 1994b, Gotts et al. 1996, Gotts 1996c) demonstrates. The task set there is to be able to distinguish a 'doughnut' (a solid, one-piece region with a single hole). It is shown how (given certain assumptions about the universe of discourse and the kinds of regions inhabiting it) all the shapes depicted in Fig.5 can be distinguished.

Here we just give a brief idea of how the task is accomplished, as it also shows some of the range of predicates that can be further defined using C alone (and thus could form the basis of RCCn (for some $n > 8$)).[11]

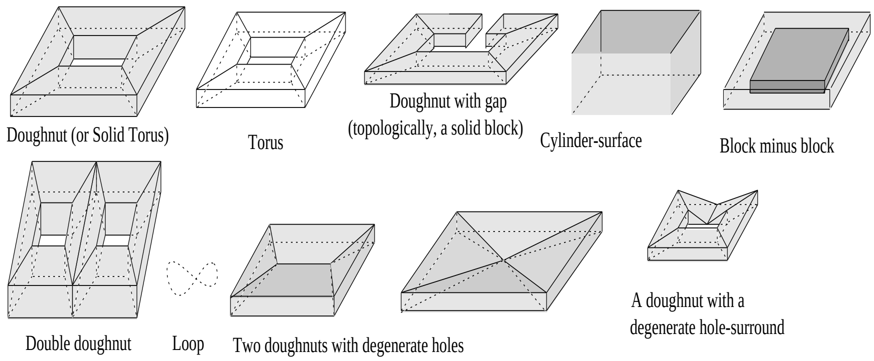

Figure 5. It is possible to distinguish all these shapes using $\mathsf{C}(x, y)$ alone.

The separation-number (SEPNUM) of a region is the maximum number[12] of mutually disconnected parts it can be divided into:

$$\mathsf{SEPNUM}(r, 1) \equiv_{\text{def}} \mathsf{CON}(r) \tag{24}$$

$$\mathsf{SEPNUM}(r, N+1) \equiv_{\text{def}} \exists s, t[[r = s + t] \wedge \mathsf{DC}(s, t) \wedge \mathsf{CON}(s) \wedge \mathsf{SEPNUM}(t, N)] \; . \tag{25}$$

The finger-connectivity (FCON) of a CON region is defined[13] in terms of its possible *dissections*, Fig.6 illustrates three different finger connectivities. Making use of an easily definable predicate $\mathsf{MAX\_P}\ (x, y)$, asserting that x is a maximal one-piece part of y, FCON can be defined as follows:

$$\begin{aligned} \mathsf{FCON}(r, N) \equiv_{\text{def}} & \mathsf{CON}(r) \wedge \\ & \exists a, x, b[[r = a + x + b] \wedge \mathsf{CON}(a) \wedge \mathsf{CON}(b) \wedge \\ & \quad \mathsf{DC}(a, b) \wedge \mathsf{SEPNUM}(x, N) \wedge \\ & \quad \forall z[\mathsf{MAX\_P}(z, x) \to \mathsf{EC}(a, z) \wedge \mathsf{EC}(z, b)]] \wedge \\ & \neg \exists a, y, b[(r = a + y + b) \wedge \mathsf{CON}(a) \wedge \mathsf{CON}(b) \wedge \\ & \quad \mathsf{DC}(a, b) \wedge \mathsf{SEPNUM}(y, N+1) \wedge \\ & \quad \forall z[\mathsf{MAX\_P}(z, y) \to \mathsf{EC}(a, z) \wedge \mathsf{EC}(z, b)]] \; . \end{aligned} \tag{26}$$

Gotts goes on to define a predicate to count the number of boundaries two regions have in common. Using these definitions a doughnut can be

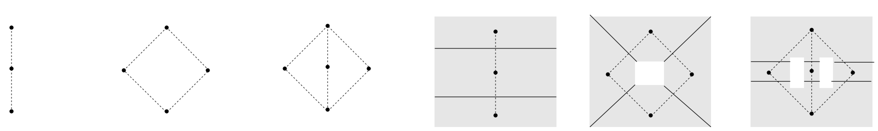

Figure 6. Dissection-graphs and dissections: finger-connectivities 1, 2 and 3